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LEVERHULME TRUST

Fault Tolerant Network Constructors

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Contents

Network Constructors

Fundamental Problem

Algorithmic distributed construction of an actual communication topology

- Distributed computing model, formed by resource limited mobile *agents*
- Agents (or *processes*) can form/delete connections between them
- on/off case: a connection either exists (active) or not (inactive)
- Initially all connections are inactive

Goal: End up with a desired stable graph

[Michail and Spirakis, PODC ‘14 and Distrib. Comput. ‘16]

Network Constructors

The model

Q: *finite set of node−states*
$q_0 \in Q$: *initial node−state*
$Q_{out} \subseteq Q$: *set of output node−states*
$\delta: Q \times Q \times \{0,1\} \rightarrow Q \times Q \times \{0,1\}$: *the transition function*
In every step, a pair uv is selected by the scheduler and u, v interact according to δ

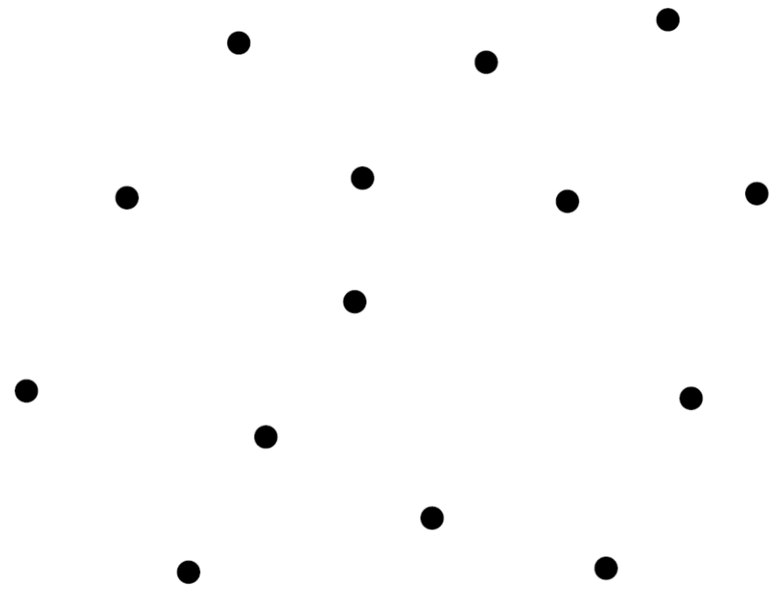
- Fair scheduler: A scheduler is fair if it always leads to fair executions. An infinite execution is fair if for every pair of configurations C and C' such that $C \rightarrow C'$, if C occurs infinitely often, then so does C'
- Output network: nodes that are in output states and edges between them that are active
- Stability: The output network cannot change in future steps

Network Constructors - Example

Spanning Star

- 2 states: black and red
- Initially all black
- Constructs a global star
- Protocol: $(b, b, 0) \rightarrow (b, r, 1)$
  $(r, r, 1) \rightarrow (r, r, 0)$
  $(b, r, 0) \rightarrow (b, r, 1)$

- Space: 2 states
- Time: $O(n^2 \log n)$
- Optimal w.r.t. both



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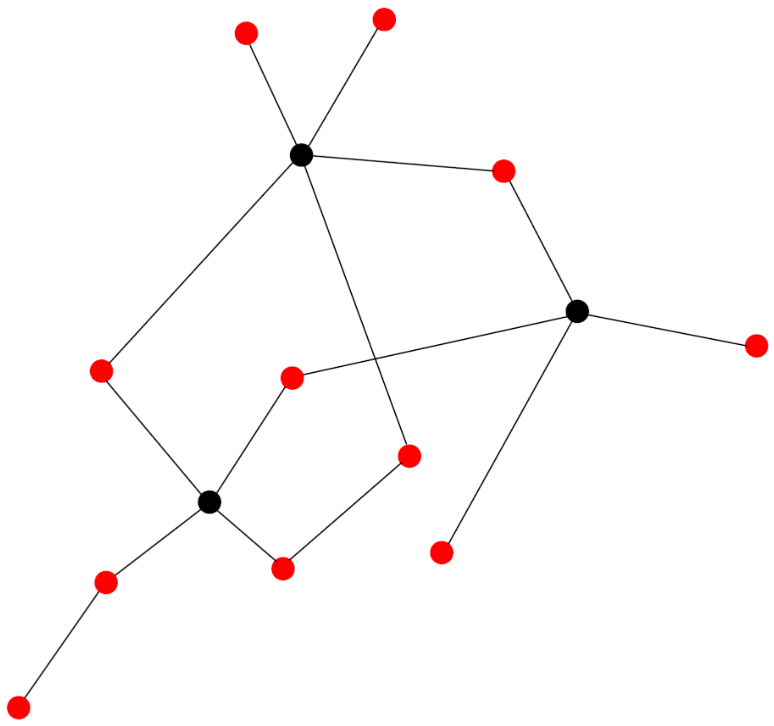

