Expressivity of first-order logic, star-free propositional dynamic logic and communicating automata

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PhD Defense – November 27, 2020

LSV, ENS Paris-Saclay, Université Paris-Saclay

Introduction

Verification

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Requirements

System

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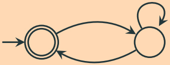
Formal specification φ

$\neg(\exists x.\mathrm{error}(x))$

System

System model $\mathcal{A}$



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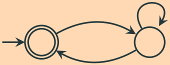

Model-checking

$\mathcal{A} \models \varphi$?

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$\xrightarrow{\text{Synthesis}}$

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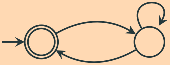

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Comparisons with automata

Given a specification, can we always construct an equivalent automaton?

Some expressivity results over finite words

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Over words, FO$[<]$ defines the same class of languages as

- LTL [Kamp 1968]
- FO$^3[<]$ [Kamp 1968]
- Star-free expressions [McNaughton, Papert 1971]
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What about more complex structures?

Outline

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Logic-automata connections

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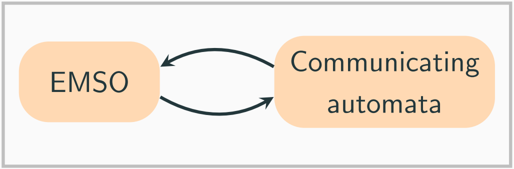

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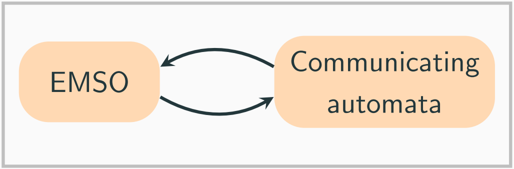

The 3-variable property

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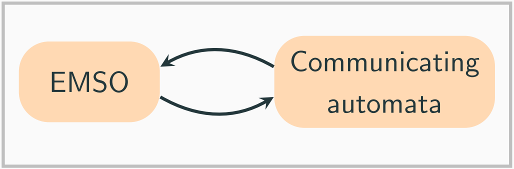

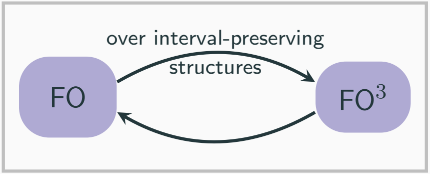

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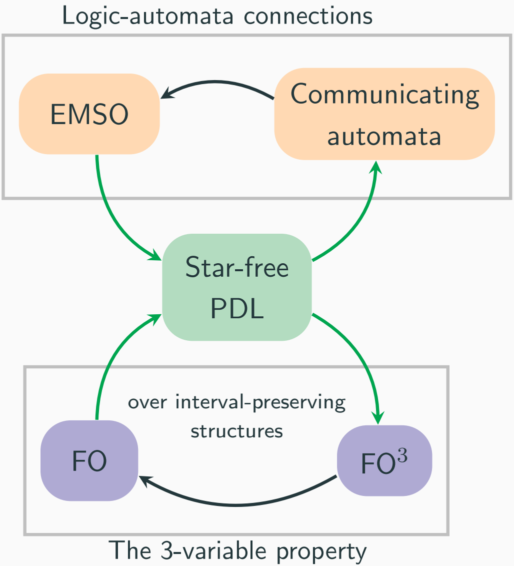
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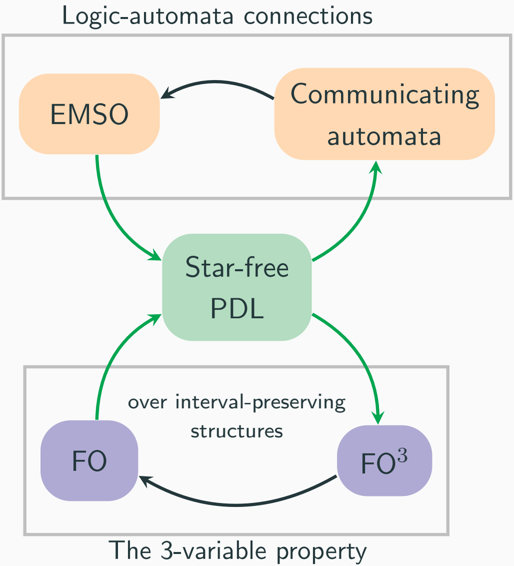

