Communicating Finite-State Machines and Two-Variable Logic

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Büchi-Elgot-Trakhtenbrot theorem ('60s)

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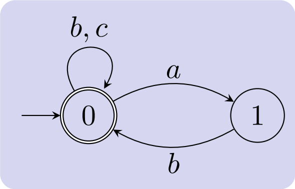
$$b \rightarrow a \rightarrow b \rightarrow c \rightarrow a \rightarrow b \rightarrow c$$

Words

$$\forall x.\ a(x) \Rightarrow \exists y.\ (x \rightarrow y \wedge b(y))$$

Monadic Second-Order Logic



Finite automaton

MSO$[\rightarrow]$ = Finite Automata

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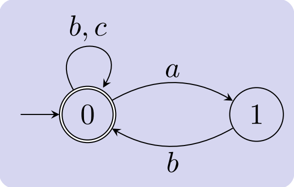

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$$b \rightarrow a \rightarrow b \rightarrow c \rightarrow a \rightarrow b \rightarrow c$$

Words

$$\begin{aligned} & \exists X_0.\ \exists X_1.\ \forall x.\ \forall y. \\ & \qquad x \rightarrow y \wedge X_0(x) \wedge a(y) \Rightarrow X_1(y) \\ \wedge & \qquad x \rightarrow y \wedge X_0(x) \wedge b(y) \Rightarrow X_0(y) \\ \wedge & \qquad \ldots \end{aligned}$$

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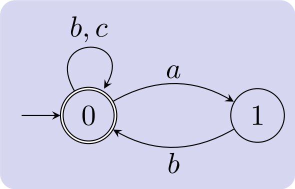

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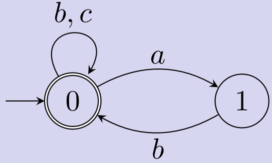

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Finite automaton

MSO$[\rightarrow]$ = Finite Automata = EMSO$[\rightarrow]$

$\exists X_0 \ldots \exists X_n.\varphi$ with $\varphi \in$ FO$[\rightarrow]$

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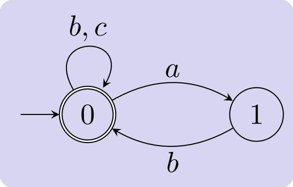

Finite automaton

MSO[$\to$] = Finite Automata = EMSO[$\to$] = EMSO^2[$\to$]

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two first-order variable names

. . . and some of its extensions

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Over trees:

MSO = Tree Automata = EMSO^2 [Thatcher-Wright '68]

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Over data words:
EMSO^2 = Data Automata $\subsetneq$ EMSO
[Bojanczyk-David-Muscholl-Schwentick-Segoufin '06]

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Goal: A Büchi-like theorem for message-passing systems

Communicating finite-state machines (CFMs)

[Brand–Zafiropulo '83]

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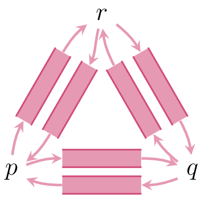
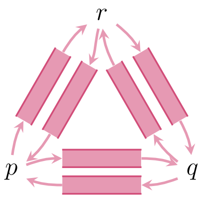


Communicating finite-state machines (CFMs)

[Brand–Zafiropulo '83]

- Finite set of processes, e.g. $Proc = \{p, q, r\}$
- Unbounded point-to-point FIFO channels
- For each process, one finite labeled transition system over

$$(\underbrace{\Sigma}_{\text{Finite alphabet}} \times \{\underbrace{!}_{\text{send}}, \underbrace{?}_{\text{receive}}\} \times \underbrace{Msg}_{\text{Finite set of messages}} \times \underbrace{Proc}_{\text{Processes}}) \cup \Sigma$$



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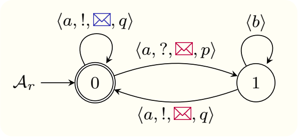