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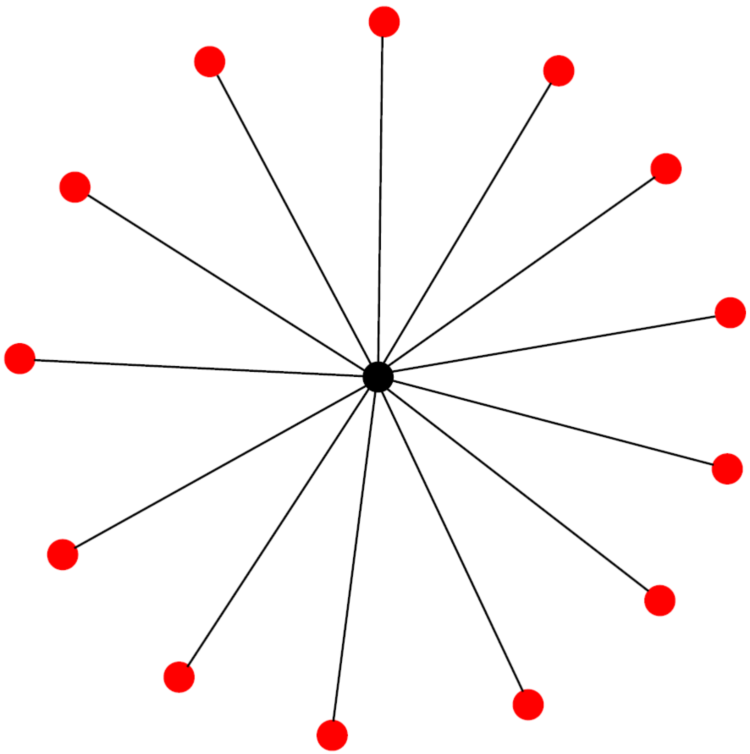

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- Time: $O(n^2 \log n)$
- Optimal w.r.t. both