Part I

Star-free PDL

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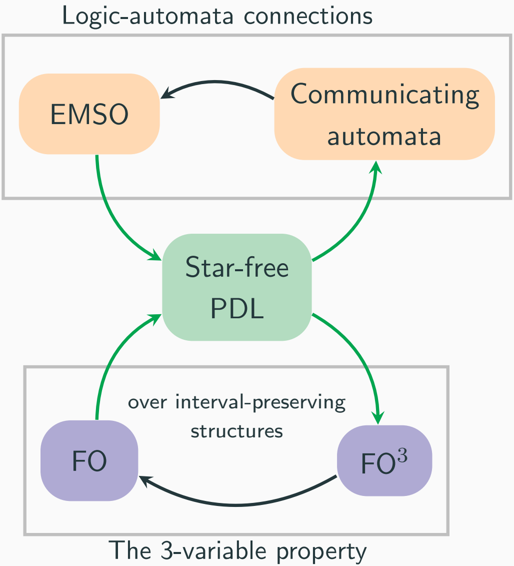

Part II

A Büchi theorem for message-passing systems

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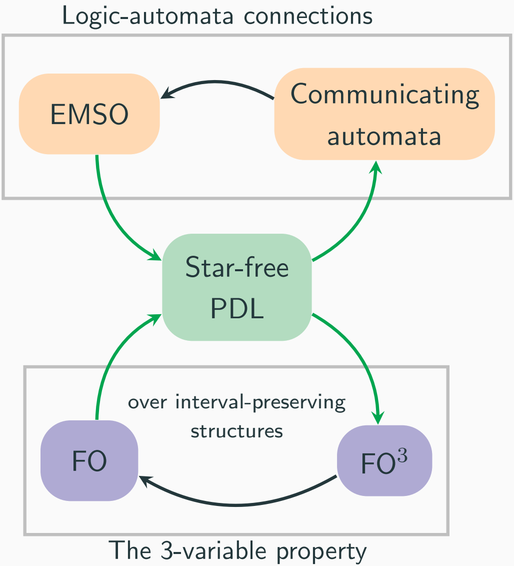

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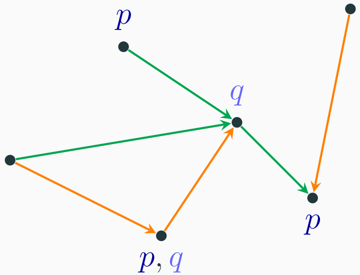
Part III

Sufficient conditions for the 3-variable property

Star-free PDL

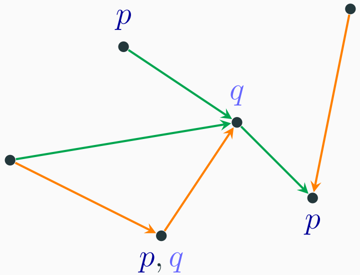
Examples



- Binary relations $\rightarrow$ and $\rightarrow$
- Unary predicates p, q

Examples

- $p \vee \langle \textcolor{orange}{\rightarrow} \rangle\, q$



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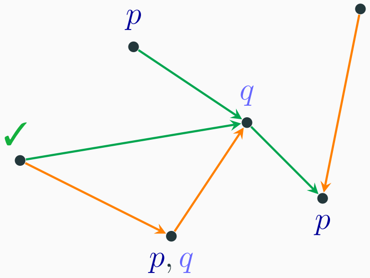

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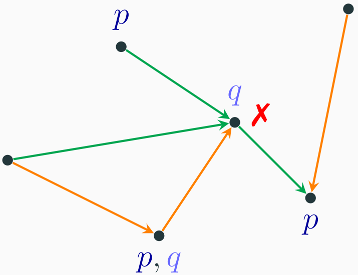

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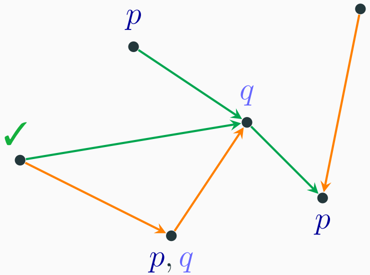

