Parameterized Linear Temporal Logics Meet Costs: Still not Costlier than LTL

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Motivation

Linear Temporal Logic (LTL) as specification language:

- Simple and variable-free syntax and intuitive semantics.
- Expressively equivalent to first-order logic on words.
- LTL model checking routinely applied in industrial settings.
- Desirable algorithmic properties.

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2. LTL cannot express all ω-regular properties.
   - Many extensions that are equivalent to ω-regular languages: add regular expression-, grammar-, or automata-operators to LTL.

Overview

LTL

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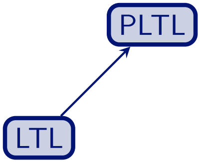

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Alur et al. '99: add parameterized operators to LTL

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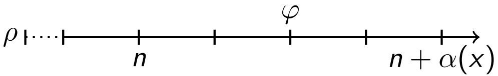
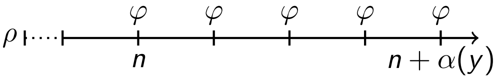
- $(\rho, n, \alpha) \models \mathbf{F}_{\leq x}\varphi$: 

- $(\rho, n, \alpha) \models \mathbf{G}_{\leq y}\varphi$: 

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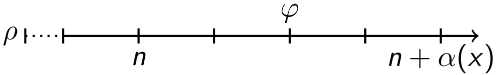
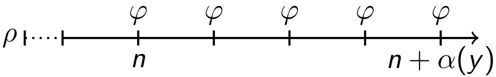
- $(\rho, n, \alpha) \models \mathbf{F}_{\leq x}\varphi$: 

- $(\rho, n, \alpha) \models \mathbf{G}_{\leq y}\varphi$: 

Example:

$$\mathbf{G}(\mathit{req} \to \mathbf{F}_{\leq x}\mathit{resp})$$

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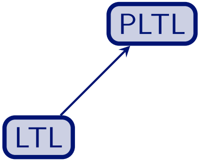

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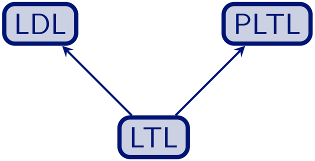

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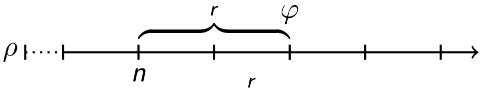

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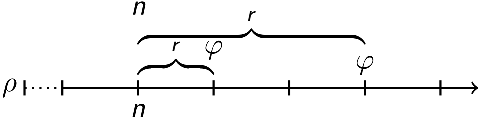

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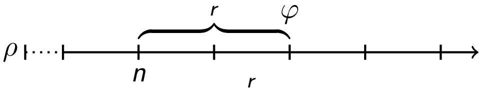
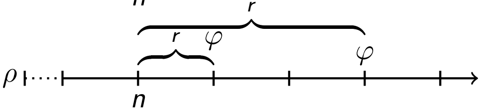
- $(\rho, n) \models \langle r \rangle \varphi$: 

- $(\rho, n) \models [r] \varphi$: 

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Expressivity can be increased for free!

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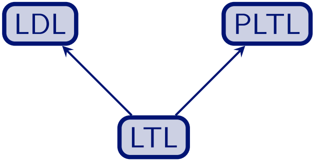

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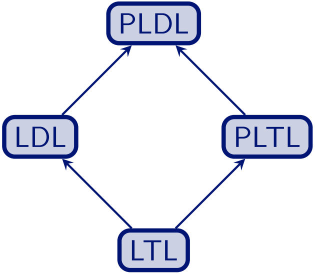

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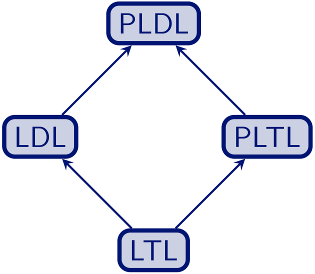

Beyond Bounding Time: Costs

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This is not expressible in PLTL or PLDL.

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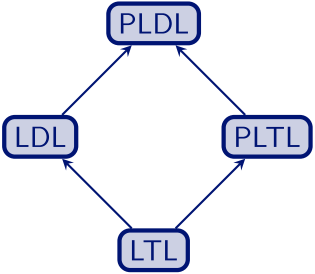

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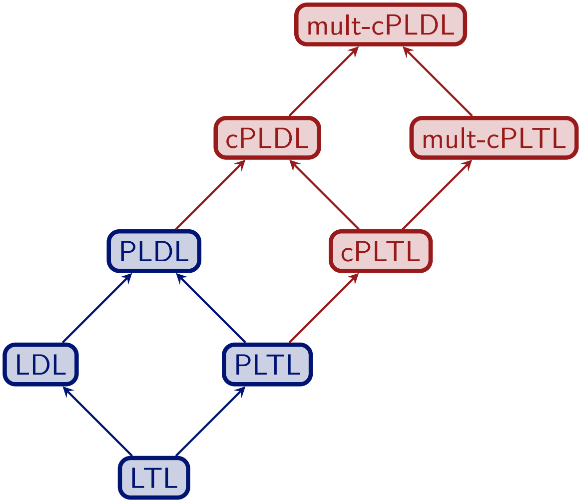

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Syntax: As for PLTL respectively PLDL.