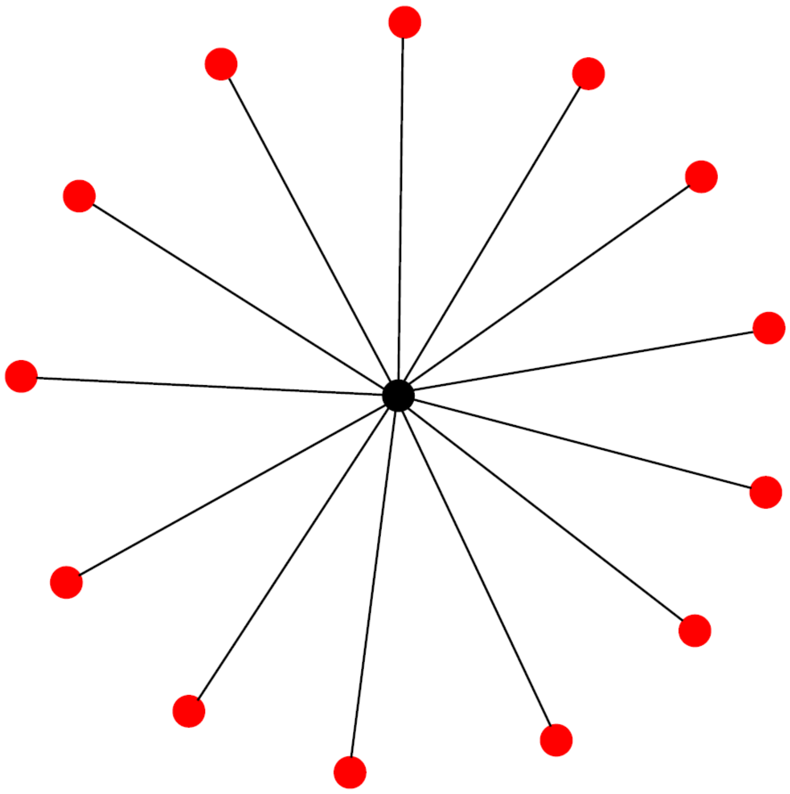
Fault Tolerance

- In each step, either two nodes are selected for interaction, or one node crashes
- During a crash failure, the node and all its edges (active or inactive) are removed from the configuration
- The goal is to find protocols that always re-stabilize to a “correct” graph

Questions

- If one or more faults can affect the formation process, can we always re-stabilize to a correct graph?
- What is the class of graph languages for which there exist fault-tolerant protocols?
- What are the additional minimal assumptions that we need to make in order to find fault-tolerant protocols for a bigger class of graph languages?

Fault Tolerance - Example



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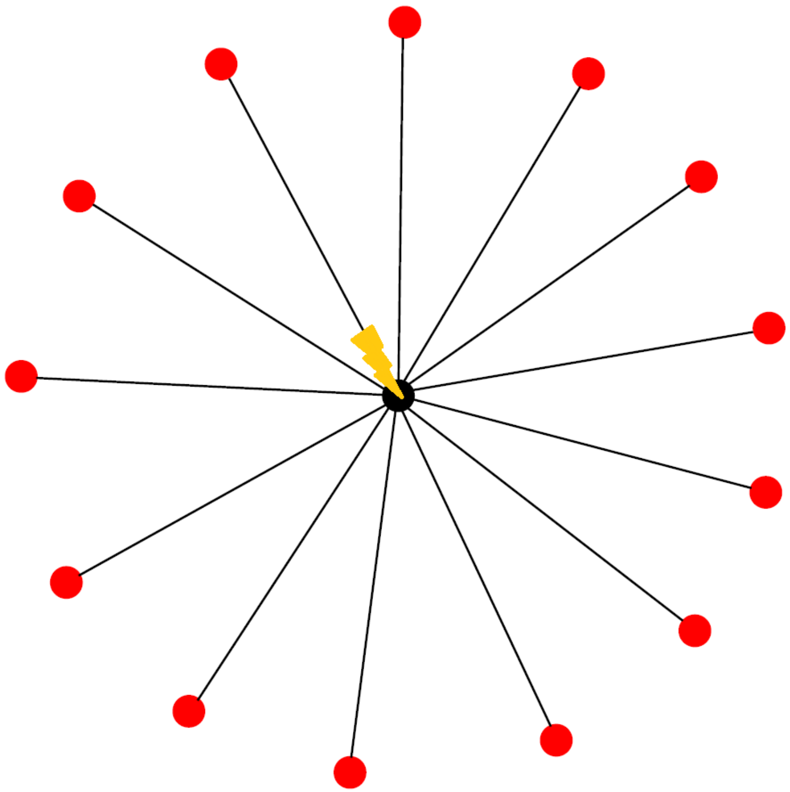

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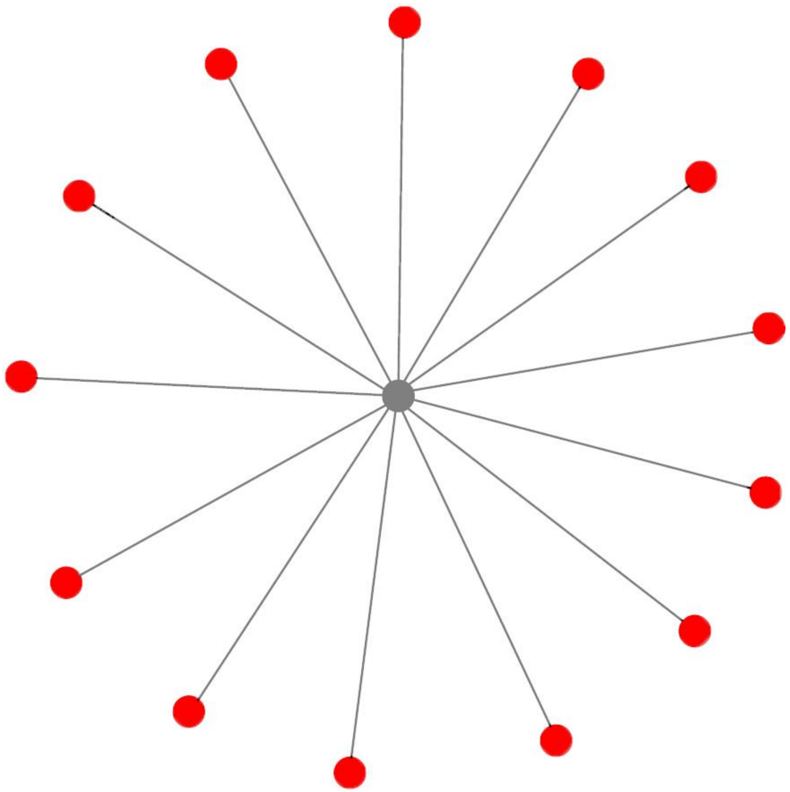