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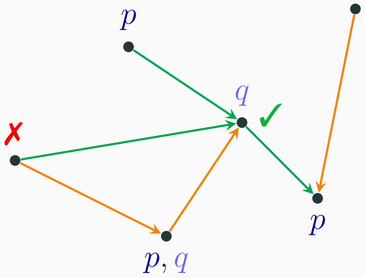

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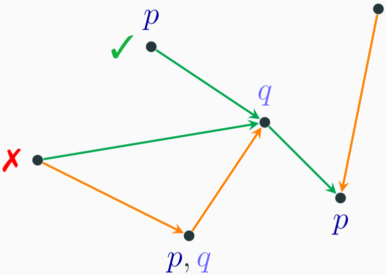

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Example: MTL modalities

Over structures with

- domain $\mathbb{R}$
- binary relations $<$ and $+q$ for every $q \in \mathbb{Q}_+$
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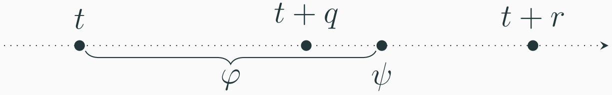

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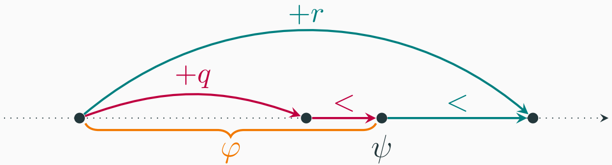
$$\varphi \,\mathsf{U}_{(q,r)}\, \psi \equiv \left\langle (+q \cdot <) \cap (+r \cdot <^{-1}) \cap (< \cdot \{\neg\varphi\}? \cdot <)^{\mathsf{c}} \right\rangle \psi$$



Syntax of Star-free Propositional Dynamic Logic

State formulas:

$$\varphi ::= p \mid \varphi \vee \varphi \mid \neg\varphi \mid \langle\pi\rangle\,\varphi$$

Path formulas:

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- Star-free regular expressions
- The calculus of relations

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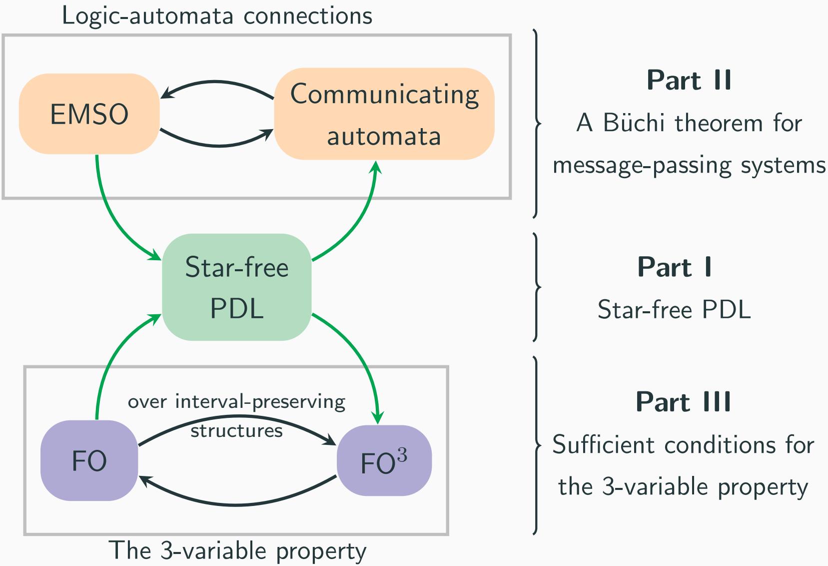
Theorem [Tarski, Givant 1987] (calculus of relations)

$\mathsf{PDL}_{\mathsf{sf}}$ and FO^3 are expressively equivalent.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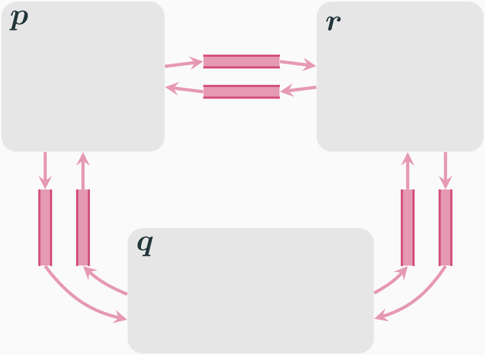
A Büchi theorem for message-passing systems

