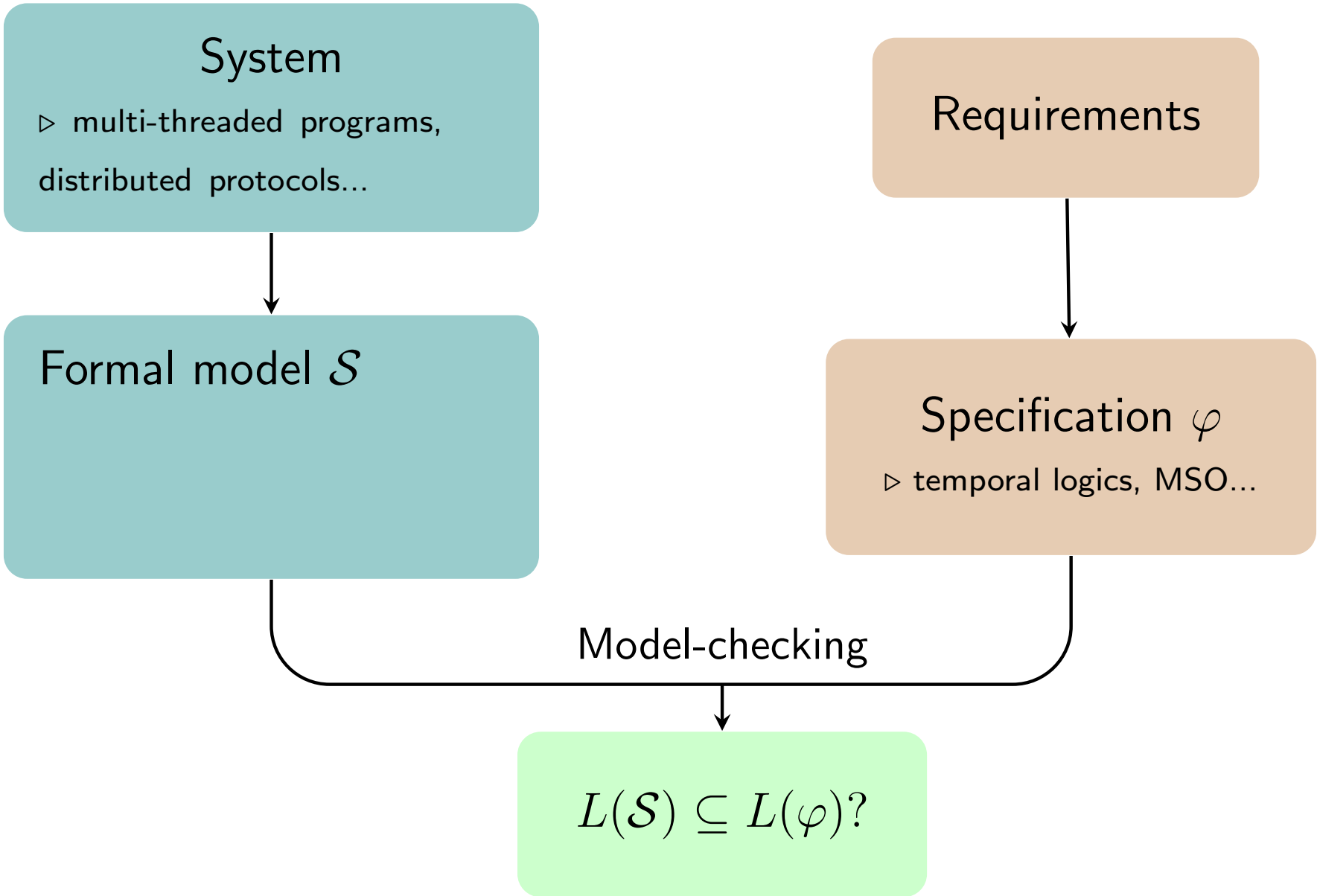
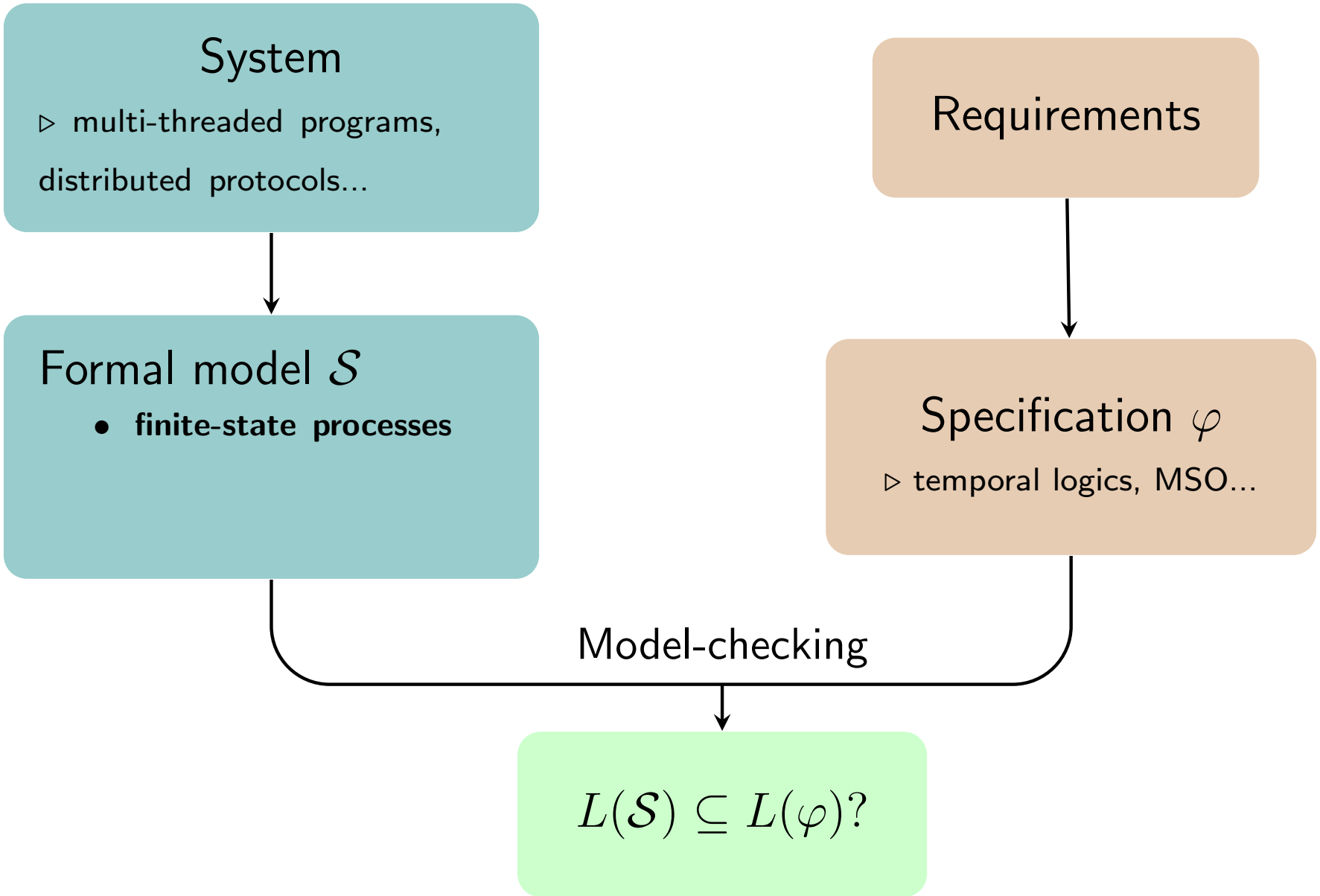
Verification of Parameterized Communicating Automata via Split-width

Marie Fortin, Paul Gastin

LSV, ENS Cachan

FoSSaCS 2016, Eindhoven


System
▷ multi-threaded programs,
distributed protocols...
Requirements
Formal model $\mathcal{S}$
Specification φ
▷ temporal logics, MSO...
Model-checking
$L(\mathcal{S}) \subseteq L(\varphi)$?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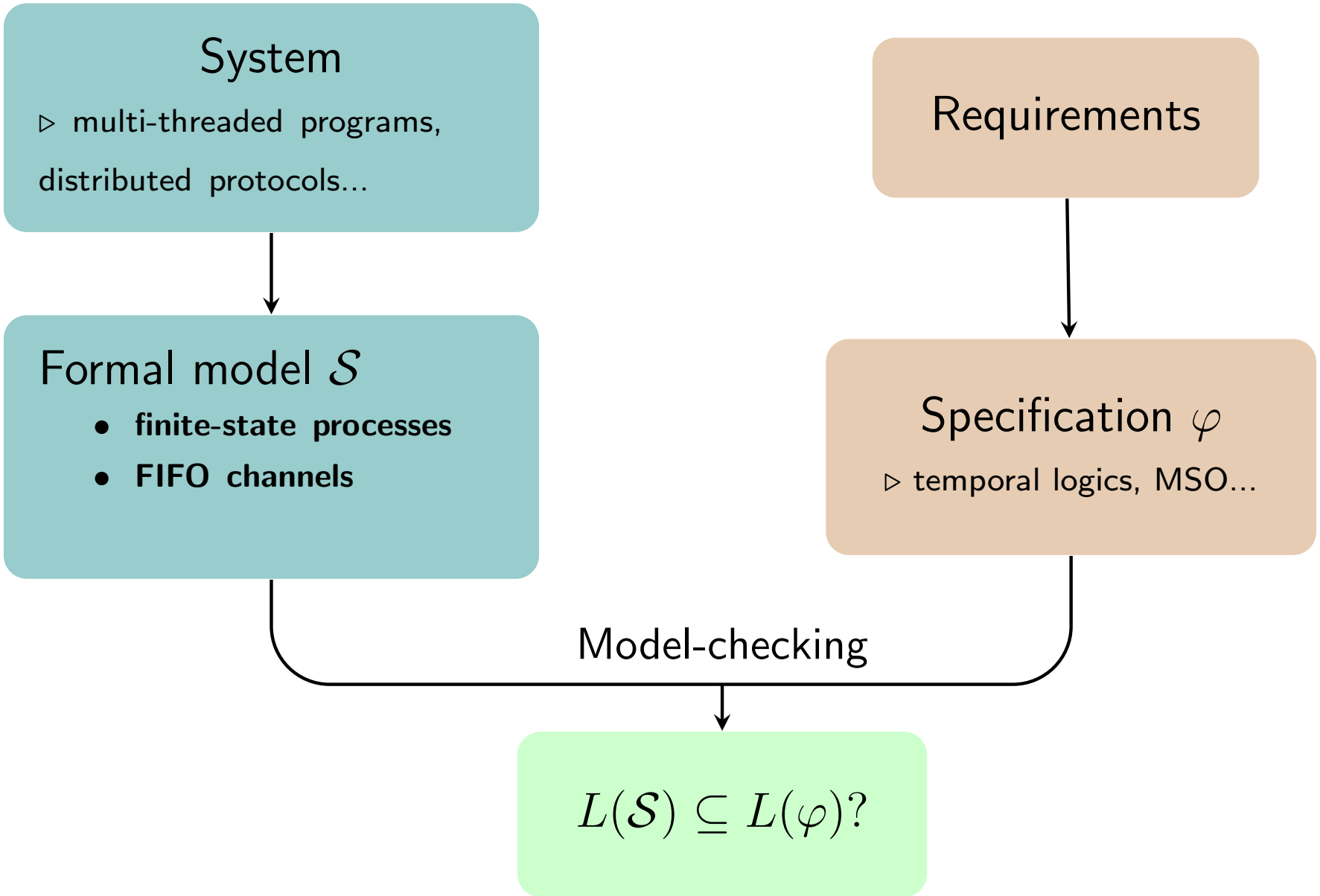
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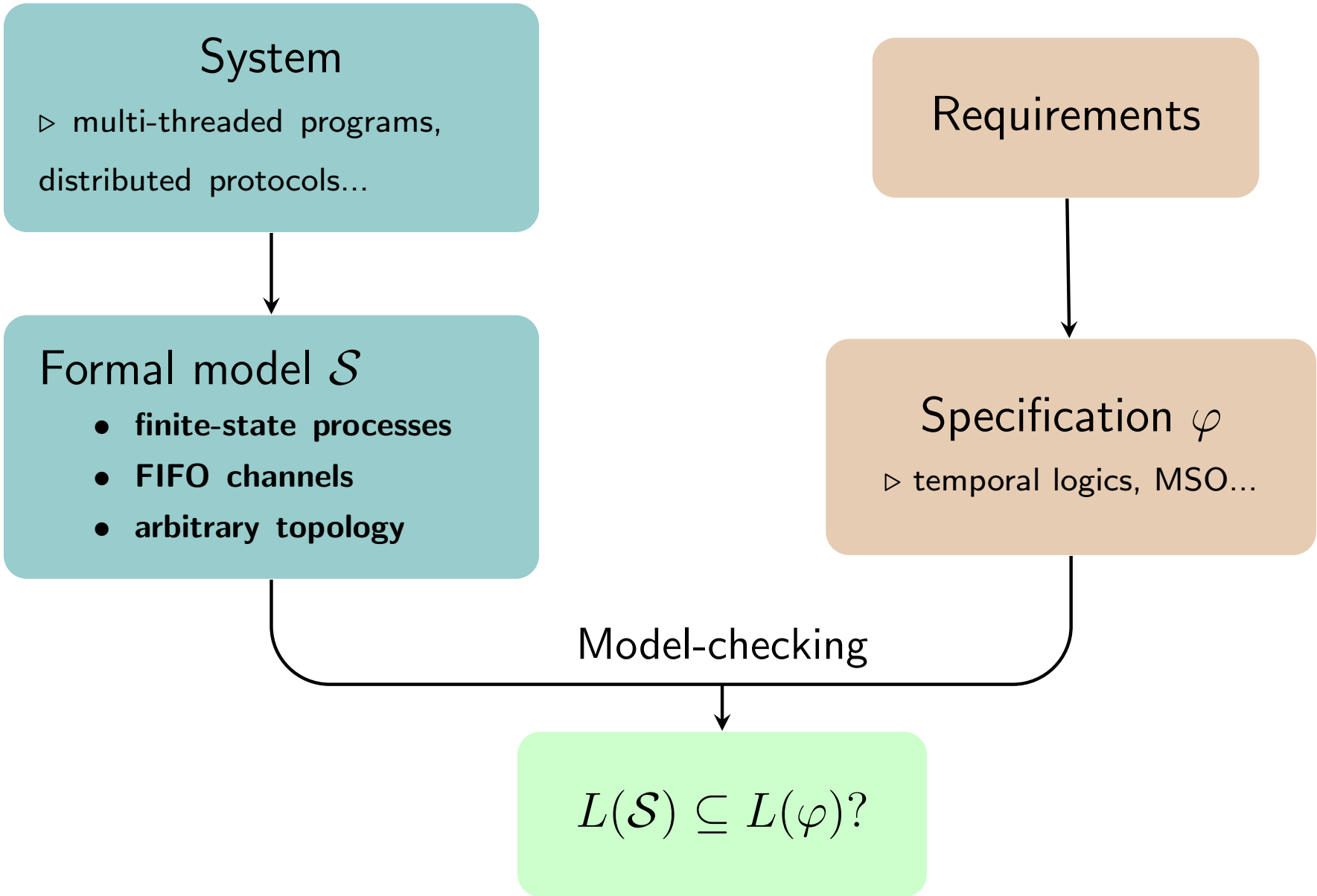
System
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Requirements
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• finite-state processes
• FIFO channels
• arbitrary topology
Specification φ
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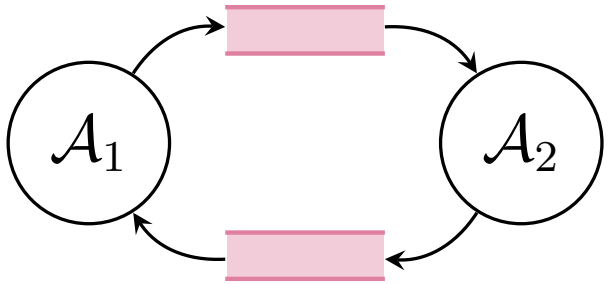
Sources of undecidability

- Unbounded FIFO channels

  $\rightarrow$ Undecidable even for 2 processes



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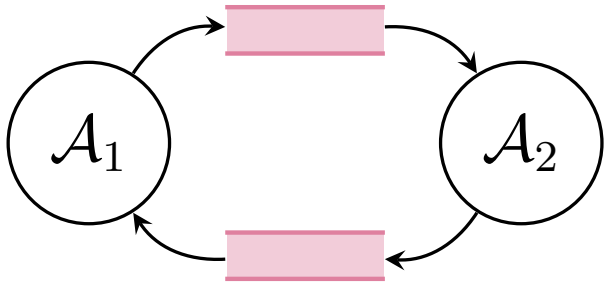

