On Parameterized Verification of Asynchronous Shared-Memory Pushdown Systems

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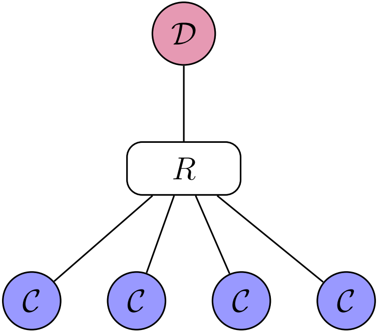
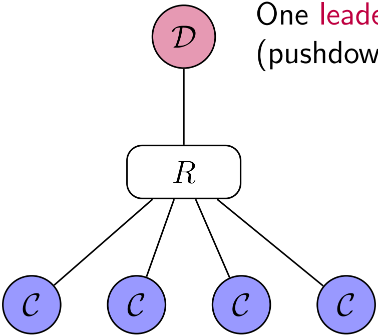
$(\mathcal{C}, \mathcal{D})$-systems



$(\mathcal{C}, \mathcal{D})$-systems



One leader process $\mathcal{D}$
(pushdown system)

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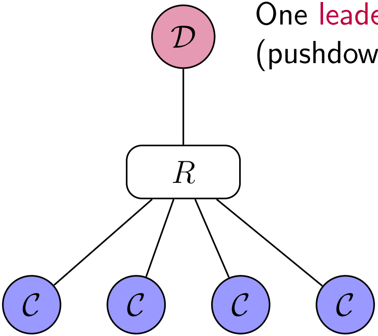

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Arbitrarily many identical and **anonymous** contributors $\mathcal{C}$
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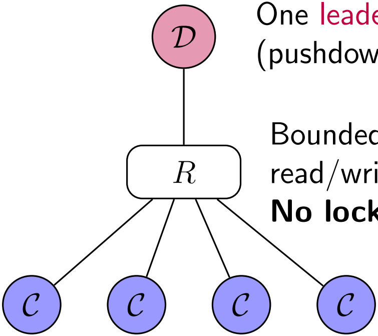

One leader process $\mathcal{D}$
(pushdown system)

Bounded shared register
read/write
No lock

Arbitrarily many identical and
anonymous contributors $\mathcal{C}$
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Previous Work

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Reachability

Is there a run of the $(\mathcal{C}, \mathcal{D})$-system where the leader performs a special action $\top$, for some number of contributors?

[Hague, 2011] EXPSPACE

[Esparza, Ganty, Majumdar, 2013] PSPACE-complete

[La Torre, Muscholl, Walukiewicz, 2015] Generalization

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Repeated reachability

Is there a run of the $(\mathcal{C}, \mathcal{D})$-system where the leader performs $\top$ infinitely often, for some number of contributors?

[Durand-Gasselin, Esparza, Ganty, Majumdar, 2015]
PSPACE-hard and in NEXPTIME

Repeated reachability

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The repeated reachability problem is PSPACE-complete.

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- ▷ Reduction to the case of finite-state contributors, by bounding the stacks of the contributors
- ▷ NP in the case of finite-state contributors $\rightarrow$ NEXPTIME for pushdown contributors

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We re-use the reduction to finite-state contributors, but change the decision procedure

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Idea: if a contributor can produce a value g once, by adding copies of this contributor we can produce as many g's as needed.

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- Define similarly $\mathcal{C}^\kappa$. A loop in $\mathcal{D}^\kappa$ corresponds to a loop in the $(\mathcal{C}, \mathcal{D})$-system if each addition to the capacity is supported by a loop in $\mathcal{C}^\kappa$ producing the necessary write.

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  $\rightarrow$ Intersection emptiness of $\ell + 1$ finite automata computable in PSPACE.

Universal reachability

Reachability

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Repeated reachability

Is there a run of the $(\mathcal{C}, \mathcal{D})$-system where the leader performs $\top$ infinitely often, for some number of contributors?

Universal reachability

Does the leader perform $\top$ in all **(finite or infinite) maximal** runs of the $(\mathcal{C}, \mathcal{D})$-system, for any number of contributors ?

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Motivation

Correctness problems for distributed algorithms: is the outcome correct (does the leader execute $\top$) for an arbitrary number of participants and for every run of the algorithm?

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Specificity

We consider finite maximal runs as well as infinite ones
$\rightarrow$ deadlock detection

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The universal reachability problem is co$\textsc{Nexptime}$-complete.

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- For infinite runs: reduction to repeated reachability
- For finite runs: use the reduction to the case of finite-state contributors, show that it is NP-complete
- $\textsc{Nexptime}$-hardness: tiling of the $2^n \times 2^n$ square.

Generalization

Until now: verification of properties on leader actions only.

We consider the verification of regular properties $\mathcal{P} \subseteq (\Sigma_C \cup \Sigma_D)^* \cup (\Sigma_C \cup \Sigma_D)^\omega$ on both

- finite **and** infinite traces
- leader **and** contributor actions

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Restriction

$\mathcal{C}$-expanding properties: if $u \in \mathcal{P}$ and u' is obtained by repeating some contributor actions in u, then $u' \in \mathcal{P}$.

Main result

Theorem

The following problem is $\textsc{Nexptime}$-complete:

Input: a $(\mathcal{C}, \mathcal{D})$-system, and a regular $\mathcal{C}$-expanding property $\mathcal{P} \subseteq (\Sigma_\mathcal{C} \cup \Sigma_\mathcal{D})^* \cup (\Sigma_\mathcal{C} \cup \Sigma_\mathcal{D})^\omega$.

Question: Is there a maximal trace of the $(\mathcal{C}, \mathcal{D})$-system that belongs to $\mathcal{P}$?

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Steps of the proof:

- Reduction to a property on leader actions, by transforming the $(\mathcal{C}, \mathcal{D})$-system into one where all contributor actions are reflected in leader writes

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Steps of the proof:

- Reduction to a property on leader actions, by transforming the $(\mathcal{C}, \mathcal{D})$-system into one where all contributor actions are reflected in leader writes
- Reduction to our previous results
  - Infinite traces: reduction to repeated reachability
  - Finite traces: results about universal reachability

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- We improved the complexity upper bound for repeated reachability from NEXPTIME to PSPACE
- We introduced universal reachability, and showed that it is coNEXPTIME-complete
- Verification of regular $\mathcal{C}$-expanding properties is also NEXPTIME-complete

Thank you !