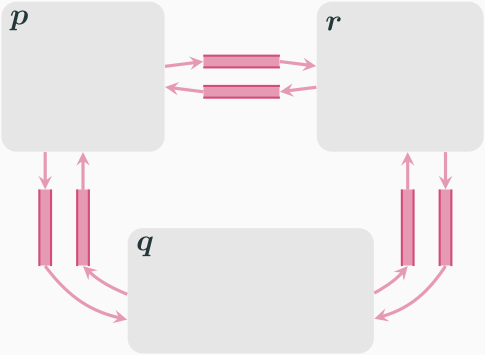
Communicating finite-state machines (CFMs)[1]



[1] [Brand, Zafiropulo 1983]

Communicating finite-state machines (CFMs)[1]

Fixed, finite set of processes, e.g. $\{p, q, r\}$



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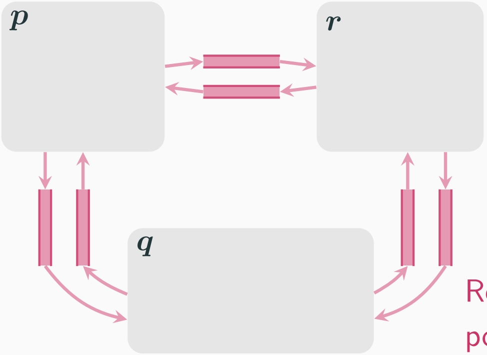

Reliable unbounded point-to-point FIFO channels

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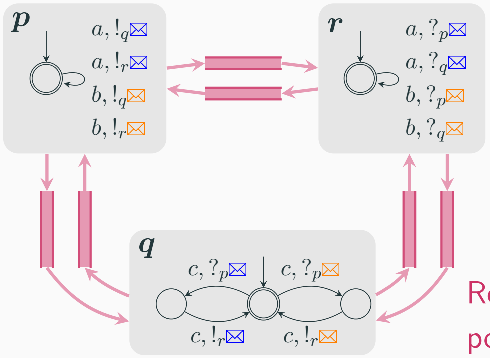

One finite automaton for each process

- Finite input alphabet, e.g. $\Sigma = \{a, b, c\}$
- Sends/receives from a finite message alphabet, e.g. {✉, ✉}

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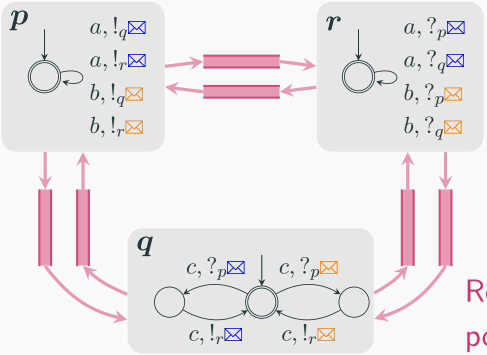

Global acceptance condition

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Message Sequence Charts (MSC)

The language of a CFM is a set of MSCs.

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Partial order consisting of

- One sequence of events for each process

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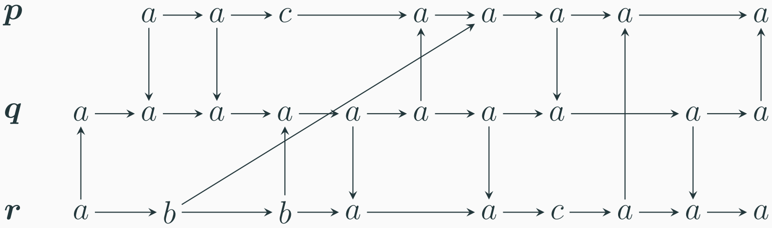
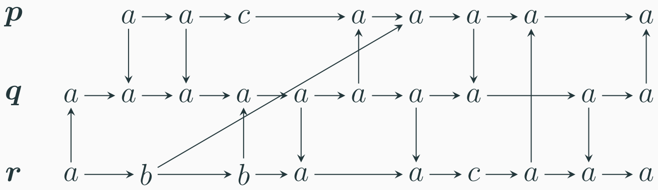


Partial order consisting of

- One sequence of events for each process
- Message relation connecting matching sends and receives