- Arbitrarily many processes

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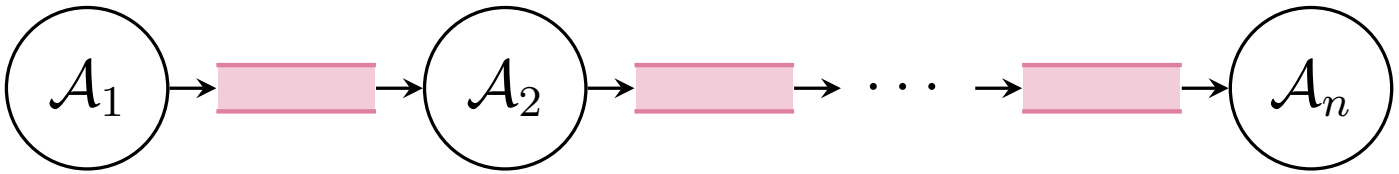

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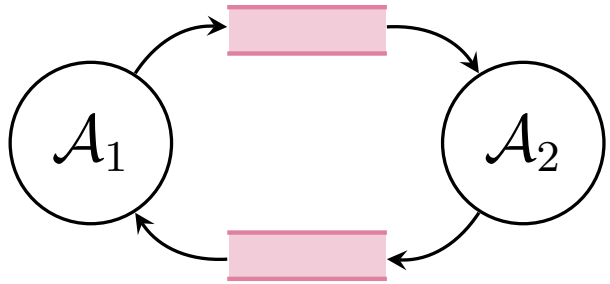

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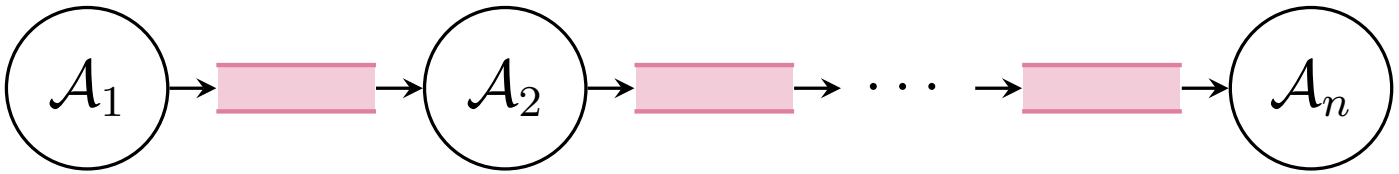

- Arbitrary topology

  $\rightarrow$ Undecidable even for locally bounded runs

Goal

A generic approach to define

- decidable restrictions
- efficient decision procedures

using graph decompositions and reductions to tree-automata.

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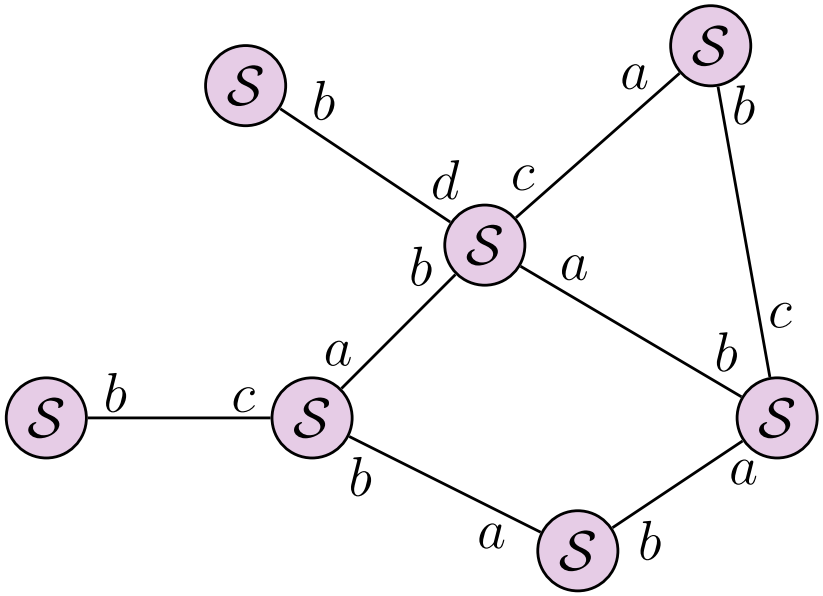
New: arbitrary topology, unknown number of processes

Parameterized Communicating Automata

[B. Bollig 2014]



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- Communication via FIFO channels
- Arbitrary topology of bounded degree

Parameterized Communicating Automata

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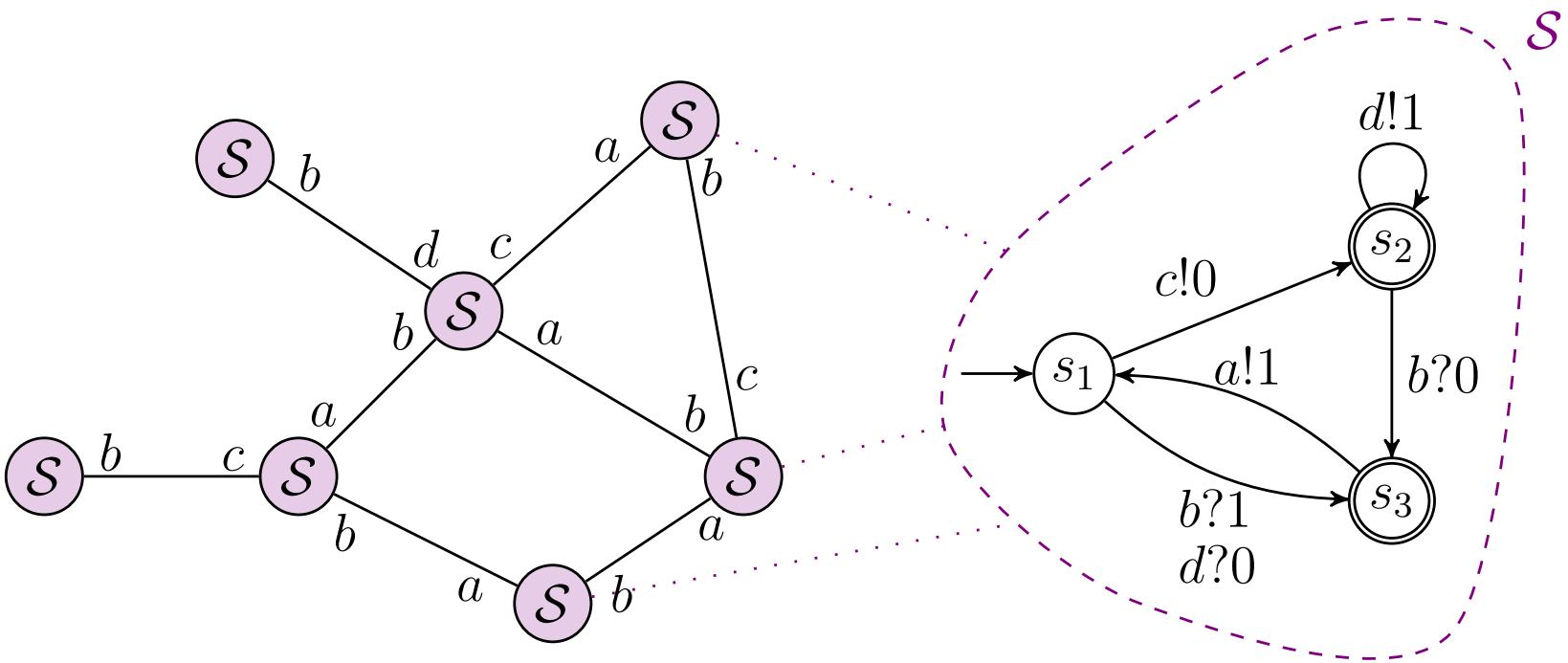

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Semantics : Message Sequence Charts (MSC)

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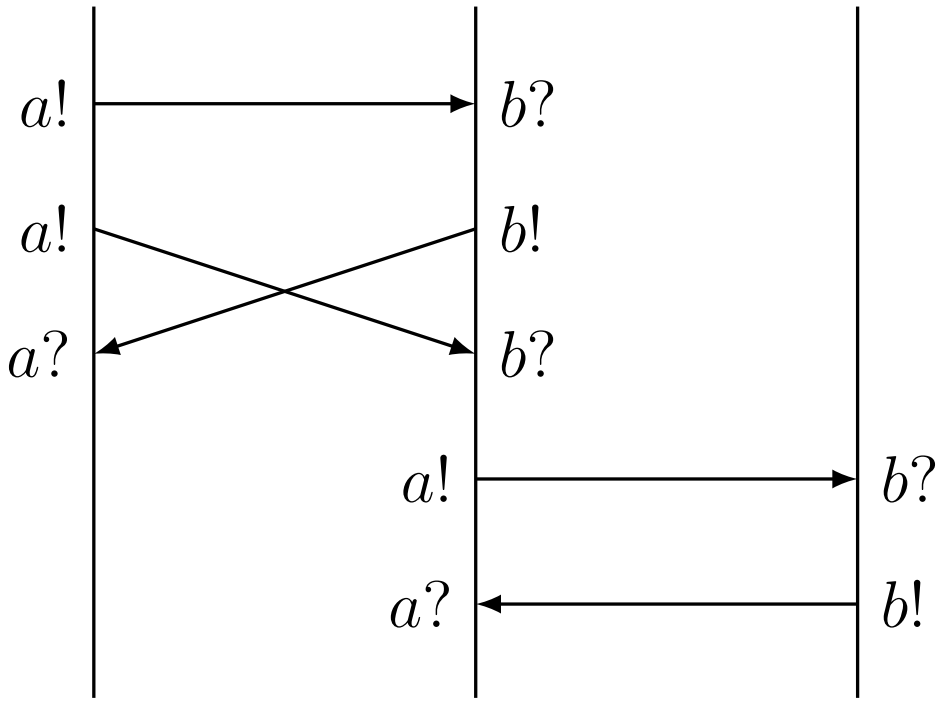

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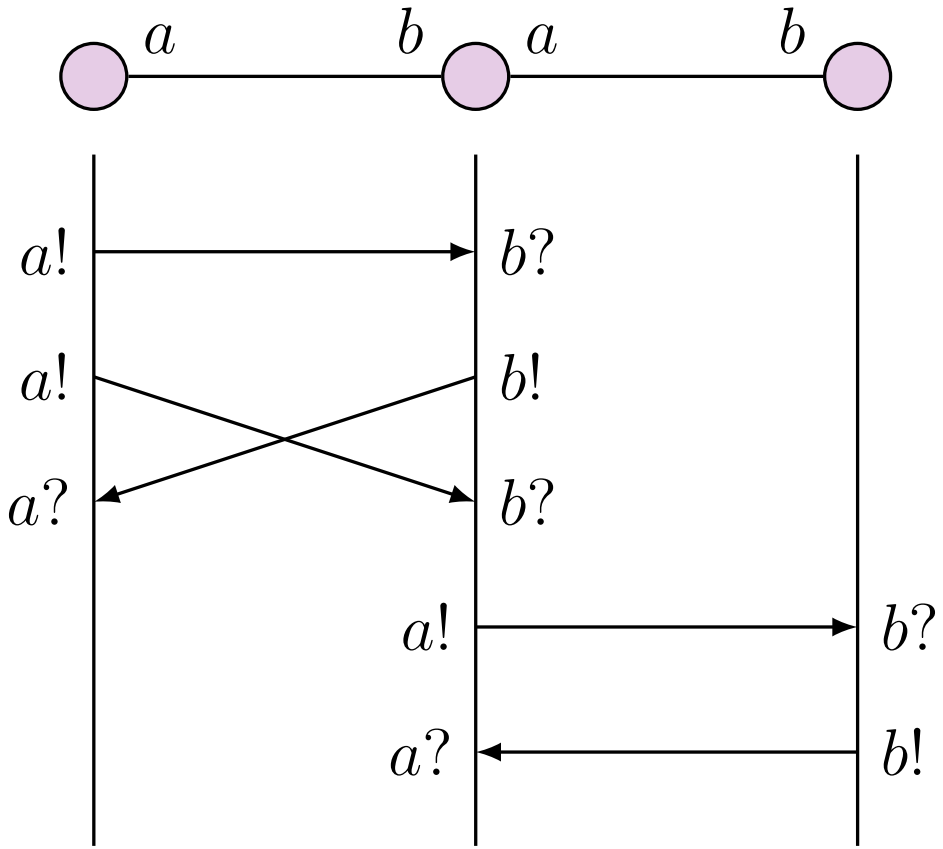

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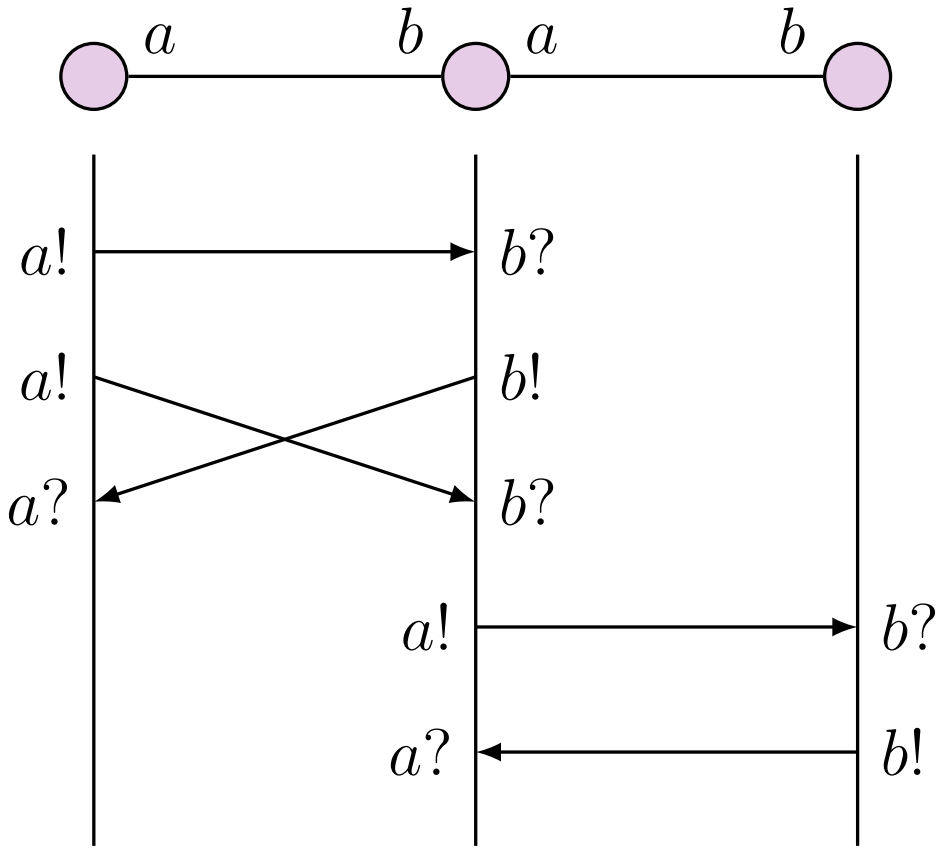
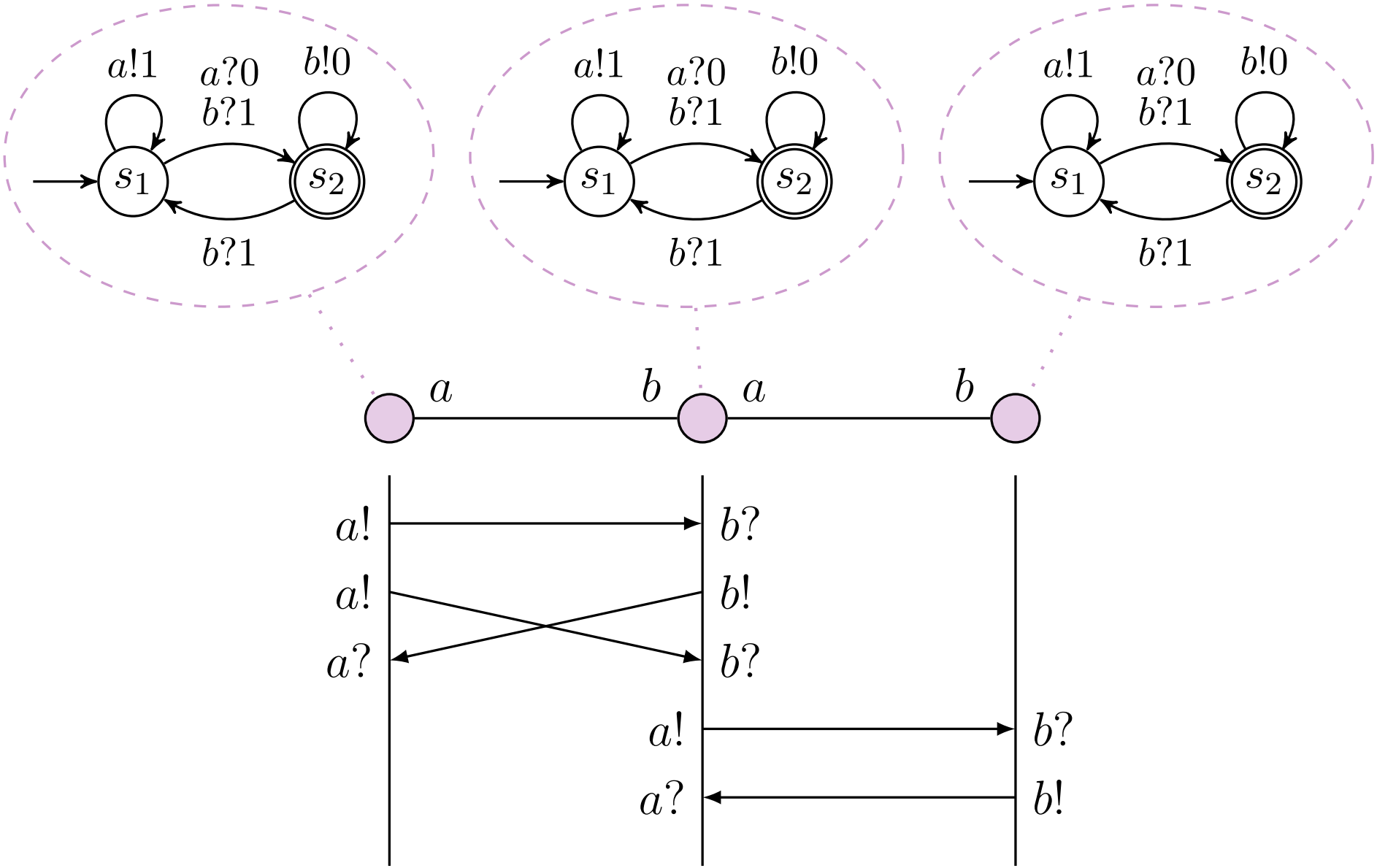


$L(\mathcal{S})$ is a set of MSCs on **arbirary topologies**.