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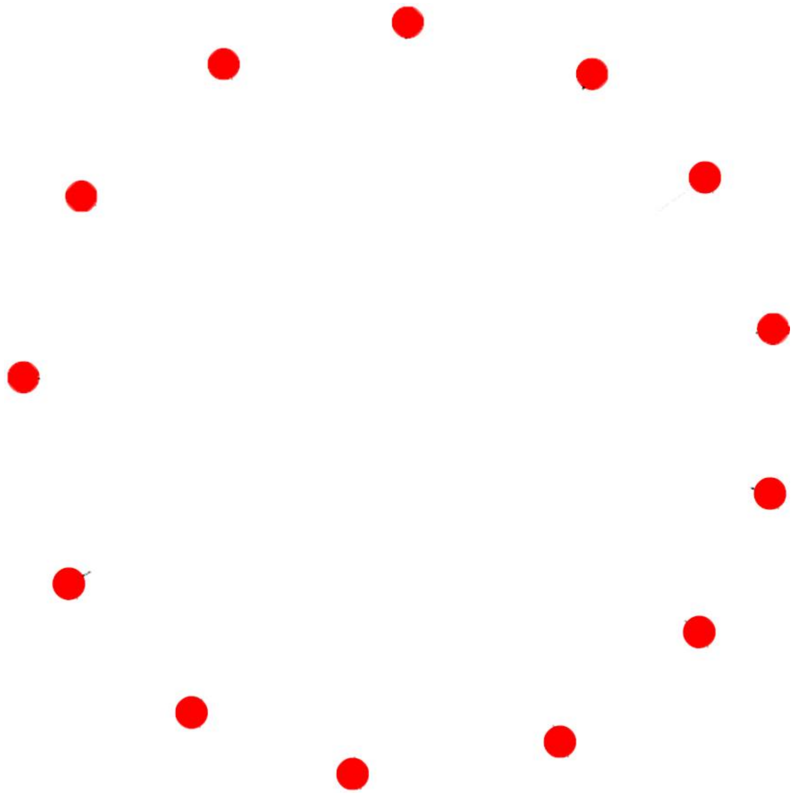

Some Definitions

Constructibility

We say that a protocol Π constructs a graph language L, if:

1. Every execution of Π on n nodes stabilizes to a graph $G \in L$ s.t. $|V(G)| = n$, and
2. $\forall G \in L$ there is an execution of Π on $|V(G)|$ nodes that stabilizes to G.

Partial Constructibility

We say that a protocol Π partially constructs a graph language L, if:

1. (1) from Definition 1 holds, and
2. $\exists G \in L$ s.t. no execution of Π on $|V(G)|$ nodes stabilizes to G.

Some Definitions

Fault-Tolerant Protocol

Let Π be a *NET* protocol that, in a failure-free setting, constructs a graph $G \in L$. Π is called f*-fault-tolerant* if for any population size $n > f$, any execution of Π constructs a graph $G \in L$, where $|V(G)| = n - f$. We also call Π fault-tolerant if the same holds for any number $f \leq n - 2$ of faults.

Constructible language

A graph language L is called constructible (partially constructible) if there is a protocol that constructs (partially constructs) it. Similarly, we call L constructible under f faults, if there is an f-fault-tolerant protocol that constructs L, where f is an upper bound on the maximum number of faults.