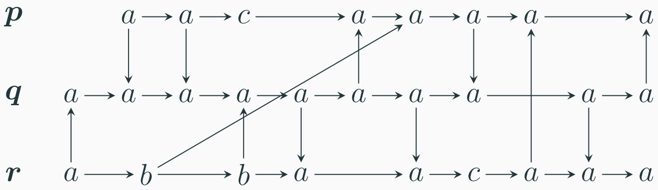
Monadic Second-Order logic (MSO) over MSCs

$\varphi ::=$



Monadic Second-Order logic (MSO) over MSCs

$\varphi ::= \quad a(x) \mid p(x)$ label/process of event x



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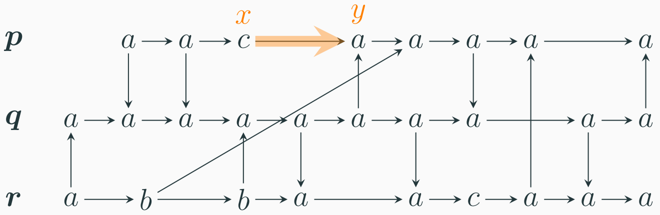

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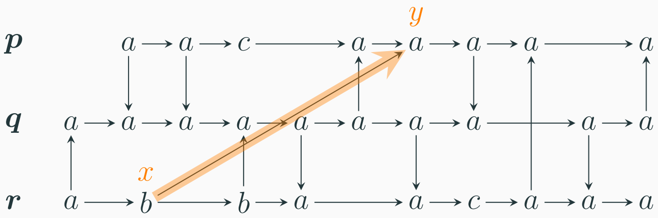

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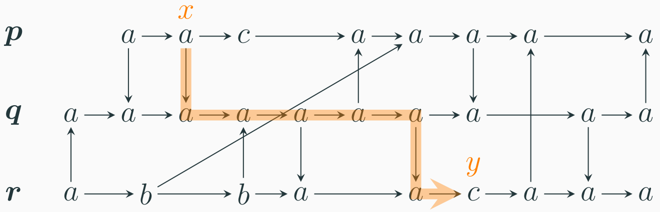
	$\mid x \leq y$	happened-before [Lamport 1978]



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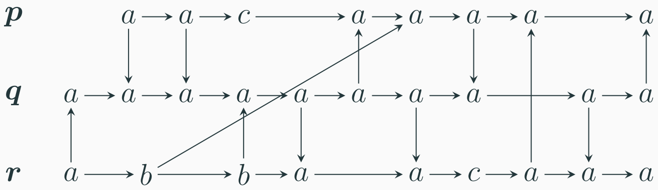
$\quad \mid \neg\varphi \mid \varphi \vee \varphi \mid \exists x.\ \varphi \mid \exists X.\ \varphi \mid x \in X$	



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$$
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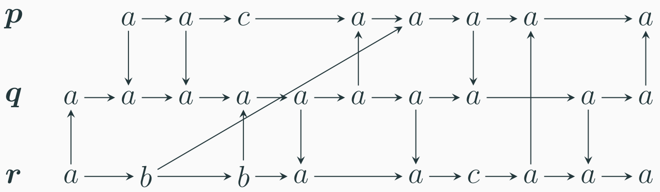
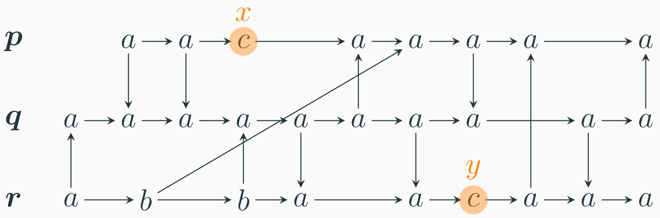


Example: mutual exclusion $\neg(\exists x.\ \exists y.\ c(x) \wedge c(y) \wedge\ x \parallel y\ )$

Monadic Second-Order logic (MSO) over MSCs

$\varphi ::= \quad a(x) \mid p(x)$	label/process of event x
$\mid x \rightarrow y$	process successor
$\mid x \lhd y$	message relation
$\mid x \leq y$	happened-before [Lamport 1978]
$\mid \neg\varphi \mid \varphi \vee \varphi \mid \exists x.\ \varphi \mid \exists X.\ \varphi \mid x \in X$	



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$\neg(x \leq y) \wedge \neg(y \leq x)$

Expressive power of CFMs

Theorem [Büchi 1960, Elgot 1961, Trakhtenbrot 1962]

If there is only one process,
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$\exists X_0 \ldots \exists X_n . \varphi$ with $\varphi \in$ FO$[\lhd, \rightarrow]$

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Note: here the happened-before relation $\leq$ is not included.