Example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b!0
s_1
s_2
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a!1
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s_1
s_2
b?1
a!1
a?0
b?1
b!0
s_1
s_2
b?1
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a!
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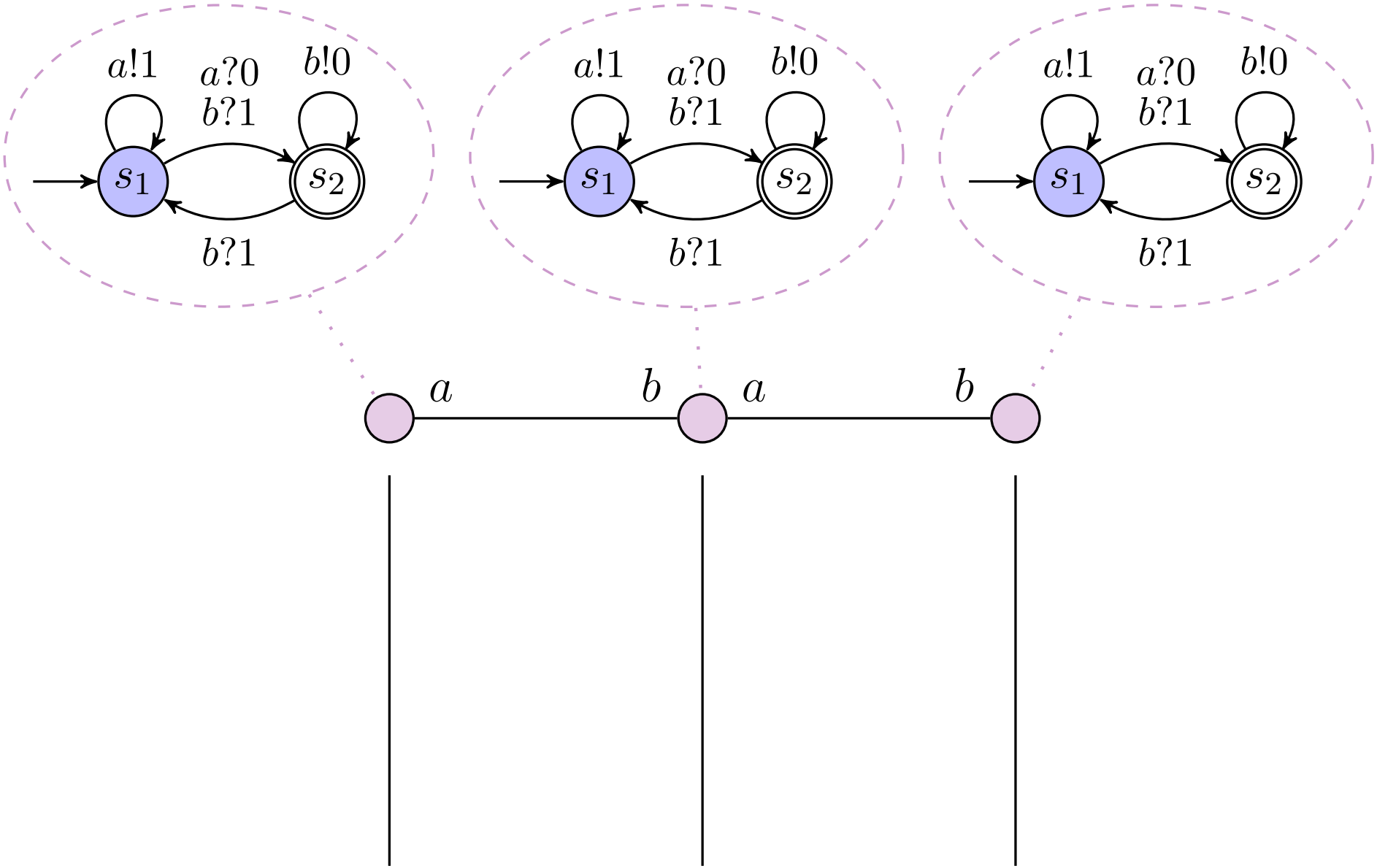
b!

Example



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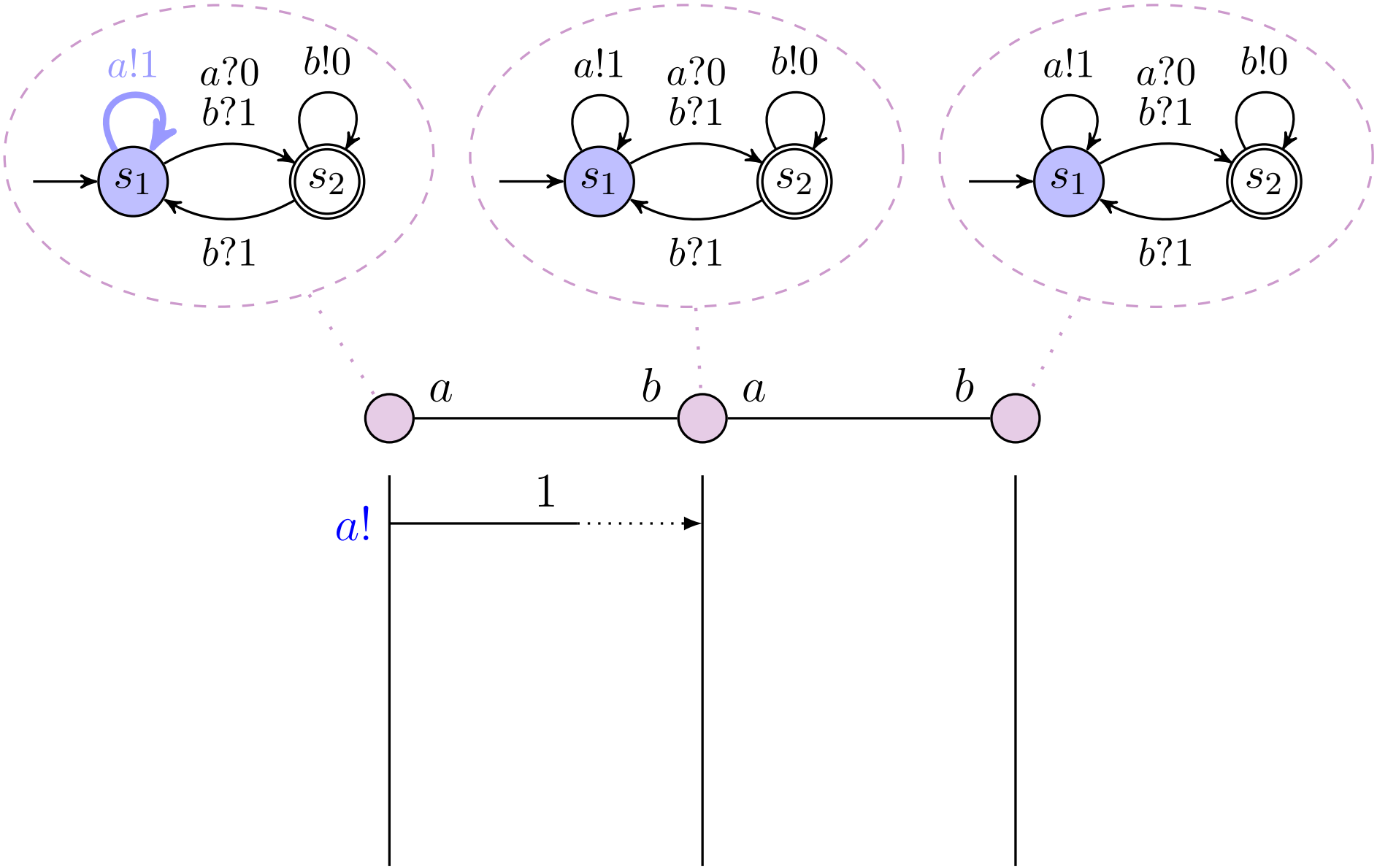

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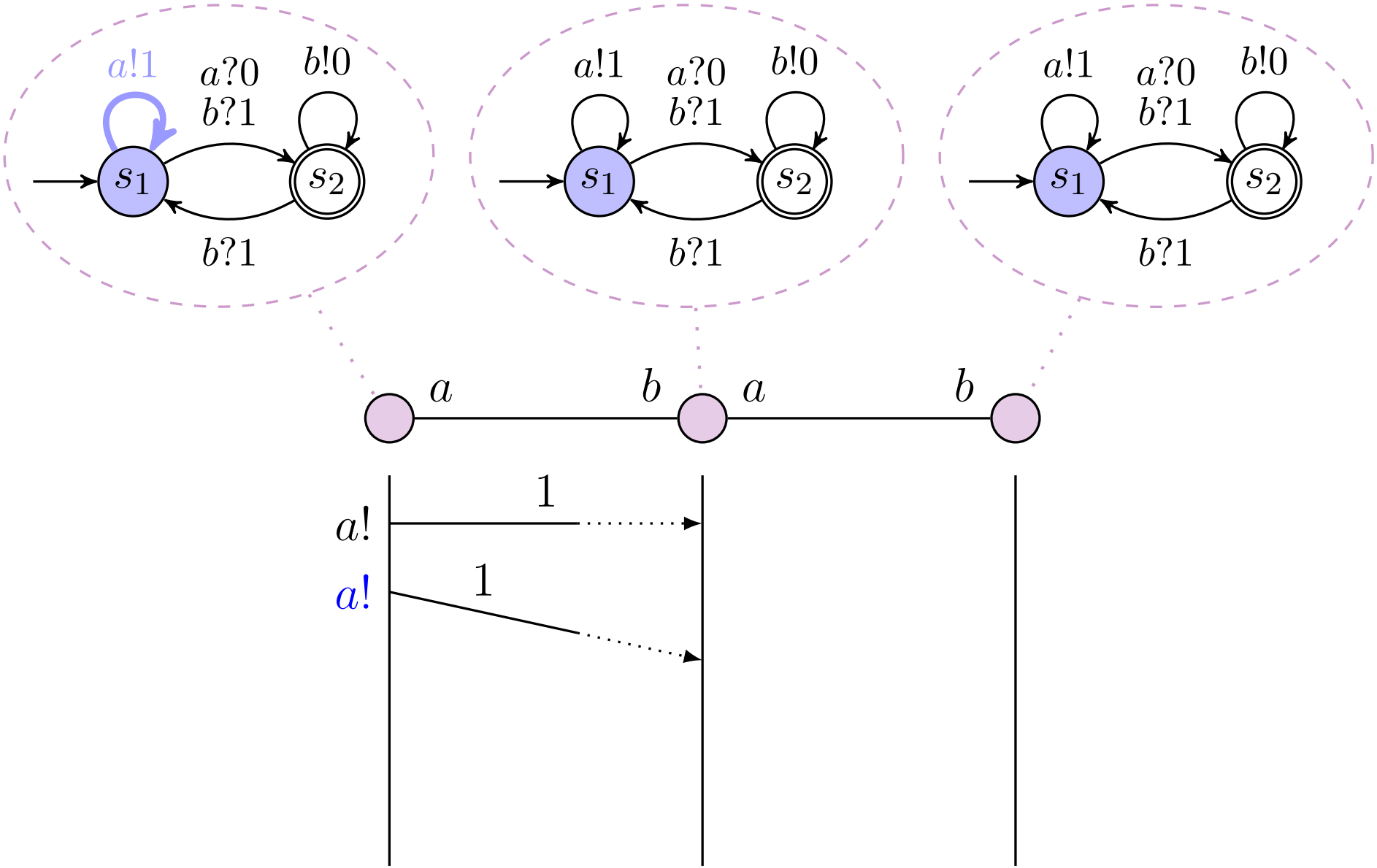