Our Results

Constructible languages		
Without notifications		With notifications
Unbounded faults	Bounded faults	Unbounded faults
Only Spanning Clique	Non-hereditary impossibility	Fault-tolerant protocols: Spanning Star, Cycle Cover, Spanning Line
Strong impossibility even with linear waste	A representation of any finite graph (partial constructibility)	Universal Fault-tolerant Constructors (with waste)
	Any constructible graph language with linear waste	Universal Fault-tolerant Restart (without waste)

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Fault Tolerant Spanning Clique

Transition function:

$$(b, b, 0) \rightarrow (b, r, 0)$$

$$(b, r, 0) \rightarrow (r, r, 0)$$

$$(r, r, 0) \rightarrow (r, r, 1)$$

- The above protocol constructs a spanning clique, tolerating any number of faults
- Spanning Clique is the only constructible graph language in the unbounded-faults case
  - Even if we allow linear waste

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Non-hereditary Graph Languages

Hereditary Language

A graph language L is called *Hereditary* if for any graph $G \in L$, every induced subgraph of G also belongs to L.

- If there exists a graph $G \in L$, such that after removing any node (crash fault), the resulting graph $G' \notin L$, then there is no protocol that stably constructs L.
- If there was a protocol that changes the configuration in order to "fix" the graph, then this would happen indefinitely and the protocol would never be stabilizing.
- This means that if a graph language in non-hereditary, it is impossible to be constructed under a single fault.

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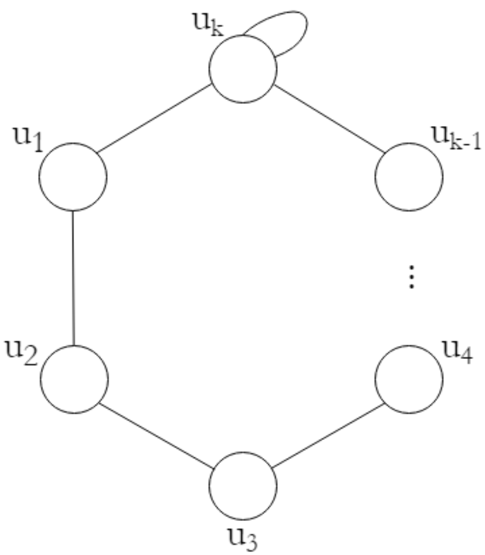
Partial Constructibility

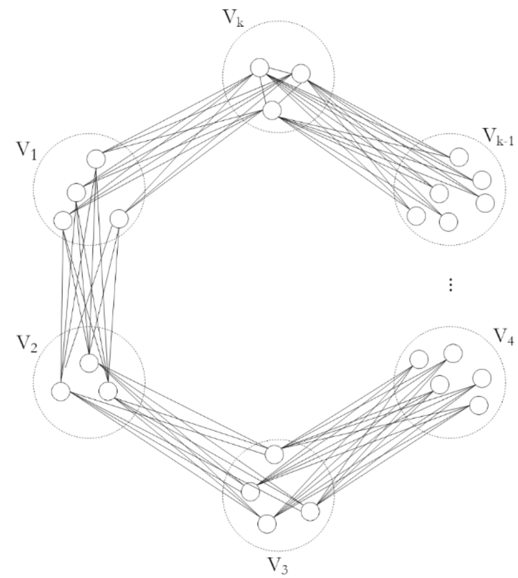
There exists a class of graph languages that is partially constructible in the case of bounded number of faults.

- Class of graph languages $L_{D,f}$
  - $D = ([k], H)$
  - $f < k$ is the finite upper bound on the number of faults
- A graph $G = (V, E)$ belongs to $L_{D,f}$ iff there are k partitions $V_1, V_2, \dots, V_k$ of V s.t. for all $1 \leq i, j \leq k$, $\left| |V_i| - |V_j| \right| \leq f + 1$
- The graph D defines a neighbouring relation between the partitions. For every $(i, j) \in H$, E contains all edges between partitions V_i and V_j.

Partial Constructibility



$D = ([k], H)$



Graph of supernodes $G = (V, E)$

- We provide a protocol which partitions the population into $k = 2^i$ groups.
- It constructs any graph language $L_{D,f}$ (as described before), where $k = 2^i$.
- The partitioning can be used in order to construct any (constructible) graph language on at least $\frac{n}{2f} - f$ nodes, where f is the number of faults

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Notified Network Constructors

- We now extend the original model with a minimal form of fault notifications.
- When a node u crashes, all the nodes that maintain an active connection with it at that time, are notified (a fault flag becomes 1).
- If no such nodes exist (i.e., u is isolated), then an arbitrary node is notified.
- In this way, we guarantee that at least one node will “sense” the crash failure.