$\rightarrow$ Harder to formalize concurrency properties such as mutual exclusion.

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The equivalence with $\mathrm{MSO}[\lhd, \leq]$ is recovered if we assume that the channels are of bounded size.

Theorem

$\mathrm{CFM} = \mathrm{MSO}[\lhd, \leq]$ over

- finite, universally bounded MSCs [Henriksen,Mukund,Narayan Kumar,Sohoni,Thiagarajan 2005]
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Remark: model-checking is undecidable in general, but decidable when restricted to bounded MSCs.

Expressive power of CFMs

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Difficulty: CFMs are not closed under complement.

Idea: Use fragments of star-free PDL as intermediate steps.

A fragment of $\mathrm{PDL}_{\mathsf{sf}}$ without explicit complements

State formulas:

$$\varphi ::= P \mid p \mid \varphi \vee \varphi \mid \neg\varphi \mid \langle\pi\rangle\,\varphi$$

Path formulas:

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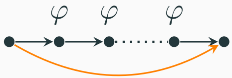

A fragment of $\mathsf{PDL}_{\mathsf{sf}}$ without explicit complements

$\mathsf{PDL}_{\mathsf{sf}}[\xrightarrow{\varphi}, \mathsf{Loop}]$

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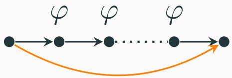
- $\xrightarrow{\varphi}$: similar to Until 

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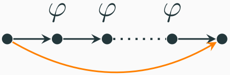

Negation occurs only at the level of state formulas (as in LTL)

Translation from first-order logic to CFMs

$\varphi \in \mathsf{FO}[\lhd, \leq]$

CFM $\mathcal{A}$ with $L(\mathcal{A}) = L(\varphi)$

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translation into star-free PDL

(non-elementary)

$\varphi' \in \mathsf{PDL_{sf}}[\xrightarrow{\varphi}, \mathsf{Loop}]$ with $\varphi \equiv \varphi'$

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($\textsc{Ptime}$)

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$\downarrow$ similar to LTL $\rightarrow$ finite automata (PSPACE)

CFM $\mathcal{A}$ with $L(\mathcal{A}) = L(\varphi)$

Translation from first-order logic to CFMs

$\psi = \exists X_1 \ldots \exists X_n . \varphi$
$\in \mathsf{EMSO}[\lhd, \leq]$

$\varphi \in \mathsf{FO}[\lhd, \leq]$

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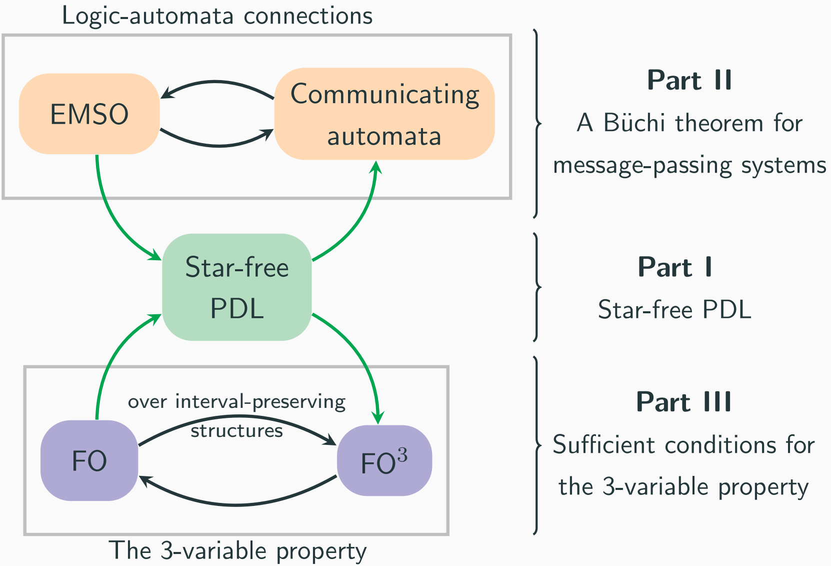
projection


Logic-automata connections
EMSO
Communicating automata
Star-free PDL
over interval-preserving structures
FO
FO^3
The 3-variable property
Part II
A Büchi theorem for message-passing systems
Part I
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Part III
Sufficient conditions for the 3-variable property

The 3-variable property for interval-preserving structures

Reminder

Theorem

Over MSCs, FO and $\mathsf{PDL}_{\mathsf{sf}}$ have the same expressive power.