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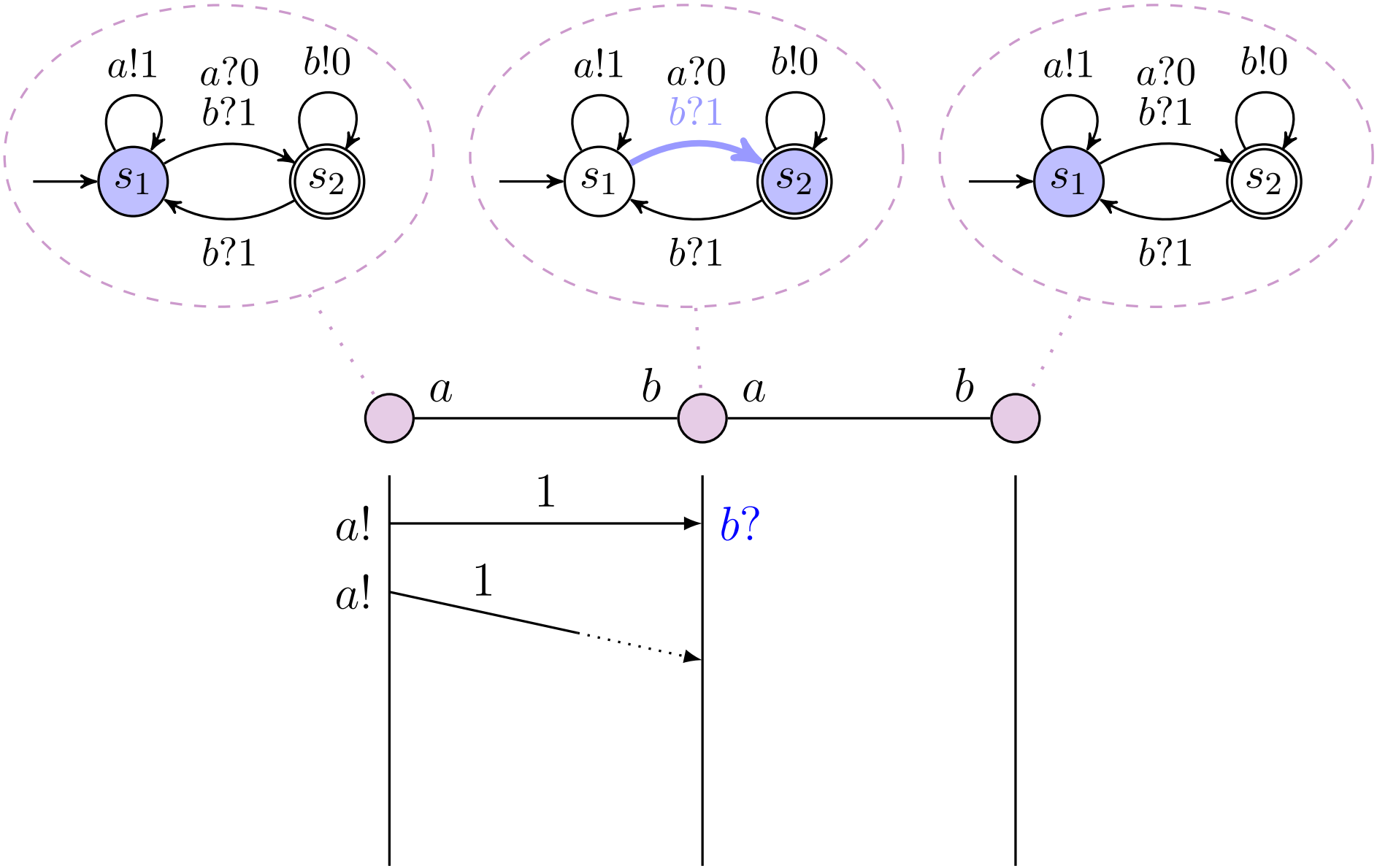
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s_1
s_2
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a!1
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a!
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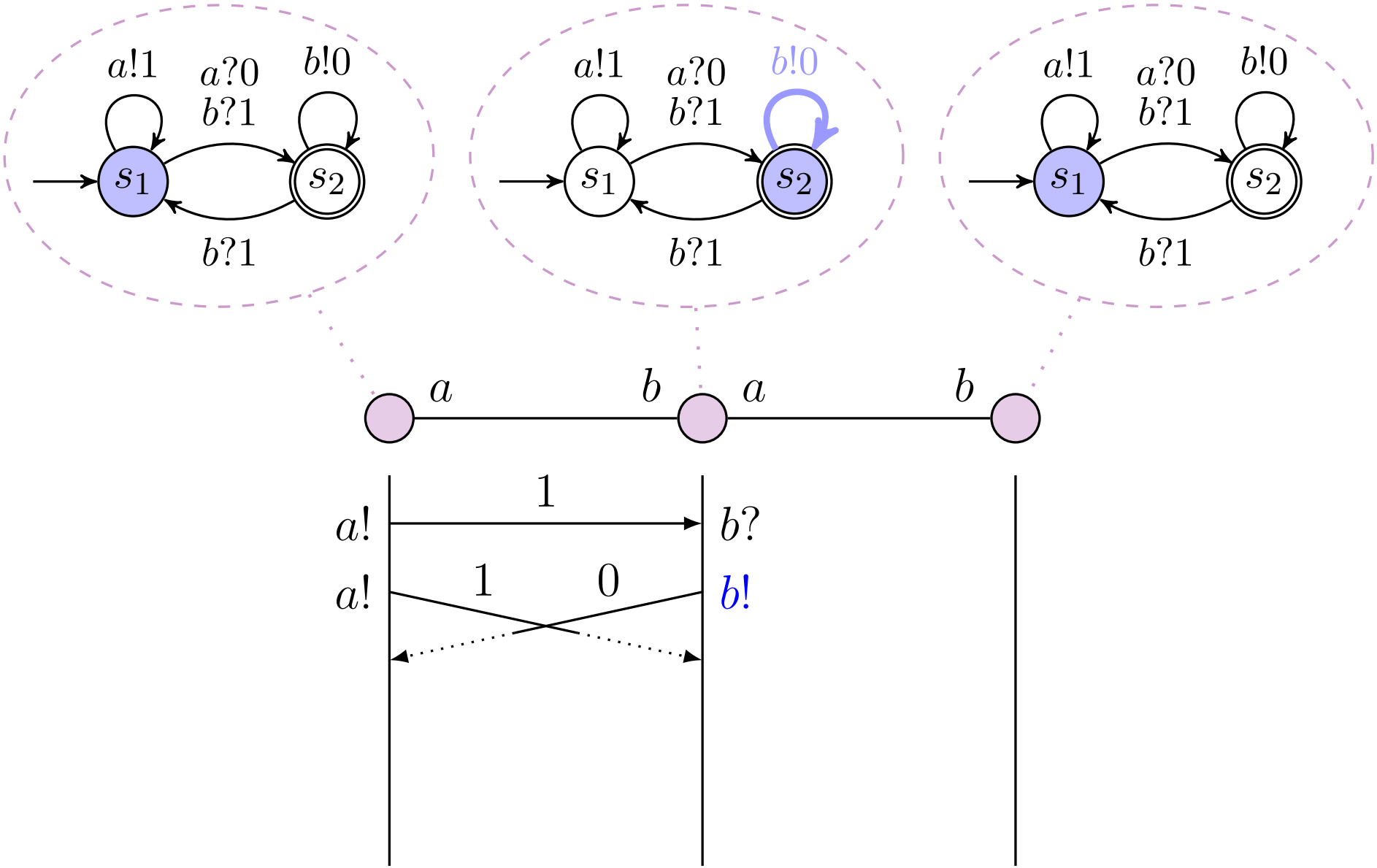
b?
a!
1

Example



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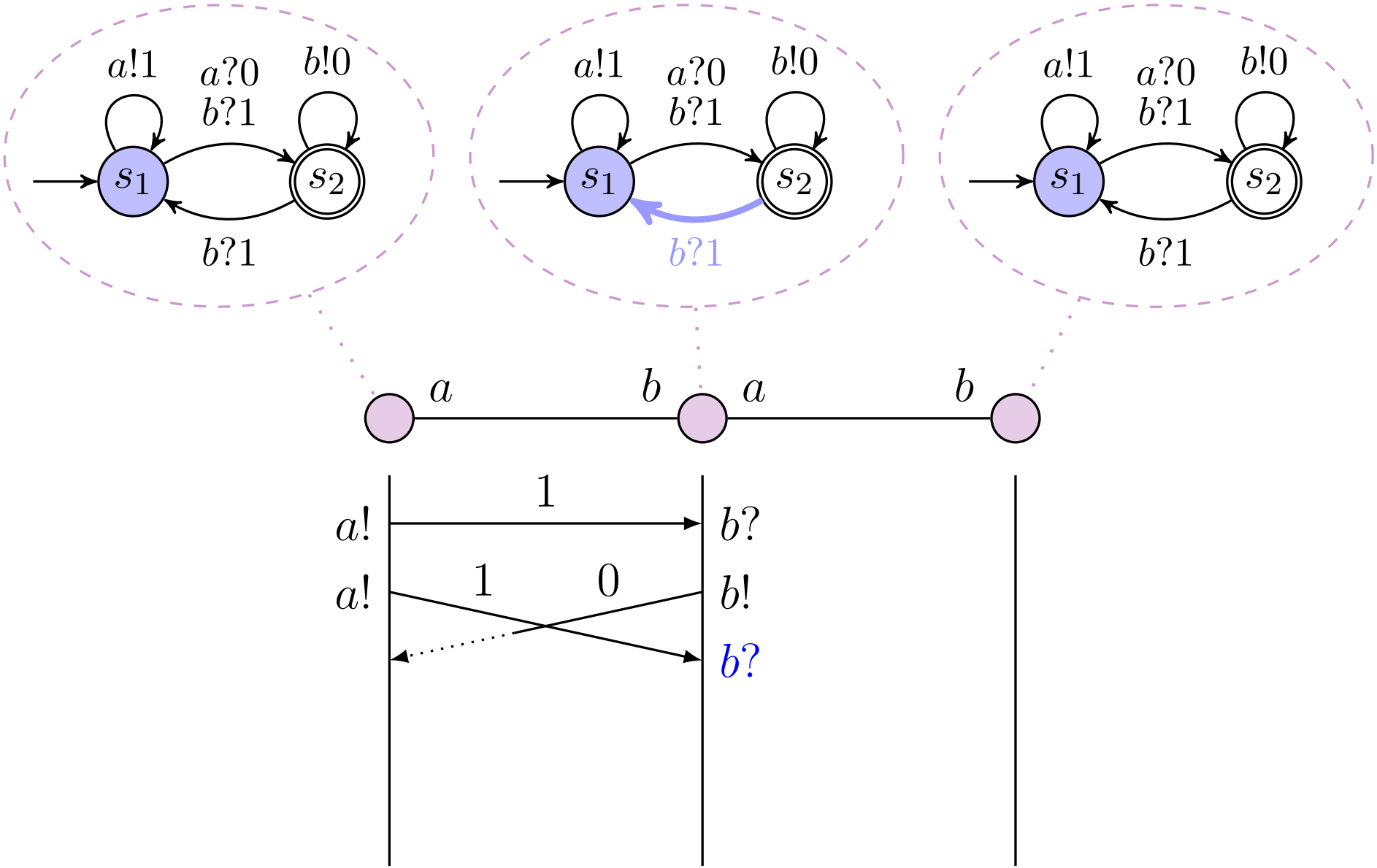
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b?1
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b?1
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b?1
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a
b
a!
a!
1
1
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b?
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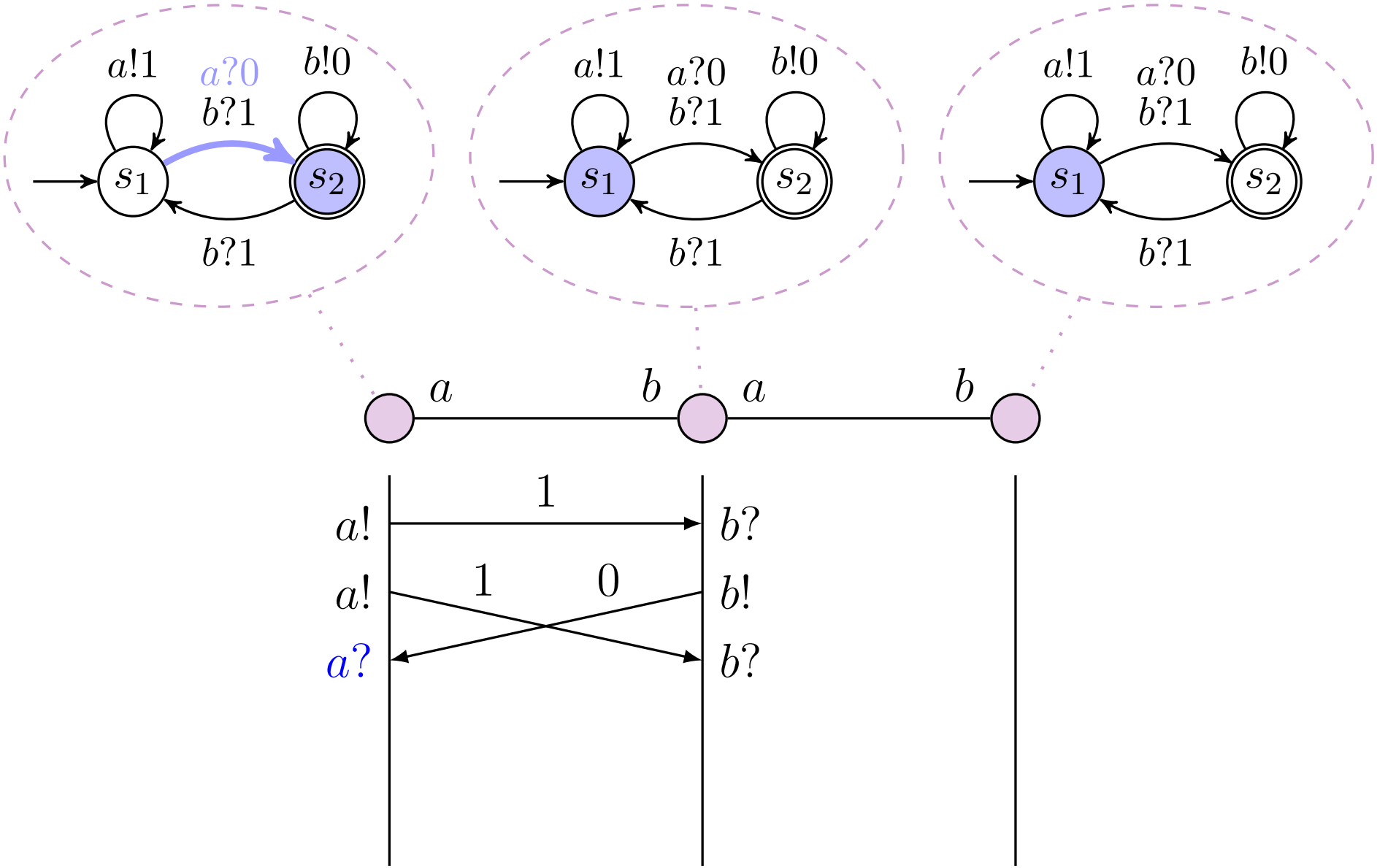
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s2
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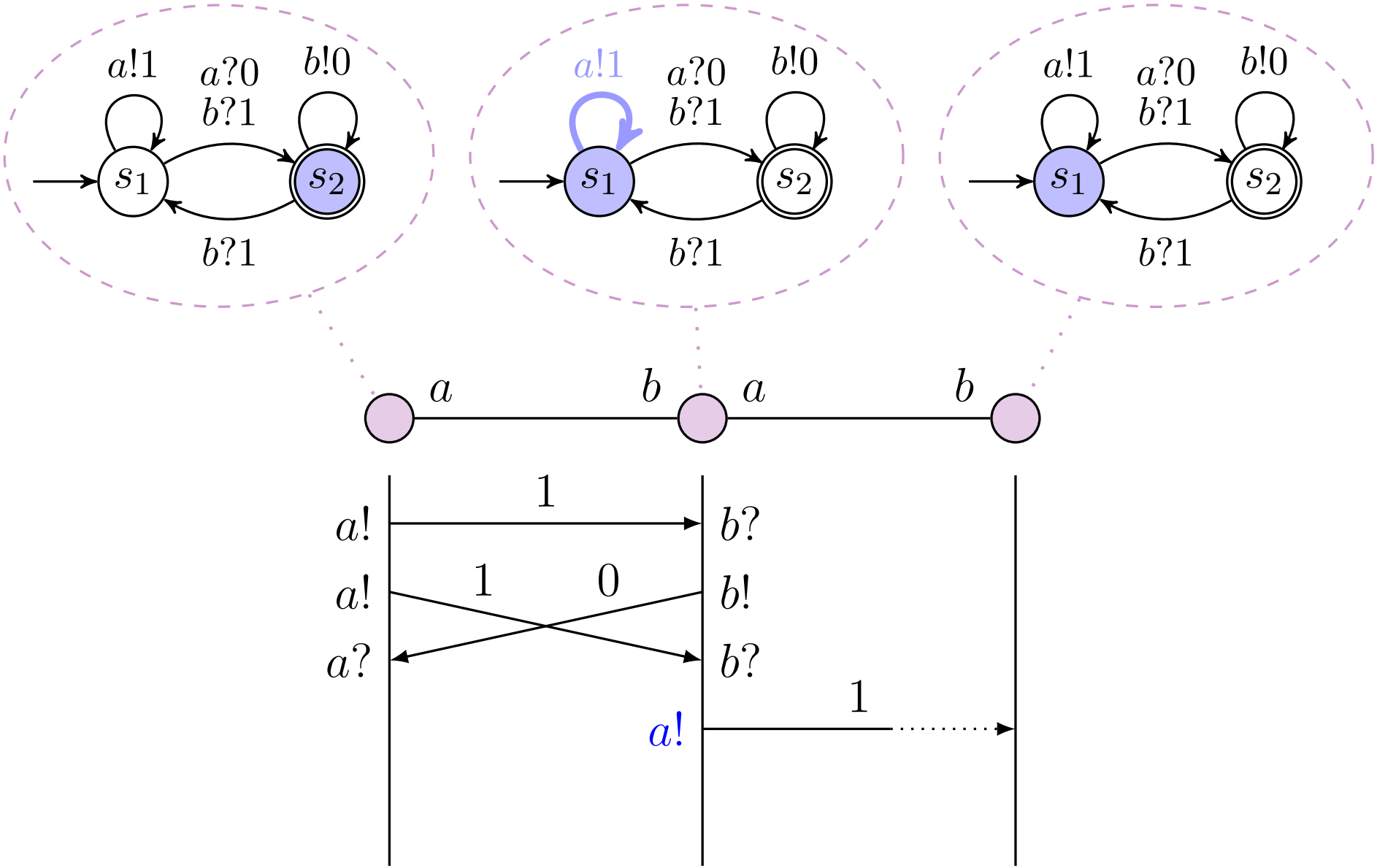
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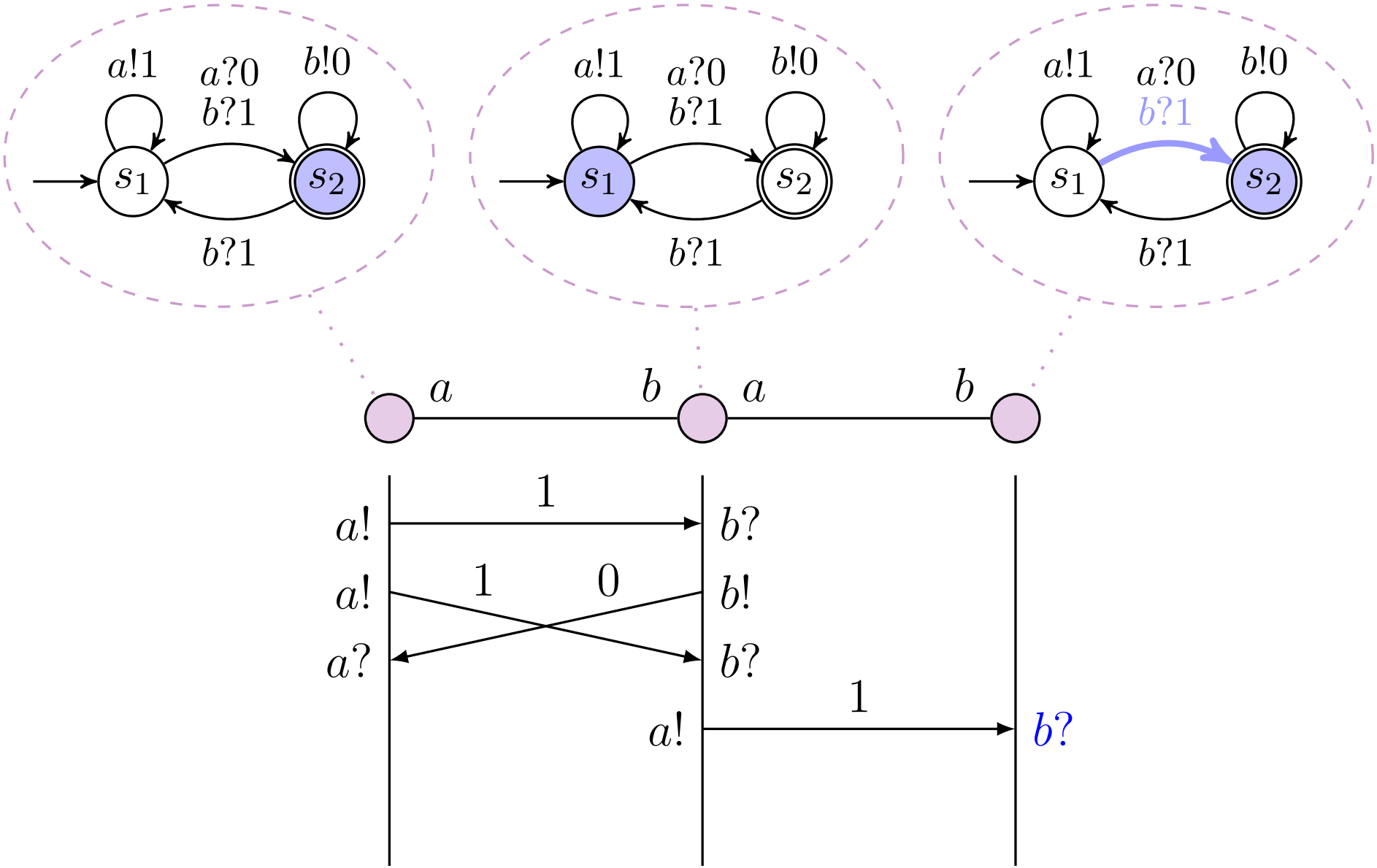
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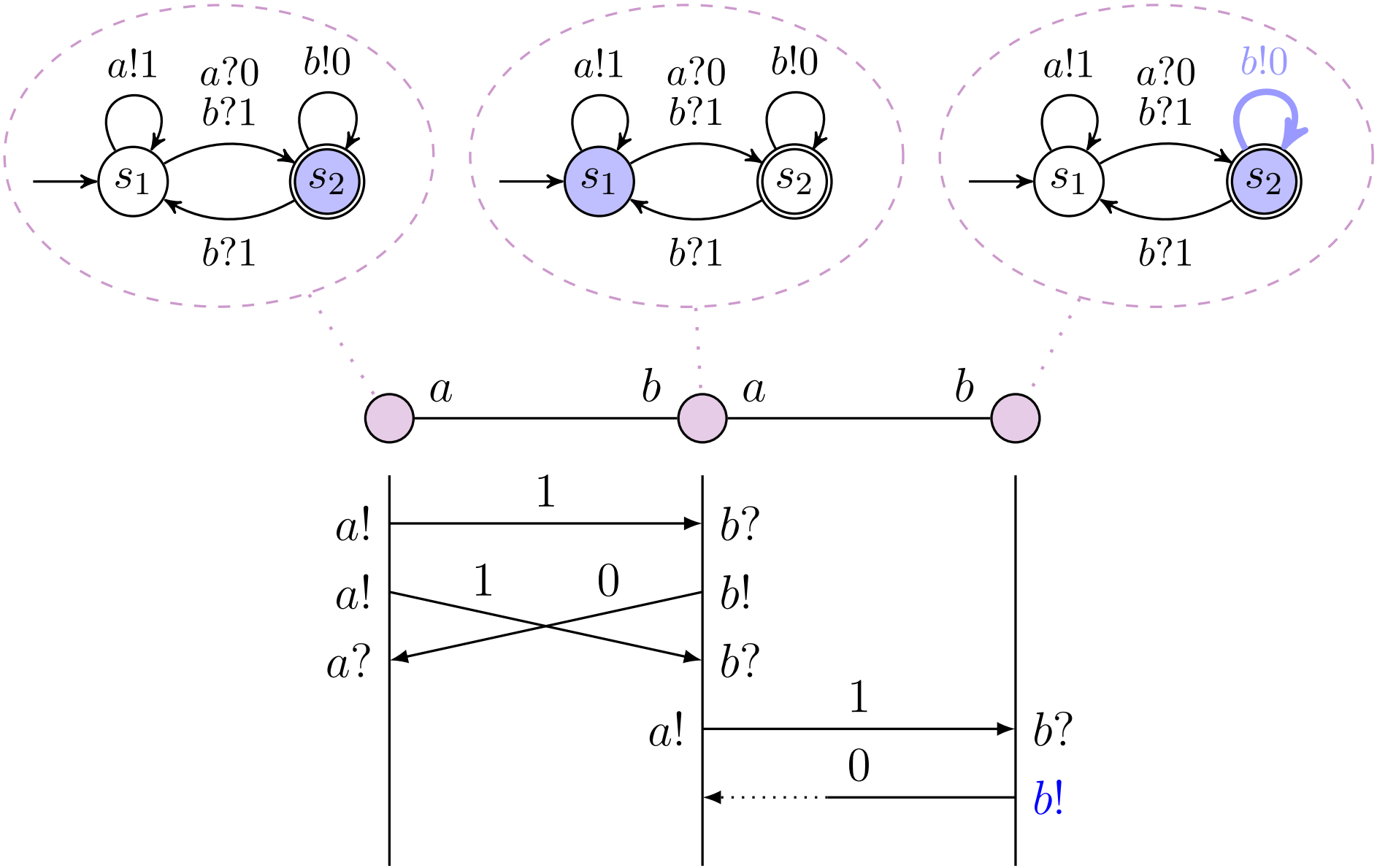