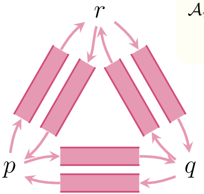

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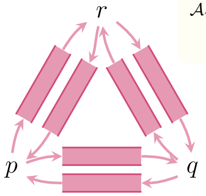

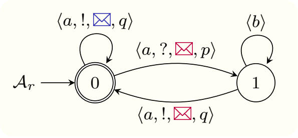

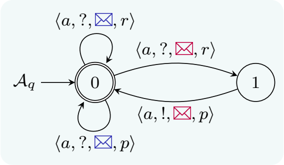

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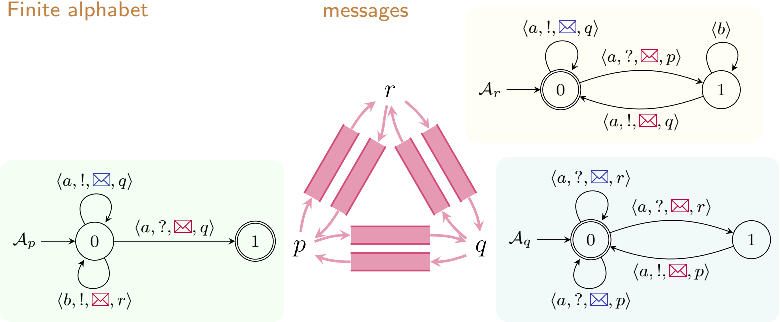

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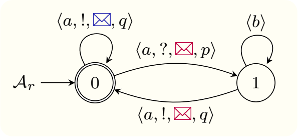
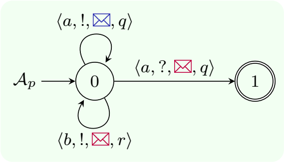
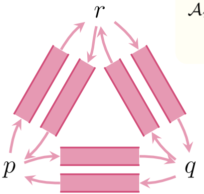
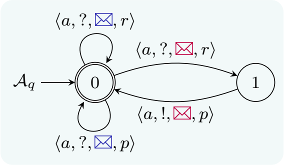
- Global acceptance condition



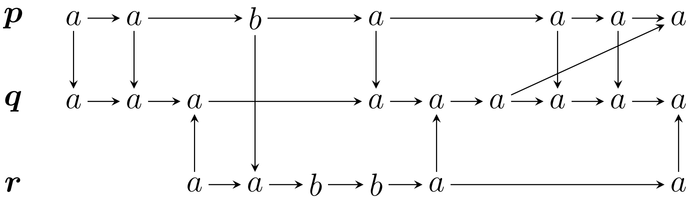






Language of a CFM

CFMs recognize languages of **Message Sequence Charts**:



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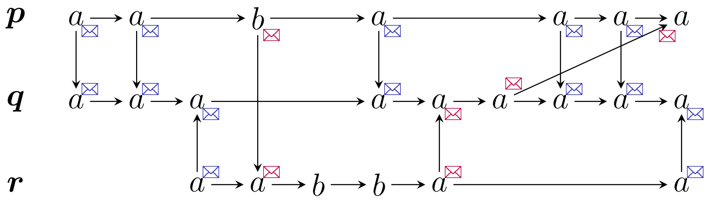

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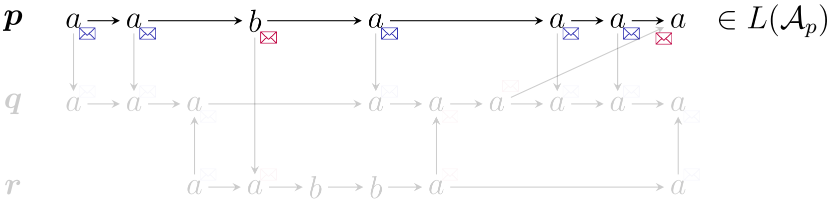