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formulas with at most 3 variables

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formulas with at most 3 variables

$\rightarrow$ **MSCs have the 3-variable property.**

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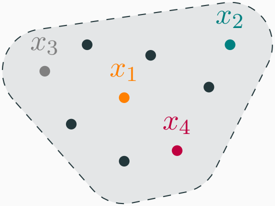

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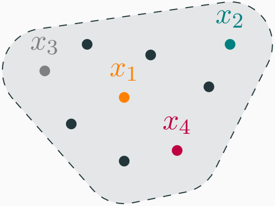

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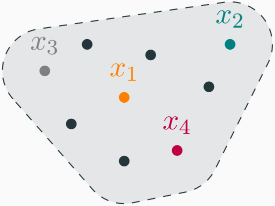

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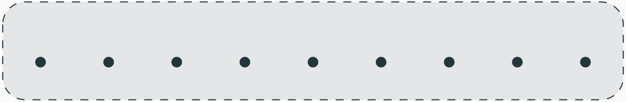

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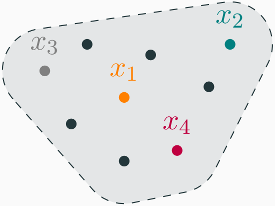

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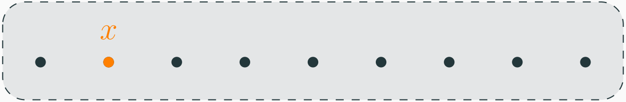

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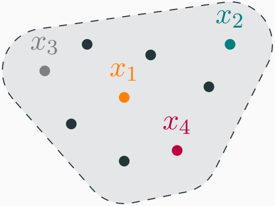

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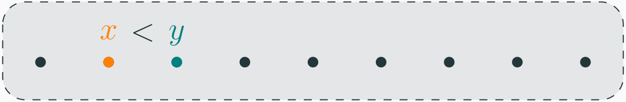

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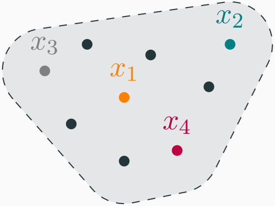

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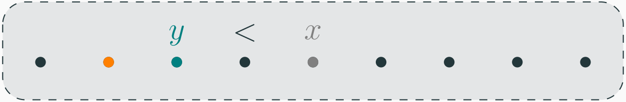

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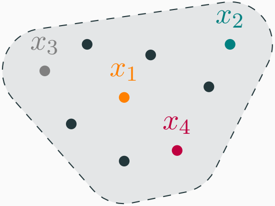

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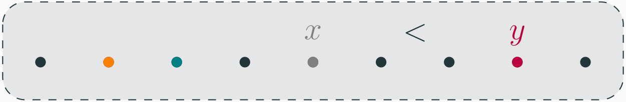

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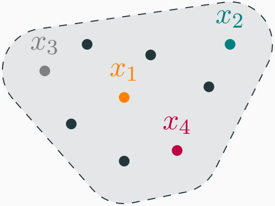

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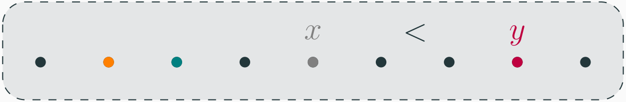

Over linear orders, FO = FO^3.

Examples

$\boxed{\begin{array}{c}\text{Over words,}\\ \mathsf{FO} = \mathsf{FO}^3\\ \text{[Kamp'68]}\end{array}}$ ✓

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What do the positive results have in common?