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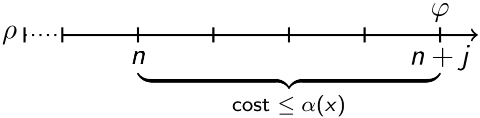

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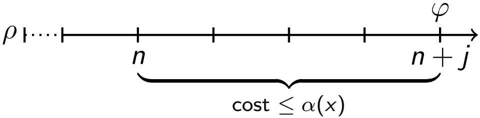

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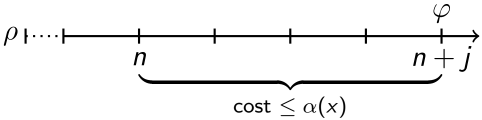

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A multi-dimensional setting: mult-cPLTL and mult-cPLDL

- $\mathrm{cst}\colon E \to \mathbb{N}^d$, $d \in \mathbb{N}$.
- Label parameterized operators with coordinate $i \in \{1, \ldots, d\}$, e.g., $\mathbf{F}_{\leq_i x}$ and $\langle r \rangle_{\leq_i x}$

Results

Let $\mathcal{L} \in \{\text{cPLTL}, \text{cPLDL}, \text{mult-cPLTL}, \text{mult-cPLDL}\}$.

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$\mathcal{L}$ model checking is $\textsc{PSpace}$*-complete.*

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Going from bounding time to bounding (multi-dimensional) costs for free!

Optimization Problems

- Unipolar formulas: at most one type of parameterized operator
- Then: ask for optimal variable valuations
  - For $\mathbf{F}_{\leq x}$ and $\langle r \rangle_{\leq x}$: minimize $\alpha(x)$
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2. *Game optimization in triply-exponential time.*

Proof Sketch (for PLTL Games)

1. Replacing $\mathbf{G}_{\leq y}\psi$ by ψ preserves satisfiability (monotonicity).
2. Apply alternating color technique **(Kupferman et al. '06)**:
   - Add new proposition p and replace every $\mathbf{F}_{\leq x}\psi$ by

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3. Emptiness for game with condition φ equivalent to Player 0 winning LTL game with condition $c(\varphi) \wedge \mathbf{GF}p \wedge \mathbf{GF}\neg p$, as finite state strategies bound distance between color changes.
4. Yields doubly-exponential upper bound.

Conclusion

Weighted extensions of parameterized linear temporal logics that retain the attractive algorithmic properties of LTL:

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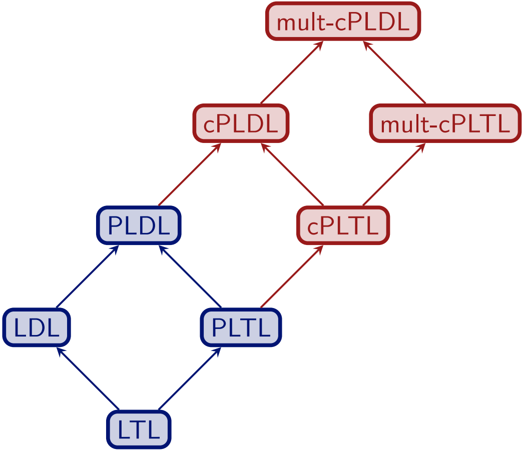
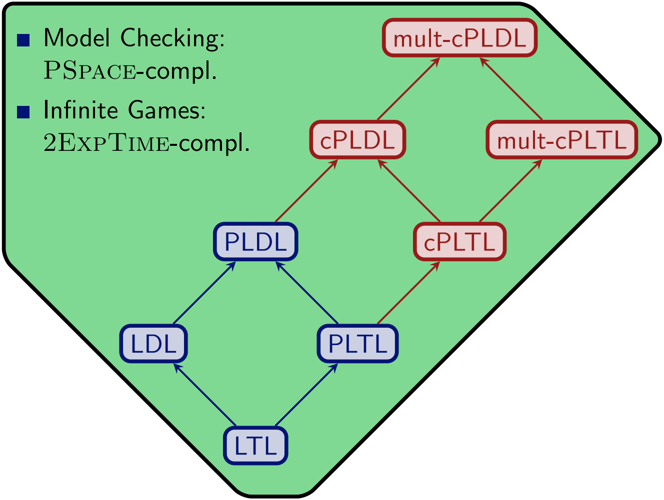
Open problems:

- Game optimization in doubly-exponential time.
- Multi-dimensional optimization problems.
- More general weight structures, e.g., negative weights, semi-rings, etc.

Overview

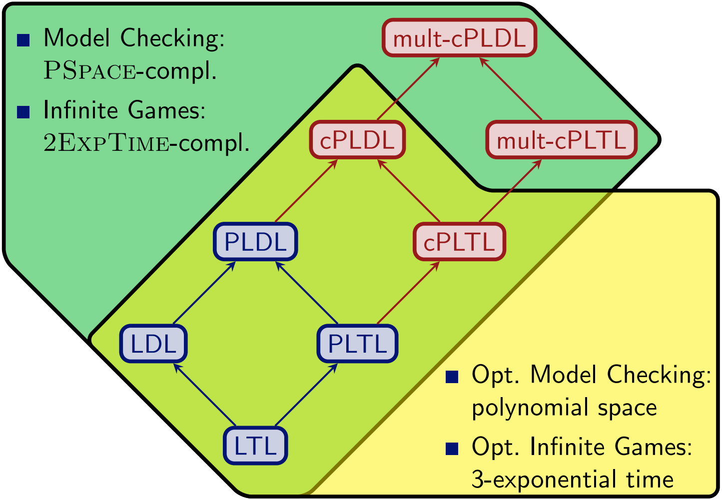


Overview



Model Checking: PSpace-compl.
Infinite Games: 2ExpTime-compl.
mult-cPLDL
cPLDL
mult-cPLTL
PLDL
cPLTL
LDL
PLTL
LTL

Overview


Model Checking: PSpace-compl.
Infinite Games: 2ExpTime-compl.
mult-cPLDL
cPLDL
mult-cPLTL
PLDL
cPLTL
LDL
PLTL
LTL
Opt. Model Checking: polynomial space
Opt. Infinite Games: 3-exponential time