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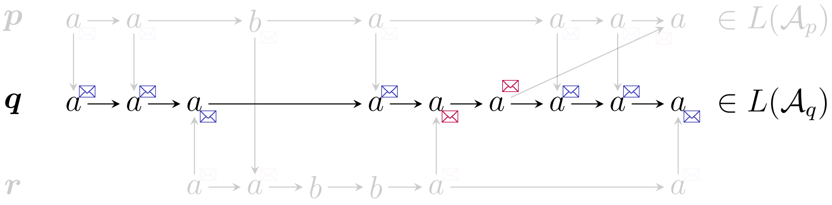

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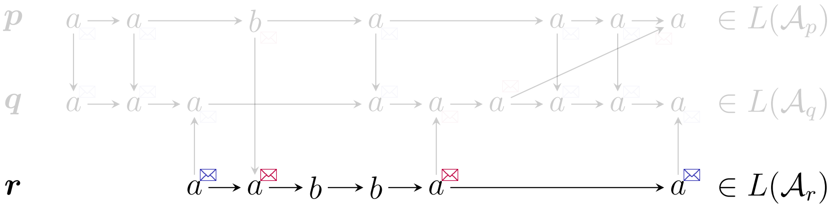

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Monadic Second-Order Logic over MSCs

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MSO$[\rightarrow, \rightarrow^*]$

$$\begin{aligned}\varphi \quad ::= \quad & a(x) \mid p(x) \mid x = y \mid x \rightarrow y \mid x \rightarrow^* y \\ & \mid \exists x.\ \varphi \mid \exists X.\ \varphi \mid x \in X \mid \varphi \vee \varphi \mid \neg\varphi\end{aligned}$$

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Monadic Second-Order Logic over MSCs

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Examples in FO$^2[\rightarrow, \rightarrow^*]$:

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Examples in FO$^2[\rightarrow, \rightarrow^*]$:

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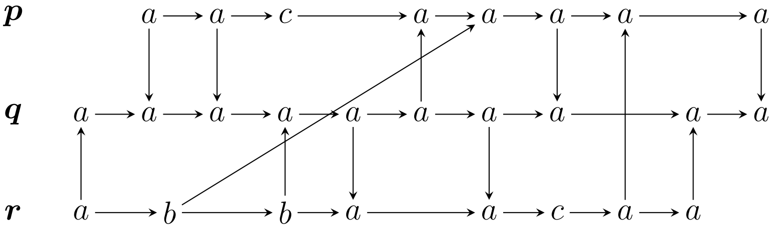
- Mutual exclusion: $\neg(\exists x.\ \exists y.\ c(x) \wedge c(y) \wedge x \parallel y)$

Monadic Second-Order Logic over MSCs



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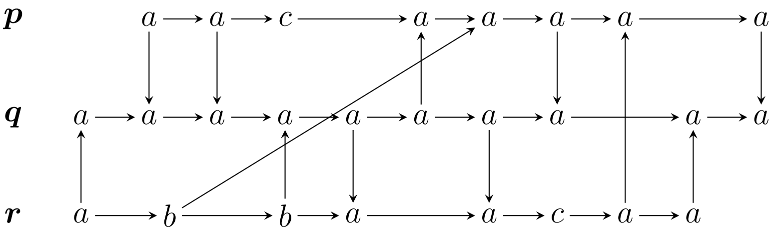

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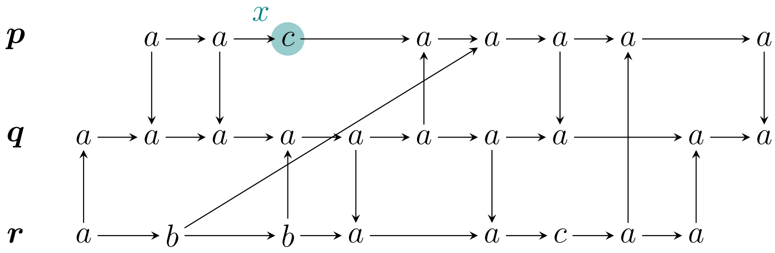