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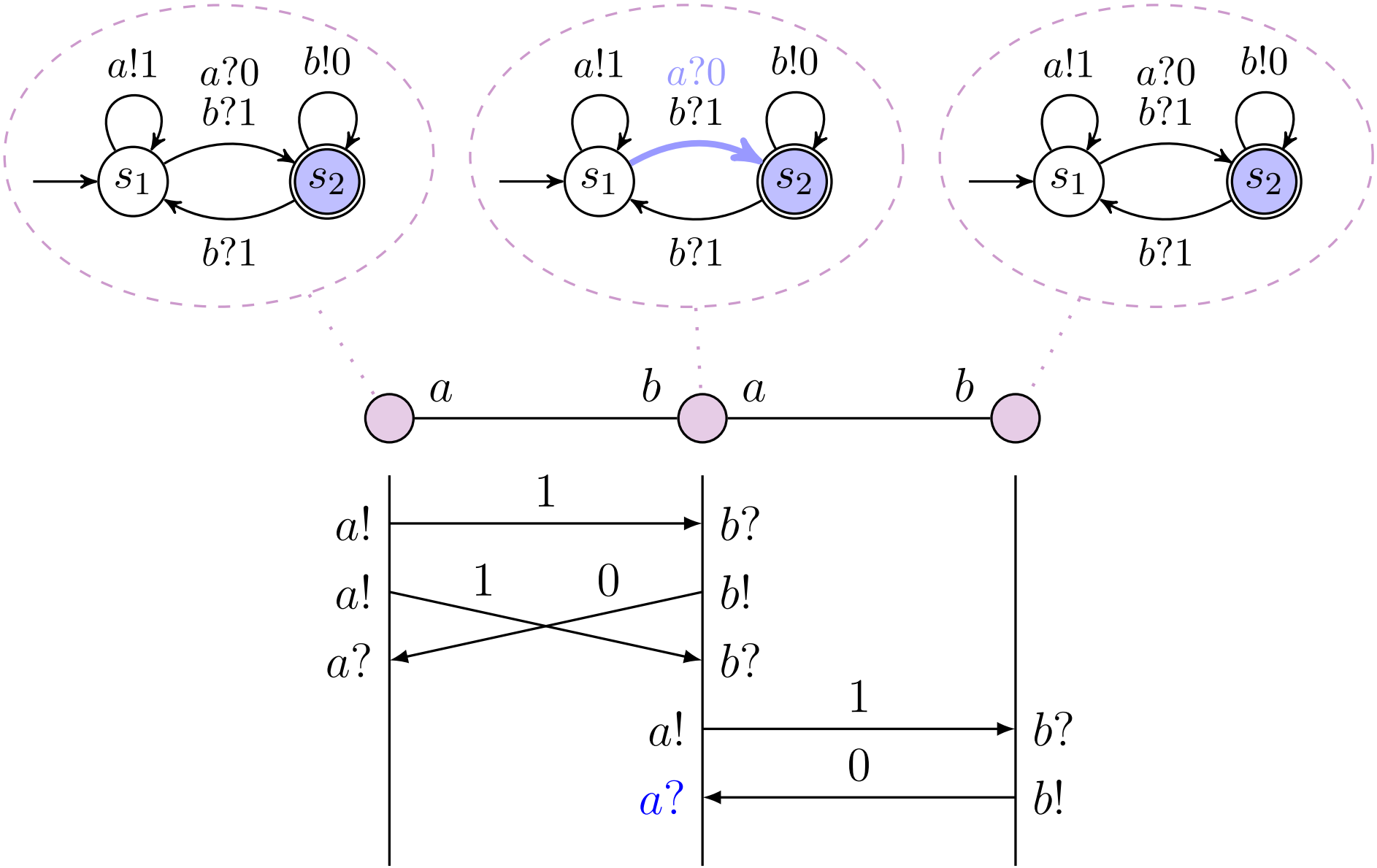

Parameterized verification

Model-checking (fixed topology)

INPUT: $\mathcal{S}$, topology $\mathcal{T}$, specification φ
QUESTION: $\forall M \in L_{\mathcal{T}}(\mathcal{S}),\ M \models \varphi$?

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Model-checking (parameterized case)

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Model-checking (parameterized case)

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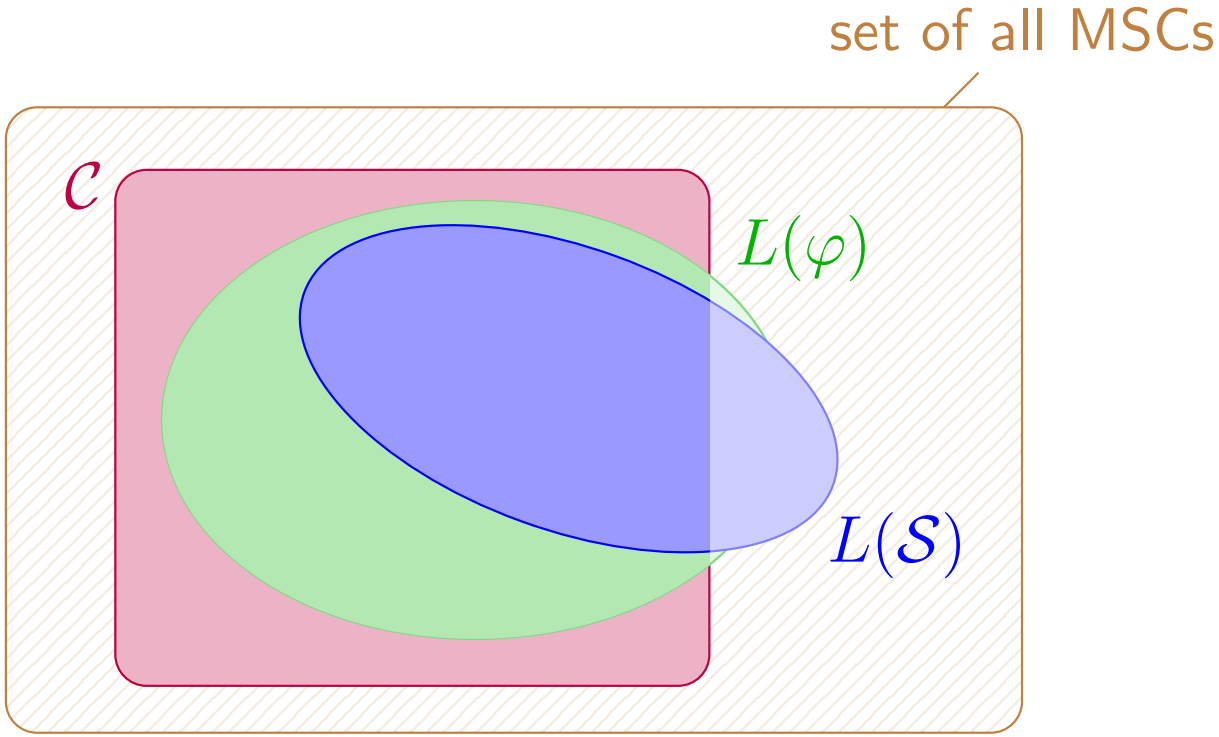
QUESTION: $\forall \mathcal{T},\ \forall M \in L_{\mathcal{T}}(\mathcal{S}),\ M \models \varphi$?