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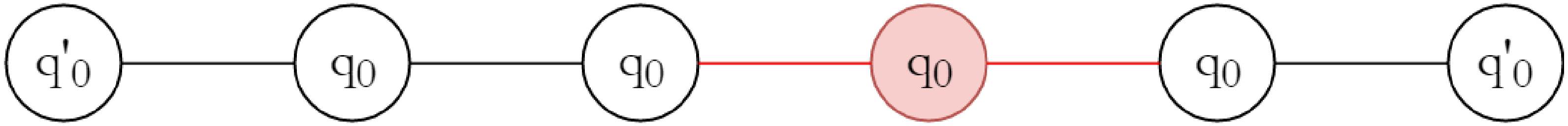

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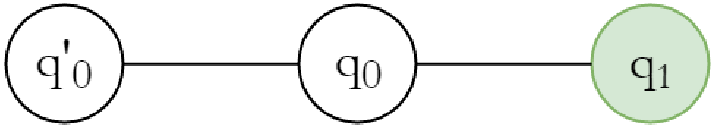

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Notified Network Constructors

- Some otherwise infeasible graph languages are now constructible under any number of faults
  - Spanning Star
  - Cycle Cover
  - Spanning Line

$Q = \{q_0,\ q_1,\ q_2\} \times \{0,1\}$
Initial state: q_0

δ_1 :
$(q_0,\ q_0,\ 0) \rightarrow (q_1,\ q_1,\ 1)$
$(q_1,\ q_0,\ 0) \rightarrow (q_2,\ q_1,\ 1)$
$(q_1,\ q_1,\ 0) \rightarrow (q_2,\ q_2,\ 1)$

δ_2 :
$(q_1, 1) \rightarrow (q_0, 0)$
$(q_2, 1) \rightarrow (q_1, 0)$

Fault Tolerant Cycle-Cover Protocol

$Q = \{b,\ r\} \times \{0,1\}$
Initial state: b

δ_1 :
$(b,\ b,\ 0) \rightarrow (b,\ r,\ 1)$
$(b,\ b,\ 1) \rightarrow (b,\ r,\ 1)$
$(r,\ r,\ 1) \rightarrow (b,\ b,\ 0)$
$(b,\ r,\ 0) \rightarrow (b,\ r,\ 1)$

δ_2 :
$(r, 1) \rightarrow (b, 0)$

Fault Tolerant Spanning Star Protocol

Notified Network Constructors

$Q = \{q_0,\ q_2,\ e_1,\ e_2,\ l_0,\ l_1,\ w,\ w_1,\ w_2\} \times \{0,1\}$
Initial state: q_0

δ_1 :
$(q_0,\ q_0,\ 0) \rightarrow (e_1,\ l_0,\ 1)$
$(l,\ q_0,\ 0) \rightarrow (q_2,\ l_0,\ 1)$
$(l_0,\ l_0,\ 0) \rightarrow (q_2,\ w,\ 1)$

\\\\w nodes perform a random walk on line
$(l_1,\ q_2,\ 1) \rightarrow (e_1,\ w_1,\ 1)$
$(w_i,\ q_2,\ 1) \rightarrow (q_2,\ w_i,\ 1)$
$(w,\ q_2,\ 1) \rightarrow (q_2,\ w,\ 1)$

$(w,\ e_i,\ 1) \rightarrow (w_i,\ e_i,\ 1)$
$(w_i,\ e_i,\ 1) \rightarrow (w_j,\ e_j,\ 1),\ i \neq j$
$(w_i,\ e_j,\ 1) \rightarrow (q_2,\ l_0,\ 1),\ i \neq j$
$(w,\ l_i,\ 1) \rightarrow (w_1,\ e_1,\ 1)$
$(w_i,\ l_i,\ 1) \rightarrow (q_2,\ l_0,\ 1)$