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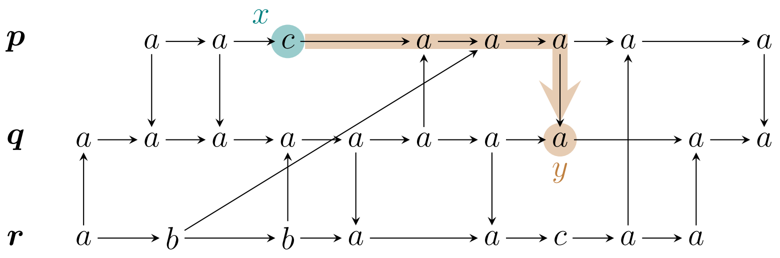

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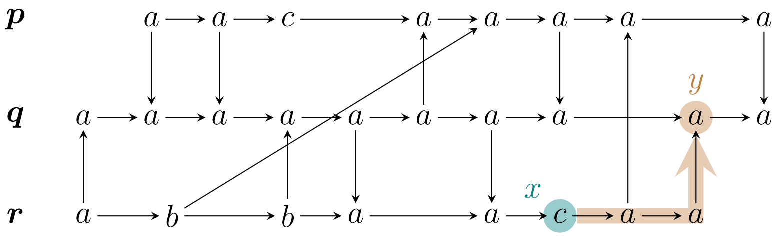

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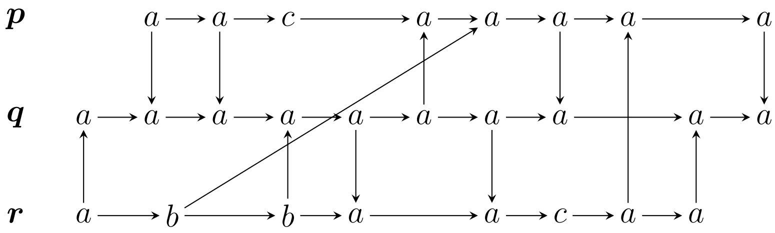

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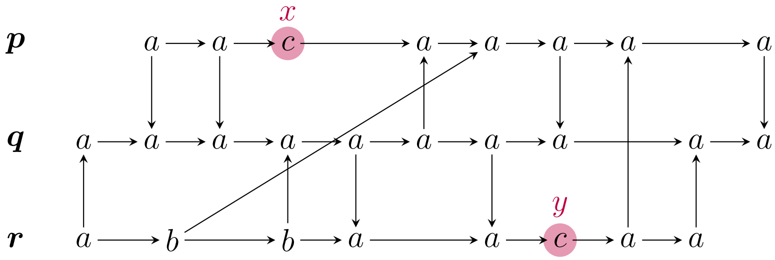

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Theorem

$\mathrm{CFM} = \mathrm{EMSO}^2[\rightarrow, \rightarrow^*]$.

Scott normal form ('65)

Scott normal form:
Any EMSO$^2[\rightarrow, \rightarrow^*]$ formula is equivalent to a formula

$$\exists X_1 . \dots . \exists X_m . \ \forall x . \forall y . \psi(x,y) \wedge \bigwedge_{i=1}^{n} \forall x . \exists y . \psi_i(x,y)$$

where $\psi(x,y)$ and each $\psi_i(x,y)$ is quantifier-free.

From Scott normal form to CFMs

$$\exists X_1 \ldots . \exists X_m . \ \forall x . \forall y . \psi(x,y) \wedge \bigwedge_{i=1}^{n} \forall x . \exists y . \psi_i(x,y)$$

From Scott normal form to CFMs

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- Guess a valuation for each X_i.

From Scott normal form to CFMs

$$\exists X_1 . \ldots . \exists X_m . \; \forall x . \forall y . \psi(x,y) \wedge \bigwedge_{i=1}^{n} \forall x . \exists y . \psi_i(x,y)$$

- Guess a valuation for each X_i.
- Compute the **type** of each event.