Both are **undecidable**

Undecidability

Topology:	**Fixed**	**Parameterized, arbitrary**	**Parameterized, pipelines**
General case	✗	✗	✗
Bounded number of actions on each process	✓	✗	✓
Synchronous communication	✓	✗	✗

Underapproximate verification



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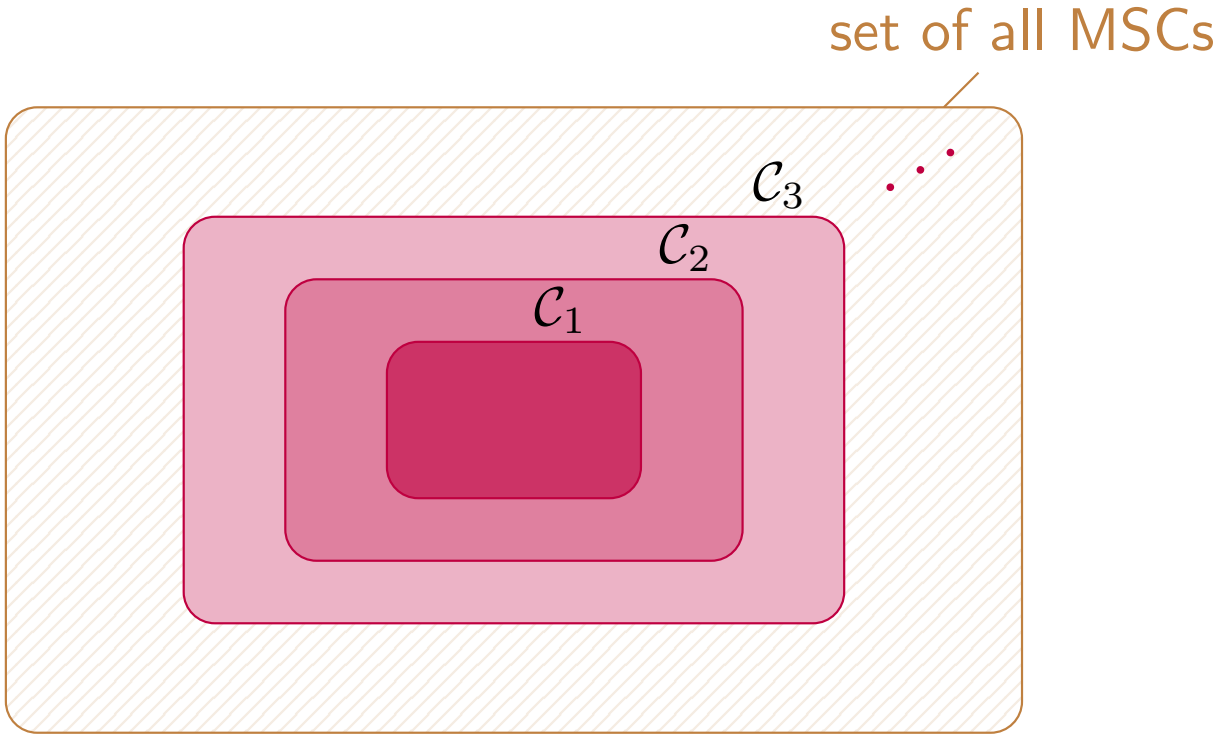
$\mathcal{C}$-Model-Checking: $\mathcal{C} \cap L(\mathcal{S}) \subseteq L(\varphi)?$

Underapproximate verification



Model-Checking: $L(\mathcal{S}) \subseteq L(\varphi)?$

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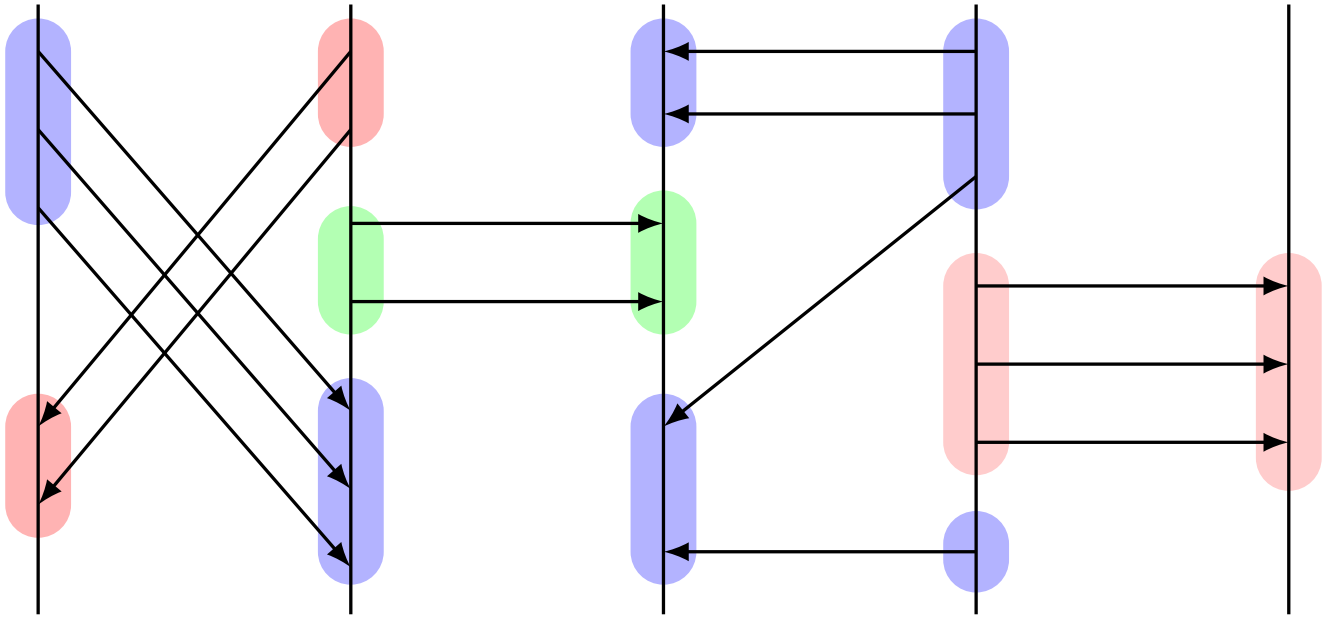
Context-bounded MSCs



A 3-context-bounded MSC

Decidability results

Fixed topology: many decidable underapproximation classes, including:

- k-context-bounded MSCs
- k-existentially-bounded MSCs
- MSCs of tree-width/split-width at most k
- ...

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Parameterized topology:

- Decidable for context-bounded MSCs over pipeline/ring/tree topologies, assuming synchronous communication.

Split-width

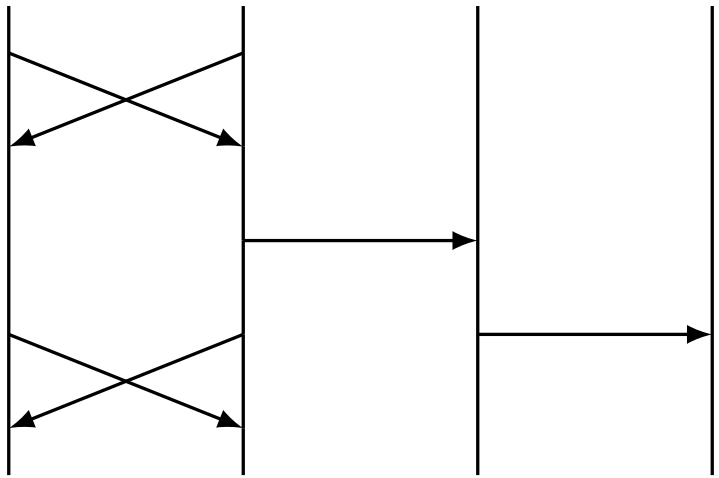
- Apply graph decomposition techniques (tree-width, clique-width...) to MSCs
- We work with split-width, equivalent but better suited for MSCs

Split-width

Two-player game on M (with budget k) :

- Eve cuts process edges, without introducing more than k blocks
- Adam choses a connected component

Example : with budget 4.



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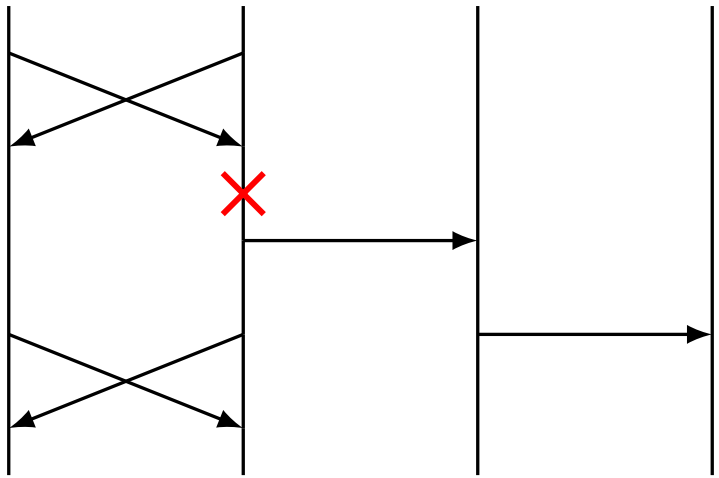

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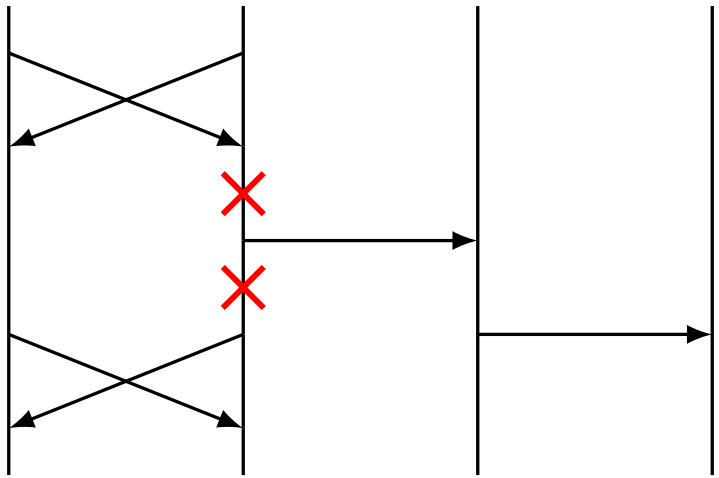

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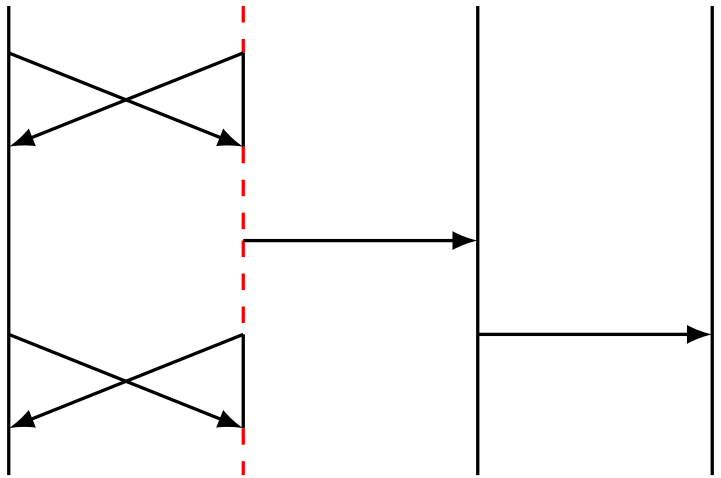

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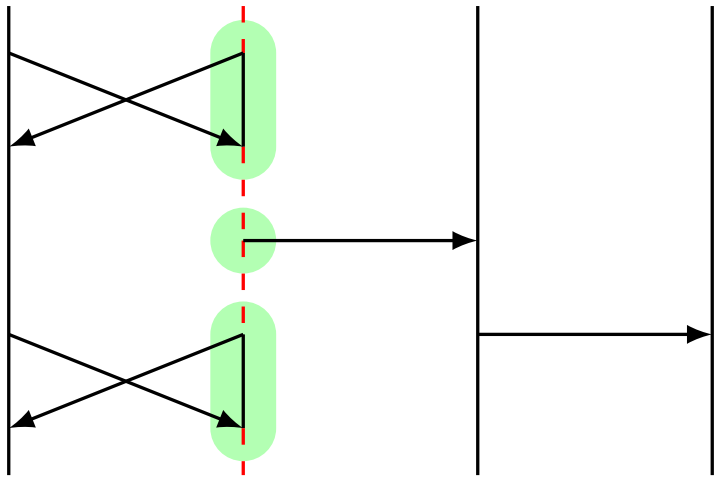

- 1 open process
- 3 blocks

Split-width

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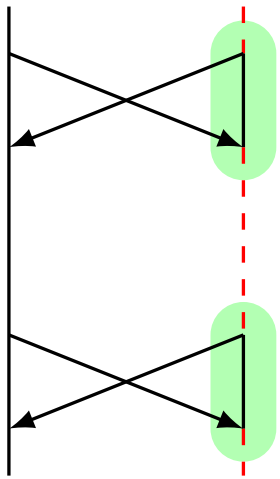
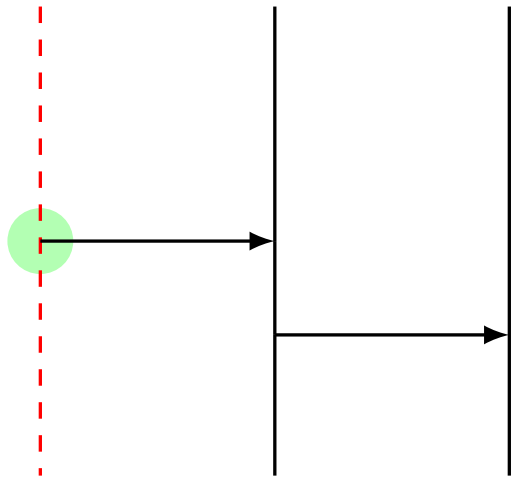


- 1 open process
- 2 blocks



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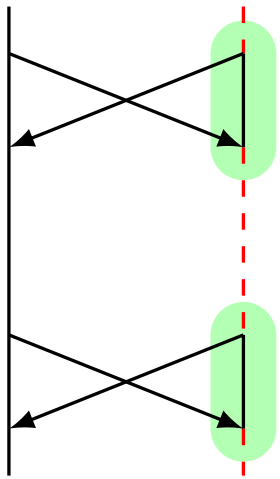

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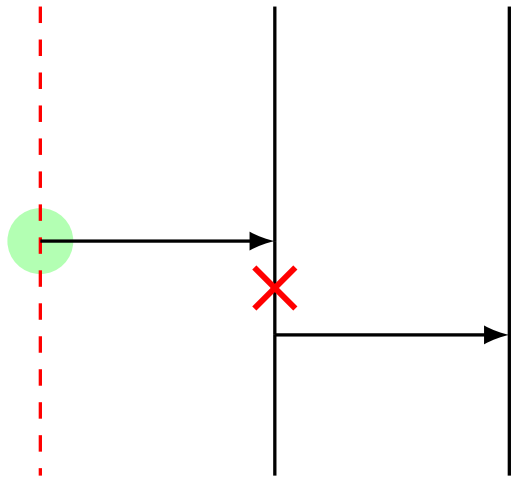

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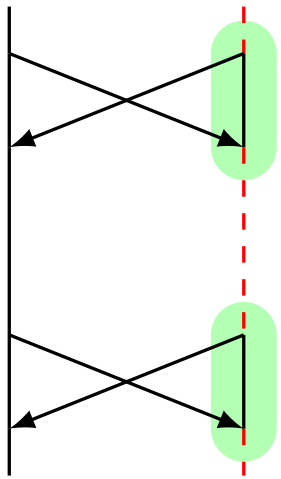

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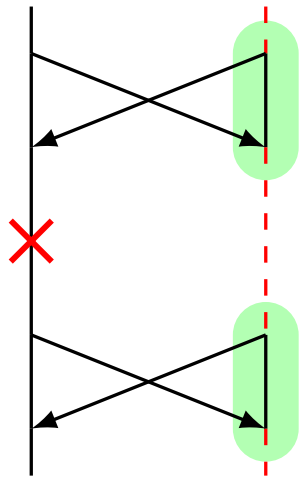

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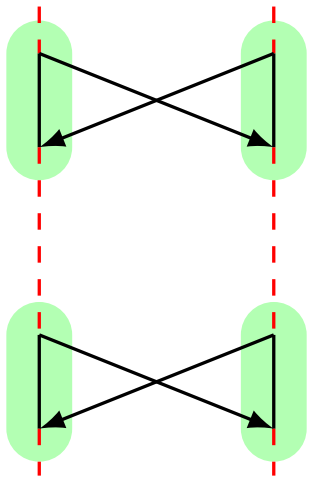

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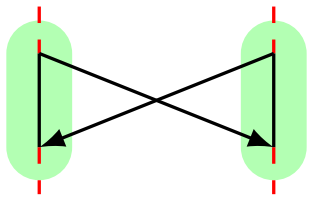

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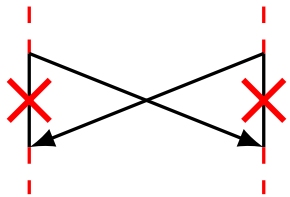

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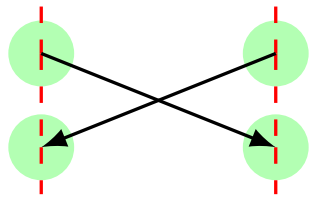

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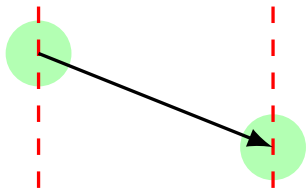

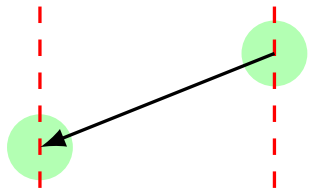

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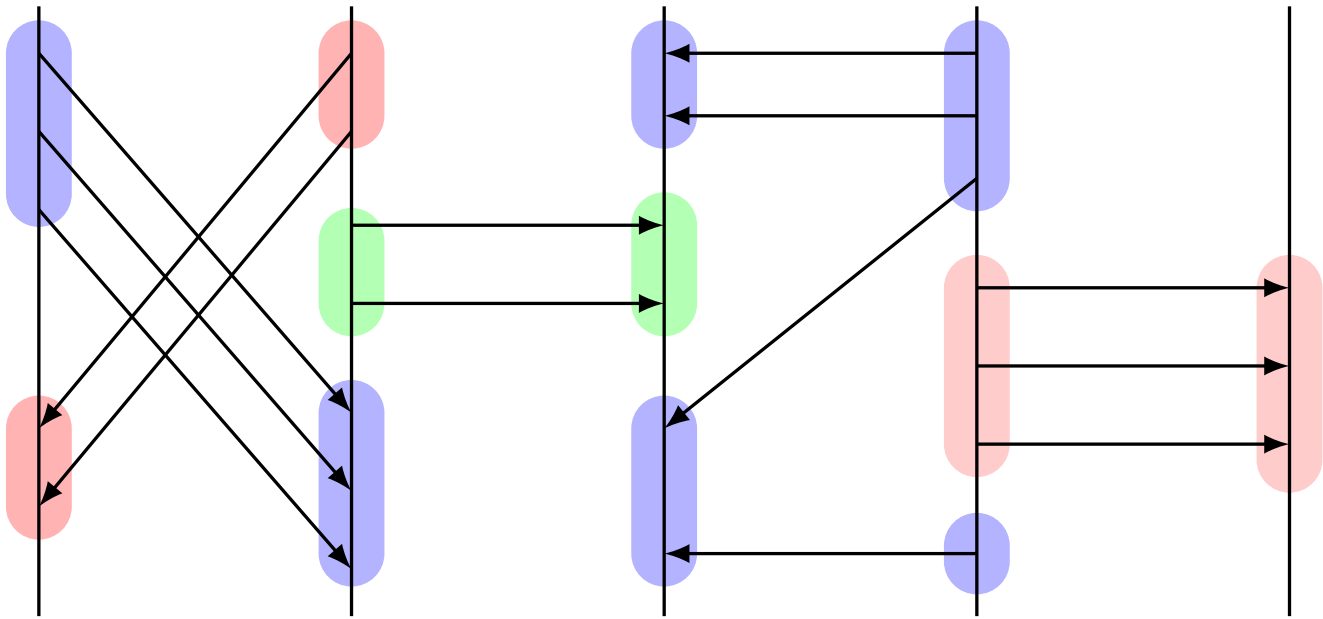
split-width$(M) =$
least k for which Eve has a winning strategy

Example

Lemma

A k-context bounded MSCs over a pipeline topology has split-width at most $2k + 2$.