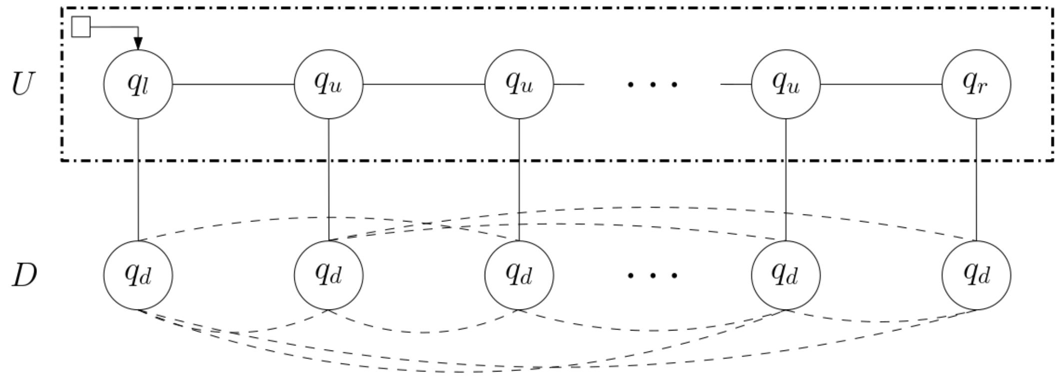
\\\\w nodes eliminate each other, until only one survives
$(w_i,\ w_j,\ 1) \rightarrow (w,\ q_2,\ 1)$
$(w,\ w_j,\ 1) \rightarrow (w,\ q_2,\ 1)$

δ_2 :
$(e_1, 1) \rightarrow (q_0, 0)$
$(e_2, 1) \rightarrow (q_0, 0)$
$(l_0, 1) \rightarrow (q_0, 0)$
$(l_1, 1) \rightarrow (q_0, 0)$
$(q_2, 1) \rightarrow (l_1, 0)$
$(w, 1) \rightarrow (l_1, 0)$
$(w_1, 1) \rightarrow (l_1, 0)$
$(w_2, 1) \rightarrow (l_1, 0)$

Fault Tolerant Spanning Line Protocol

Universal Fault-Tolerant Constructors

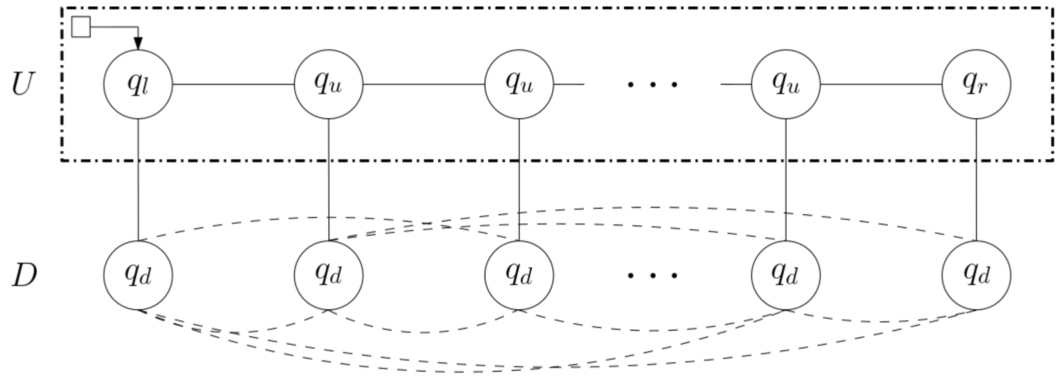
- Is there a generic fault-tolerant constructor capable of constructing a large class of graphs?
- The Fault-Tolerant Spanning Line is capable of simulating a given Turing Machine of space $O(n-k)$, where $0 \leq k < n$ is the number of faults
- We provide a fault-tolerant protocol that splits the population into two groups U and D of equal size
  - U is a spanning line with a unique leader in one endpoint and can eventually simulate a TM
  - Each node of D is connected with exactly one node of U, and vice versa