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Type of event x: set of all possible tuples $(\bowtie, p, a)$ such that for some event y:

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- x is on process p, and labeled a.

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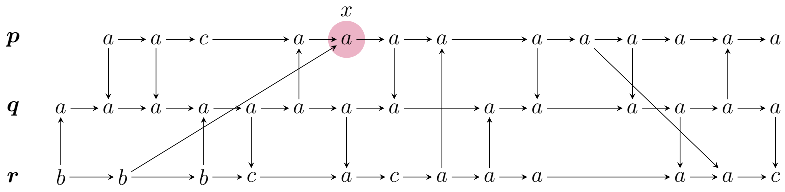
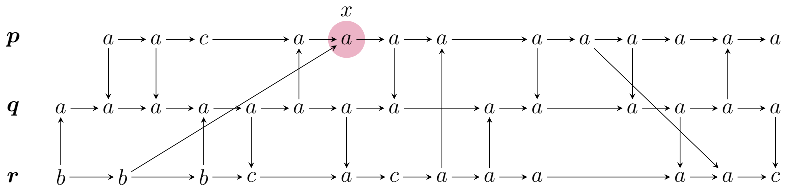
- $y \rightsquigarrow x$
- $x \parallel y$, i.e., $x \not\to^* y$ and $y \not\to^* x$

Type of an event



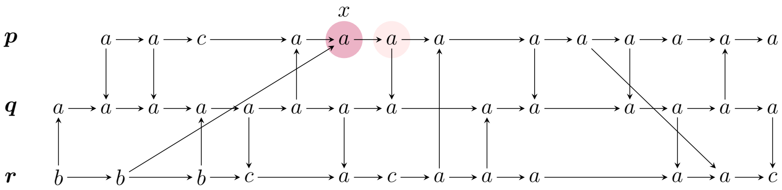
Type of an event

$x = y \ : \ (p, a)$



Type of an event

$x = y \ : \ (p, a)$ $\qquad$ $x \rightarrow y \ : \ (p, a)$



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$x = y \;:\; (p, a)$

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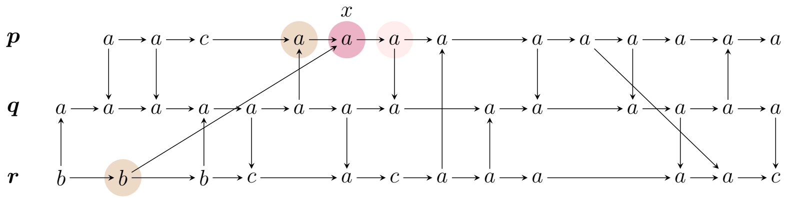
$y \rightarrow x \;:\; (p, a), (r, b)$



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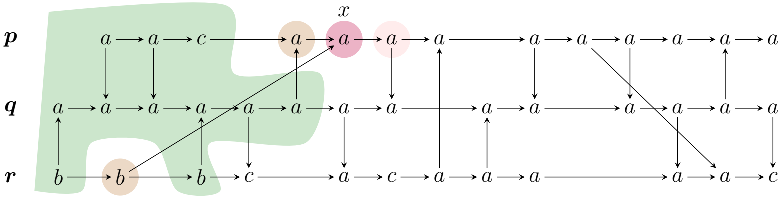
$y \rightsquigarrow x \;:\; (p, a), (p, c), (q, a), (r, b)$



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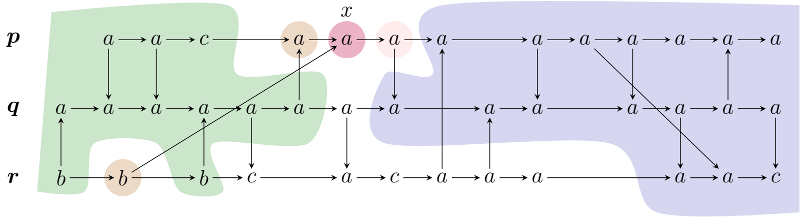
$x \rightsquigarrow y \;:\; (p, a), (q, a), (r, a), (r, c)$