Generalisation

Theorem [F. 2019]

$\mathsf{FO} = \mathsf{PDL}_{\mathsf{sf}} = \mathsf{FO}^3$ over structures with

- one linear order $\leq$
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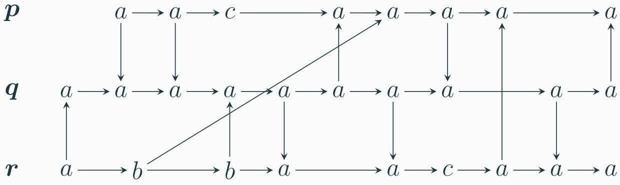

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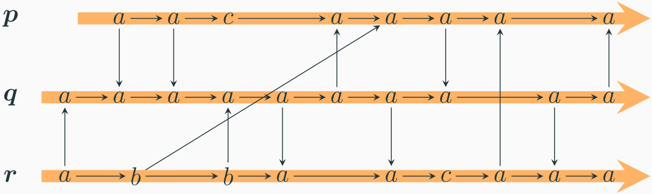

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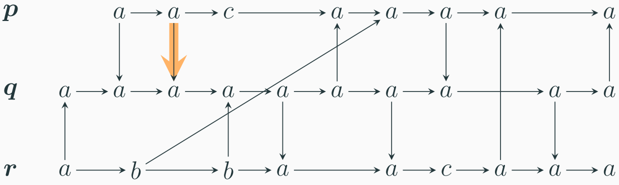

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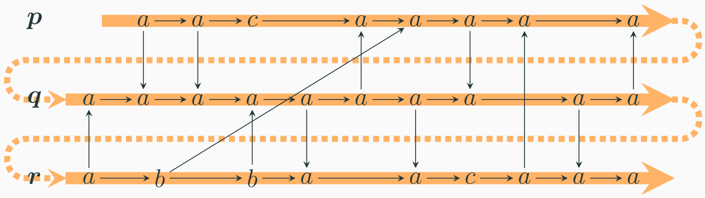

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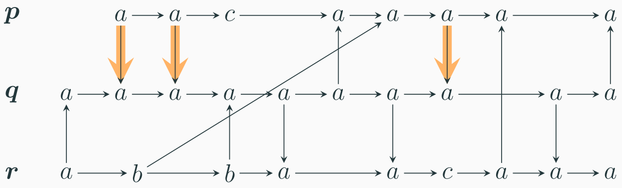

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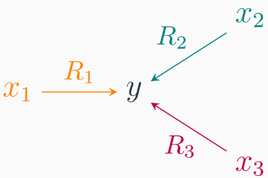

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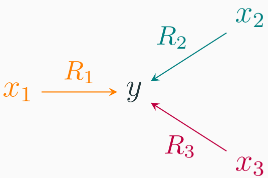

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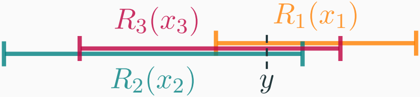

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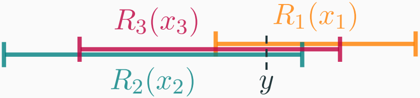

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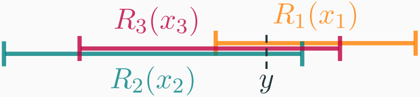

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$\mathrm{PDL}_{\mathrm{sf}}$

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$\mathsf{PDL_{sf}^{int}}$

Lemma: $\forall \pi \in \mathsf{PDL_{sf}^{int}}$, $[\![\pi]\!]$ is interval-preserving.

Equivalences over interval-preserving structures

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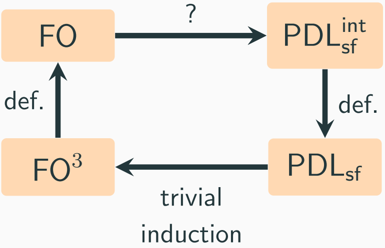

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- Negation: Express π^{c} using $(\leq \cdot \pi \cdot \leq)^{\mathsf{c}}$, $(\leq \cdot \pi \cdot \geq)^{\mathsf{c}}$, $(\geq \cdot \pi \cdot \leq)^{\mathsf{c}}$, $(\geq \cdot \pi \cdot \geq)^{\mathsf{c}}$.

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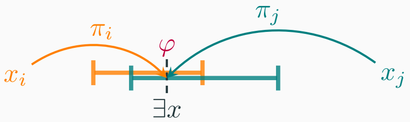

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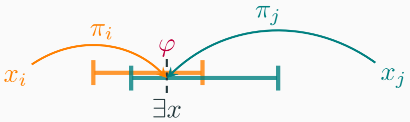

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- Over MSCs: expressively complete fragment closer to LTL and with a PSPACE translation into CFMs.

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- Is there a 1-dimensional temporal logic over MSC that is expressively complete for first-order logic?