2-Partition

Universal Fault-Tolerant Constructors

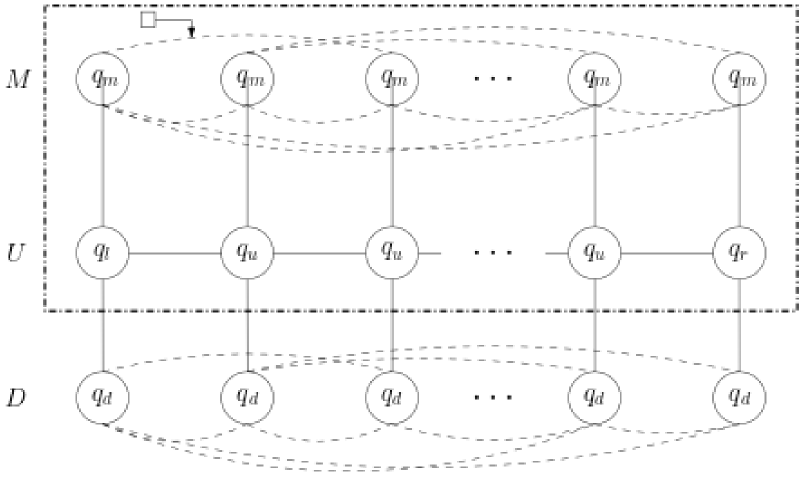
- This protocol *(Partition)* is fault-tolerant, but adds a waste of $2f(n)$, where $f(n)$ is an upper bound on the number of faults.
- We show that for any graph language L that can be decided by a *linear space* TM, there is a protocol that constructs a graph from L in D with waste at most $\min\{\frac{n}{2} + f(n), n\}$.



2-Partition

Universal Fault-Tolerant Constructors

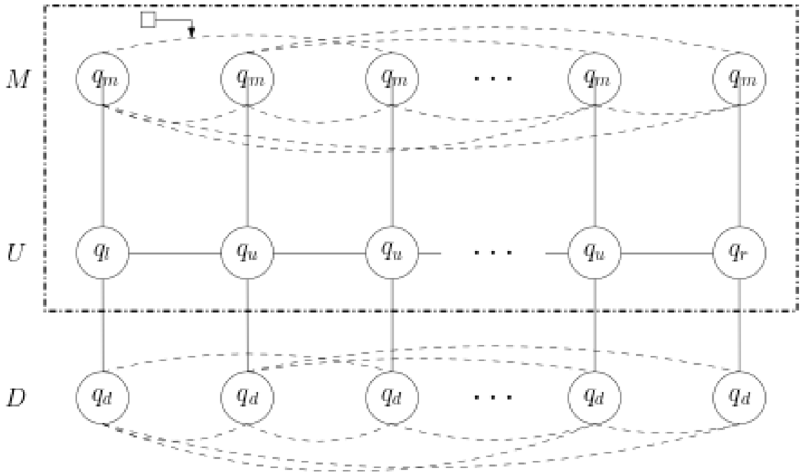
- This idea can be extended in order to increase the memory of the TM, by partitioning the population into three groups U, D and M of equal size.
- We provide a fault-tolerant protocol where
  - U is a spanning line that can eventually simulate a TM
  - Each node in $\mathrm{D} \cup M$ is connected with exactly one node of U
  - Each node of U is connected to exactly one node in D and one node in M.



3-Partition

Universal Fault-Tolerant Constructors

- This protocol is fault-tolerant, but adds a waste of $3f(n)$, where $f(n)$ is an upper bound on the number of faults.
- We show that for any graph language L that can be decided by an $O(n^2)$-*space* TM, there is a protocol that constructs a graph from L in D with waste at most $\min\{\frac{2n}{3} + f(n), n\}$.



3-Partition

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Fault-Tolerant protocols without Waste

- We increase the memory of each node to $O(logn)$ bits
  - We show that for constant memory, if the nodes can form a function of n connections with other nodes, it is impossible to restart the protocol correctly
- Each node stores two components C_1 and C_2
  - C_1 runs the restart protocol (*leader, phase, fault-flag)*
  - C_2 runs the given PP or NET protocol
- Whenever the fault-flag of a node is raised, all nodes eventually reinitialize their states in C_2
- After any re-initialization, *phase* is increased by one
- Nodes in different phases do not update their C_2 components
- We provide a protocol which guarantees that every node which enters to a new phase, has re-initialized its state correctly (all adjacent edges become inactive)

Future Work

- Are hereditary graph languages constructible if a bounded number of faults is allowed?
- Can we drop the assumption of waste and coin tossing?
- Consider other types of faults such as random, Byzantine, communication/edge faults
- Examination of fault-tolerant protocols for stable dynamic networks in models stronger than NETs.

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Thank You