Type of an event

$x = y \;:\; (p,a)$

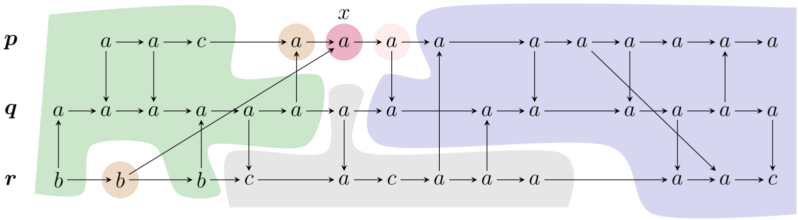
$x \rightarrow y \;:\; (p,a)$

$y \rightarrow x \;:\; (p,a),(r,b)$

$y \rightsquigarrow x \;:\; (p,a),(p,c),(q,a),(r,b)$

$x \rightsquigarrow y \;:\; (p,a),(q,a),(r,a),(r,c)$

$x \parallel y \;:\; (q,a),(r,a),(r,c)$



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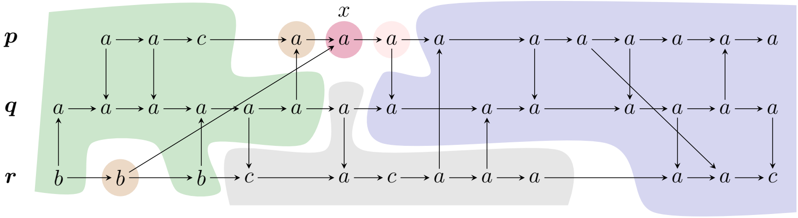
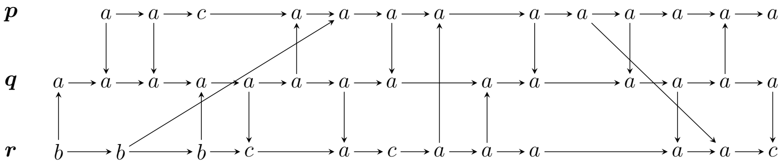


$$\models \exists y.\ \neg(x \rightarrow^* y) \wedge \neg p(y) \wedge c(y)$$

Labels of parallel events

Automaton that determines the set of events on process p that are parallel to some c on process r:



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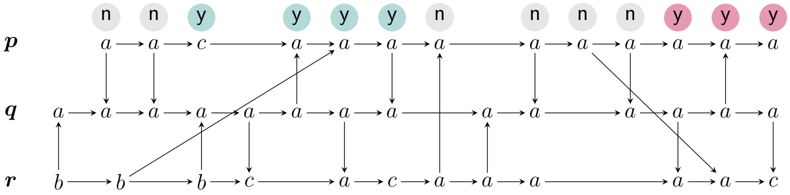

Guess yes/no for each event

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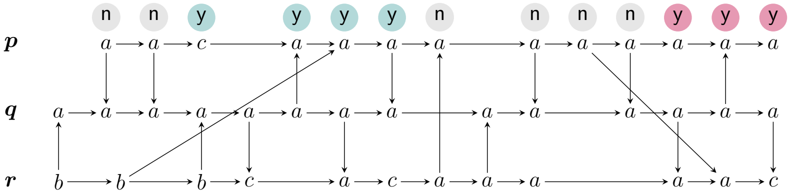

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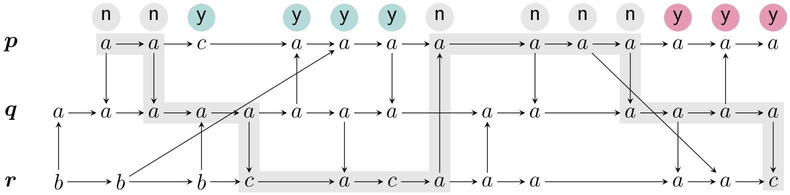