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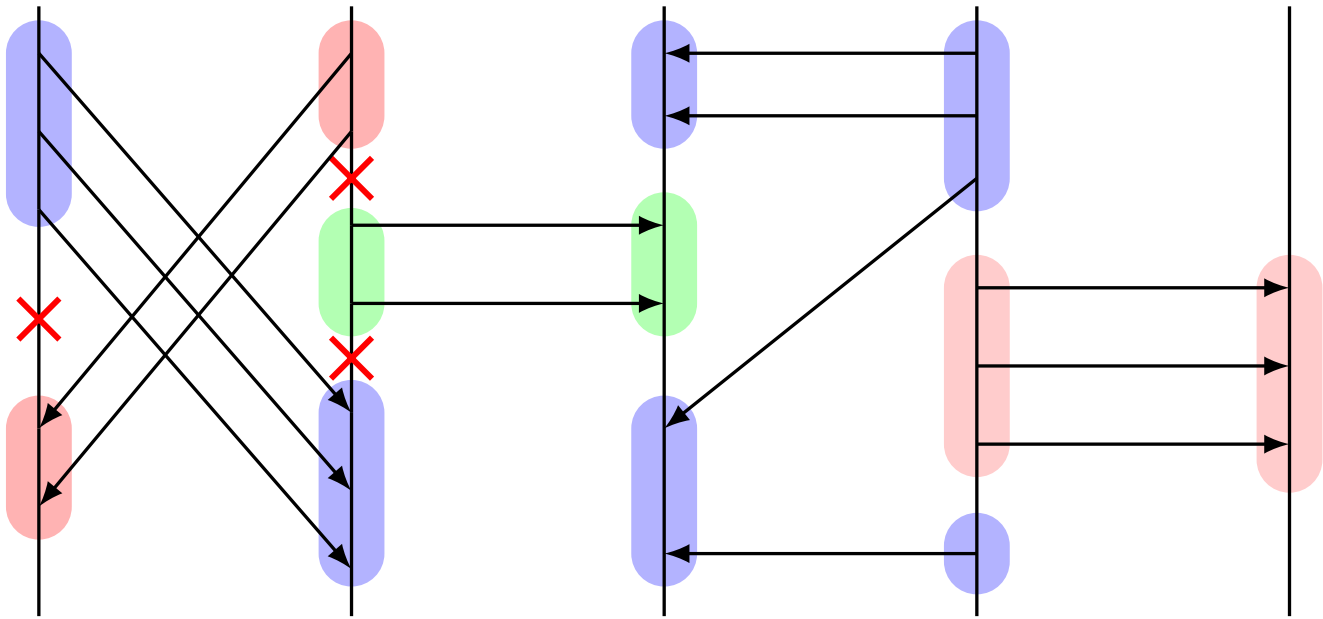

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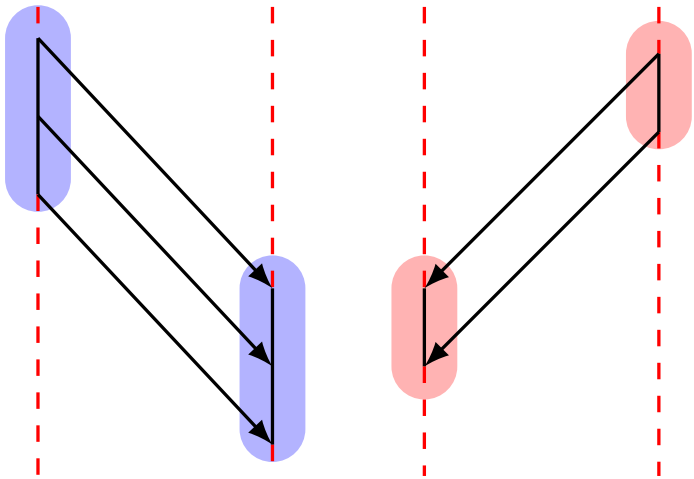

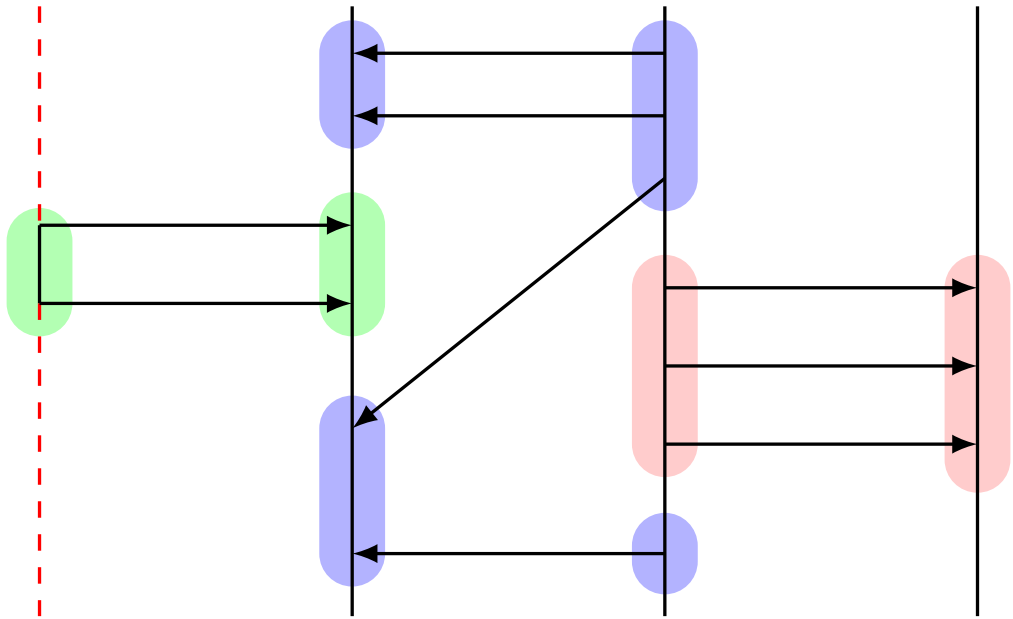

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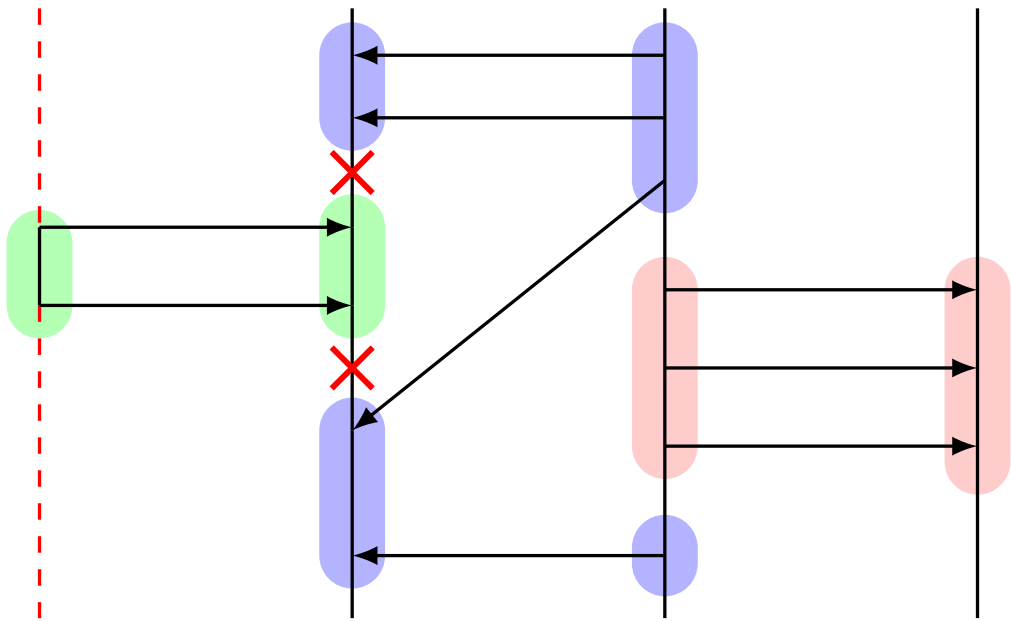

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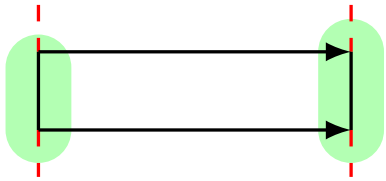

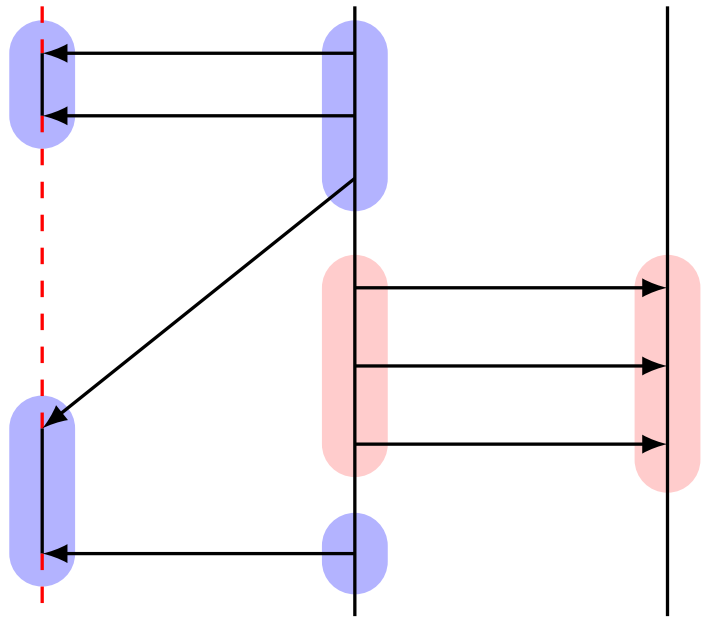

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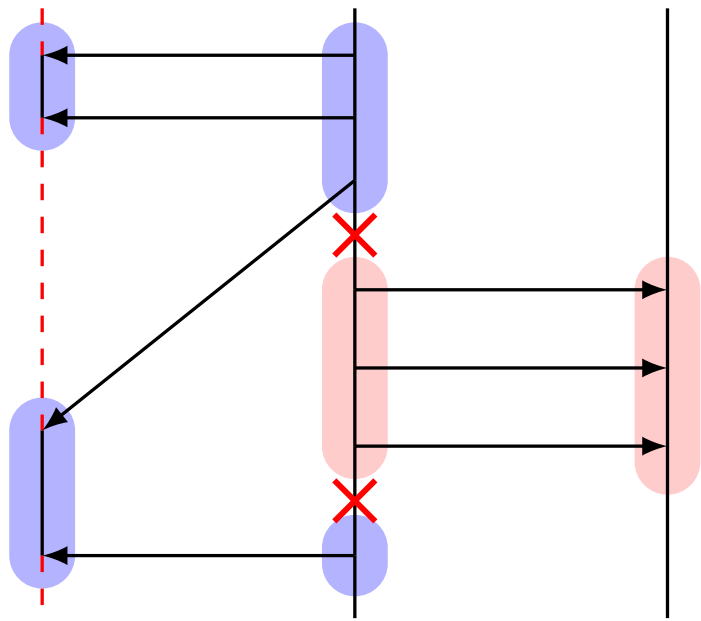

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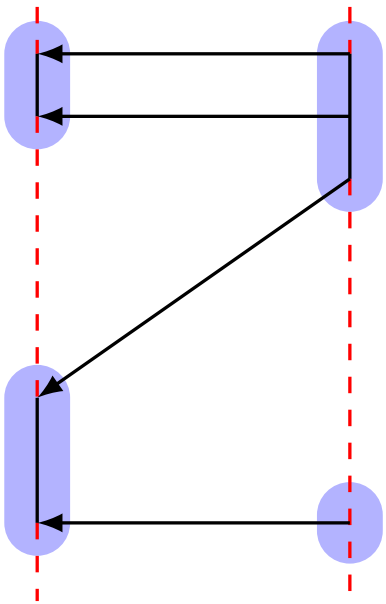

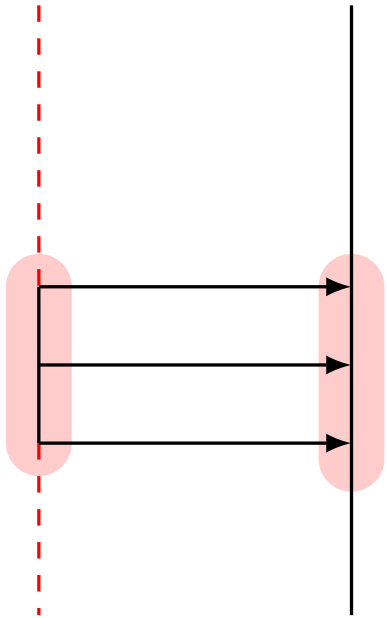

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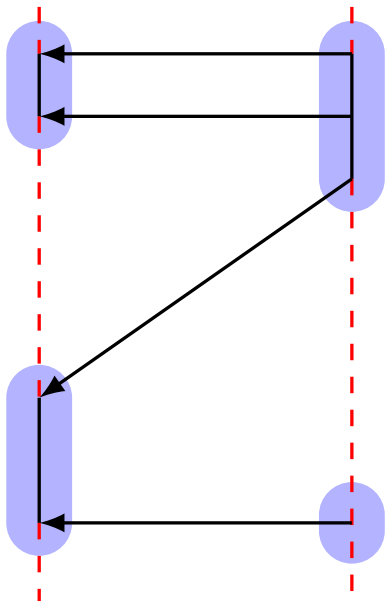

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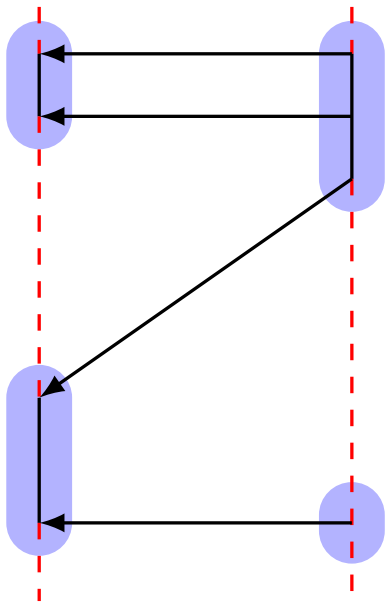