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$\to$ path containing all "no"'s on p and all c's on r

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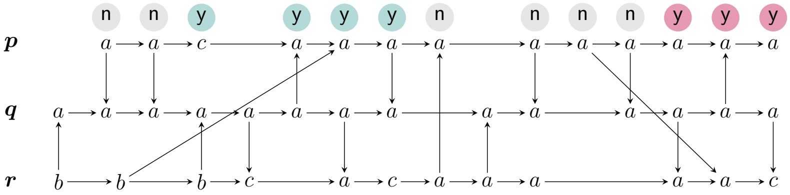

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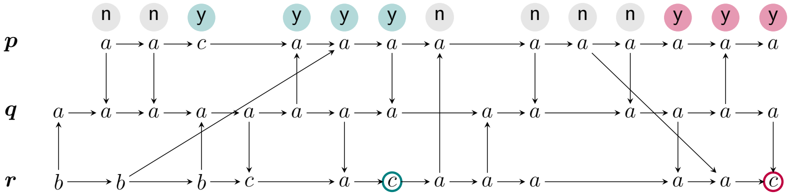

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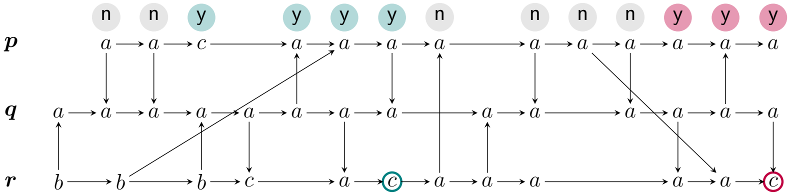

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$\rightarrow$ check y $\parallel$ c, and y $\parallel$ c

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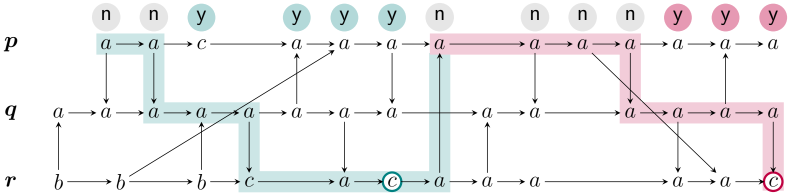

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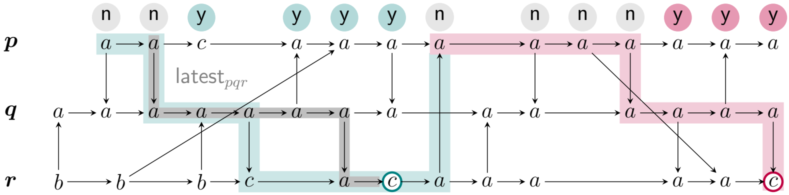

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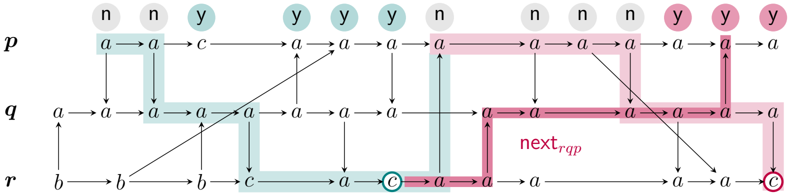

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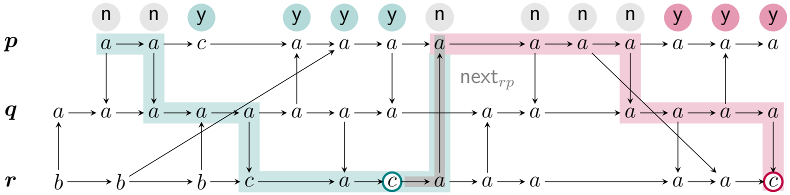

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Thank you!