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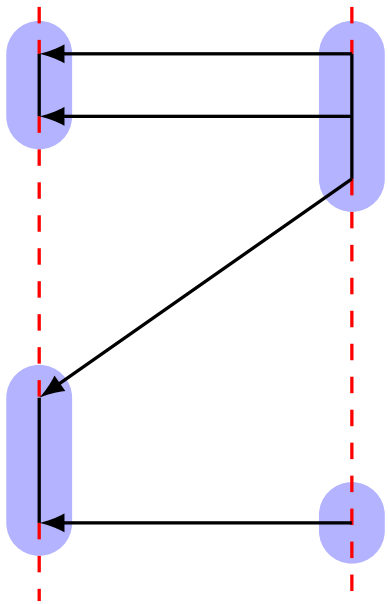
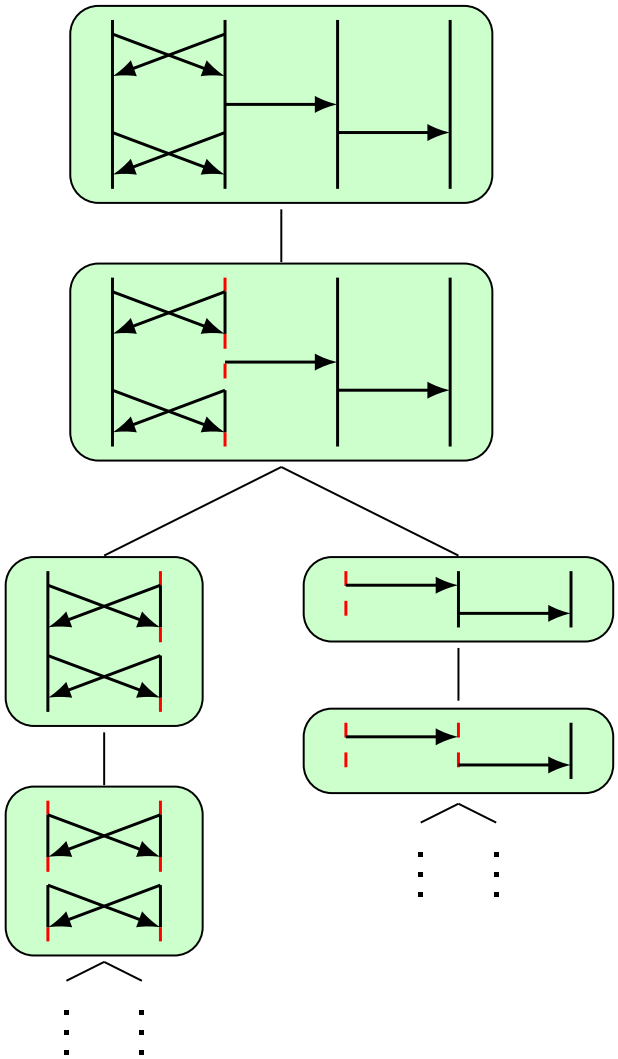


- At most k blocks on each process
- Messages detached with two extra cuts

Interpretation into binary trees : k-DSTs

Eve's strategy = a tree



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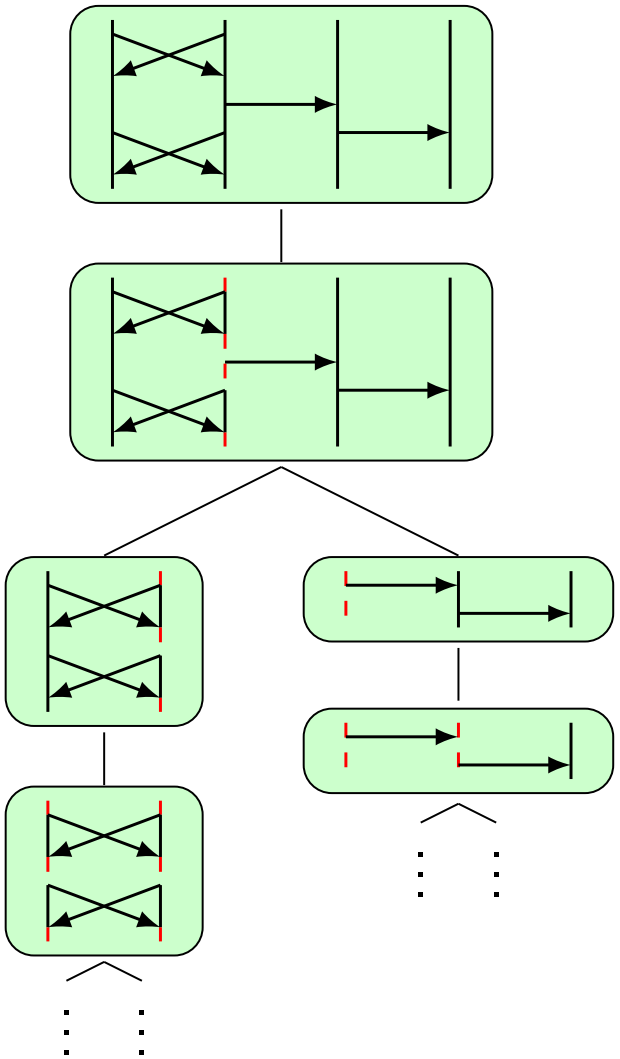

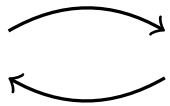

k-DST (Disambiguated Split-Term)

- Tree of identical structure
- Finite alphabet, depending only on k

Interpretation into binary trees : k-DSTs

- k-DST t $\longmapsto$ unique MSC M_t of split-width $\leq k$

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- k-DST t $\longmapsto$ unique MSC M_t of split-width $\leq k$
- MSC of split-width $\leq k$ $\longmapsto$ set of k-DSTs
  (over any topology)

Decision procedure

$\mathcal{C}$-Model-Checking (parameterized)

INPUT: $\mathcal{S}$, φ
QUESTION: $L(\mathcal{S}) \cap \mathcal{C} \subseteq L(\varphi)$?

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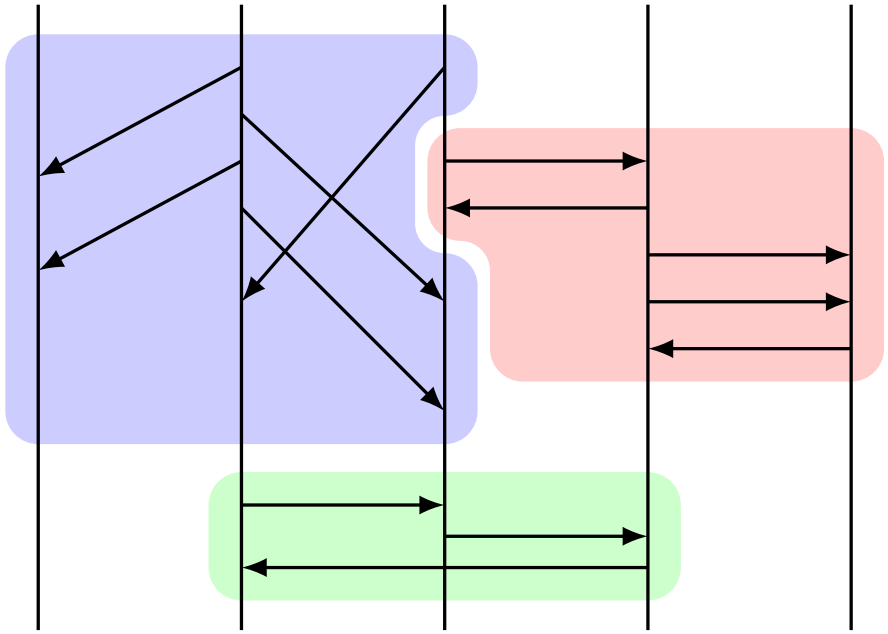
Concrete classes of bounded split-width

- Context-bounded MSCs over topologies of bounded tree-width
- Tile-bounded MSCs over topologies of bounded tree-width
- "Existentially-bounded" MSCs

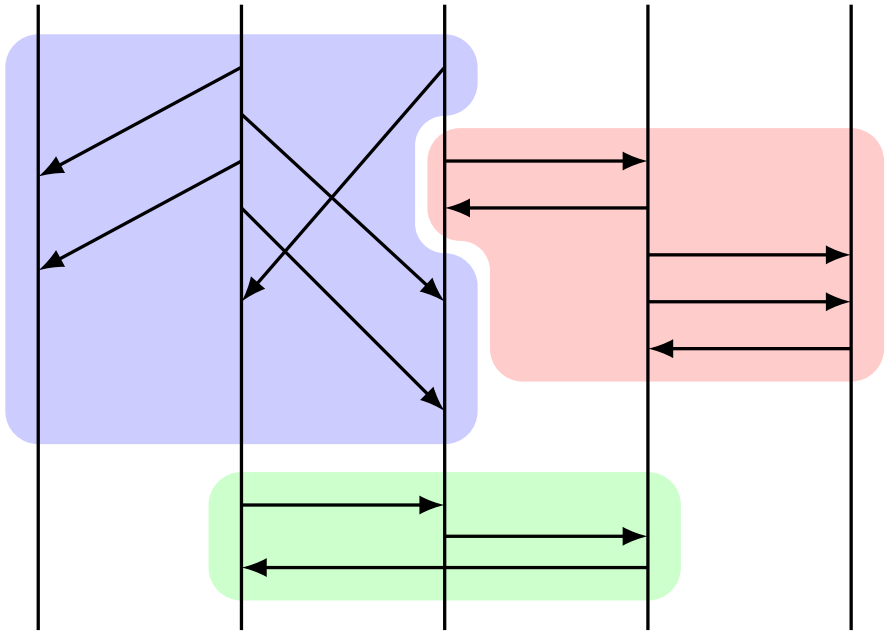
Tile-bounded MSCs



A decomposition for $k = 5, \ell = 3$

- No message edge between two distinct tiles
- A tile has at most k processes
- A tile has split-width at most k
- At most ℓ tiles on each process

Tile-bounded MSCs



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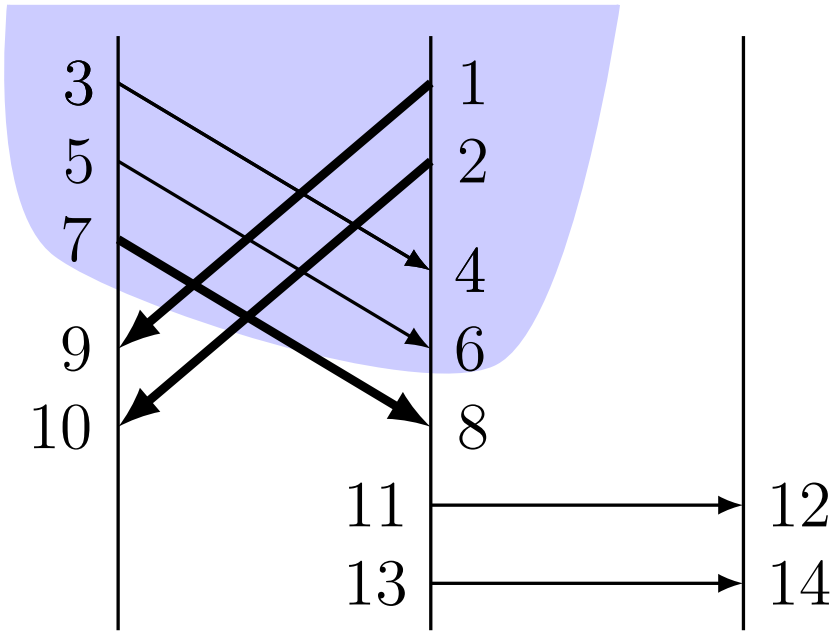
(tile-bounded + topology of bounded tree-width)
$\iff$ (bounded split-width)

Existentially bounded MSCs

Fixed topology

At any time, $\leq k$ pending messages
(for some linearization).



existentially 3-bounded

Existentially bounded MSCs

Parameterized case

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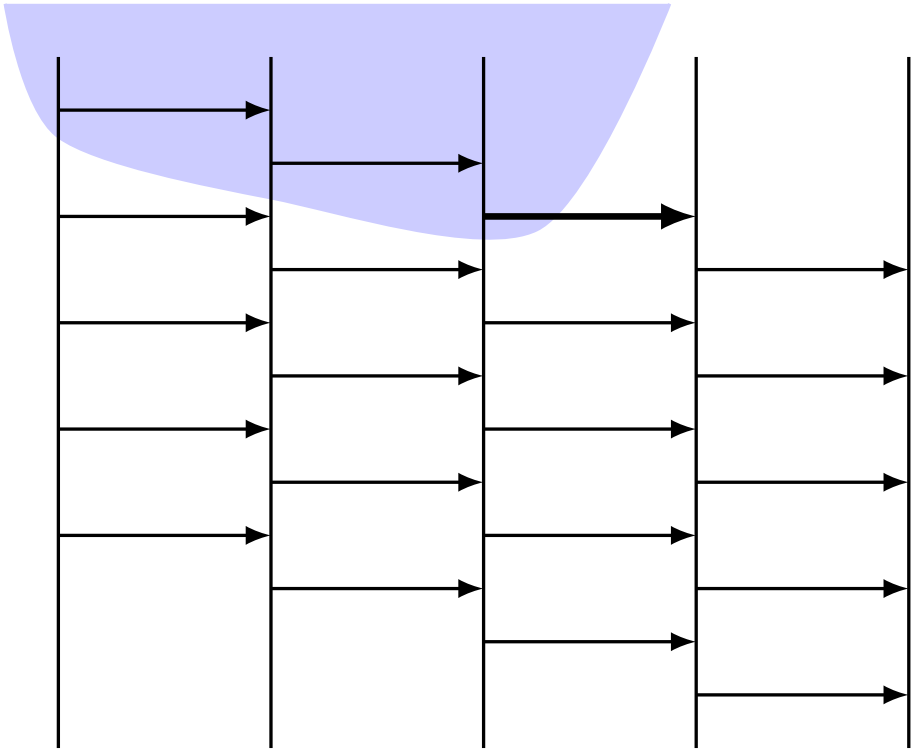

- existentially 1-bounded
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Existentially bounded MSCs

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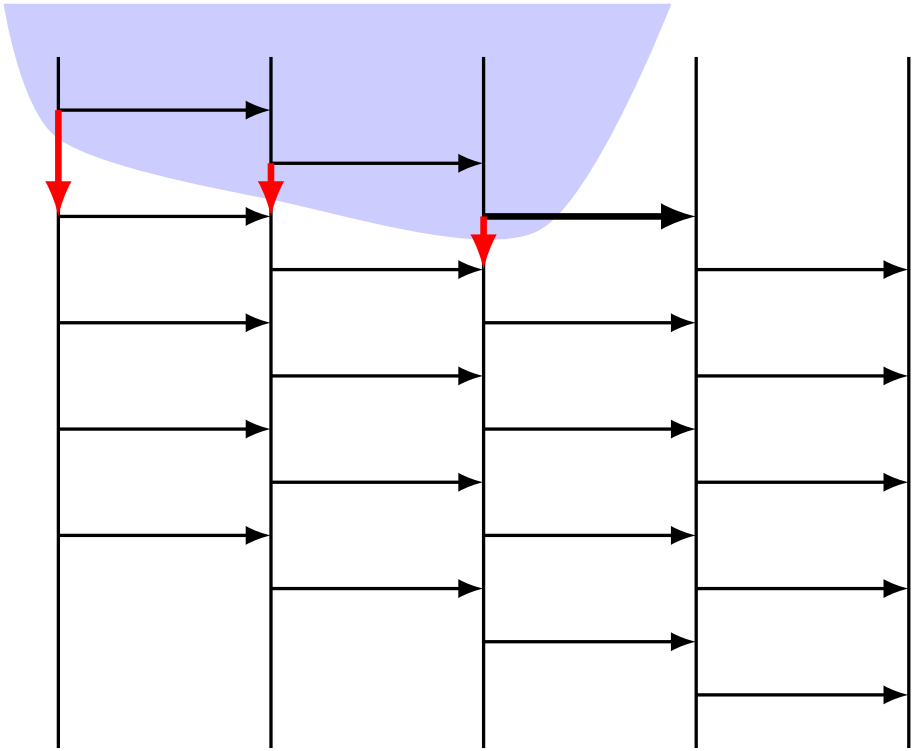

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Results

	$\mathcal{C}_k = \mathsf{EB}_k$	$\mathcal{C}_{(k,h)} = \mathsf{CB}_{k,h}$	$\mathcal{C}_k = \mathsf{SW}_k$	$\mathcal{C}_{(k,\ell,h)} = \mathsf{TB}_{k,\ell,h}$
$\mathcal{C}$-Reachability	Pspace-c	Exptime-c		
$\mathcal{C}$-Emptiness				
$\mathcal{C}$-CPDL-Sat/MC				
$\mathcal{C}$-ICPDL-Sat/MC	Expspace-c	2-Exptime-c		
$\mathcal{C}$-MSO-Sat/MC	Non elementary			

Polynomial in $|\mathcal{S}|$.

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- Other classes of bounded split-width?
- Data, process identifiers?
- Dynamic creation of processes?