

Bitonic and Merging sorting networks

Efficient Parallel Algorithms COMP308

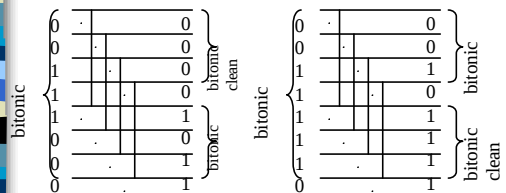
Bitonic sequence

- **Bitonic sequence** is a sequence that
 - either monotonically increases and then monotonically decreases,
 - or else monotonically decreases and then monotonically increases.
- For example
 - $\langle 1, 4, 6, 8, 3, 2 \rangle$
 - $\langle 9, 8, 3, 2, 4, 6 \rangle$ are both *bitonic*
- The zero-one sequences that are bitonic have a simple structure. They have the form $0^i 1^j 0^k$ or the form $1^i 0^j 1^k$, for some $i, k, j \geq 0$.

The half-cleaner

- The bitonic sorter is comprised of several stages, each of which is called a **half-cleaner**.
- Each half-cleaner is a comparison network of depth 1 in which input line i is compared with line $i + n/2$ for $i = 1, 2, \dots, n/2$.
- When a bitonic sequence of 0's and 1's is applied as input to a half-cleaner, the half-cleaner produces an output sequence in which smaller values are in the top half, larger values are in the bottom half, and both halves are bitonic.

The comparison network Half-cleaner



In fact at least one of the halves is clean – consisting all 0's or all 1's. The next lemma proves these properties of half-cleaners.

- **Lemma.** If the input to a half-cleaner is a bitonic sequence of 0's and 1's, then the output satisfies the following properties:
 - both the top and the bottom half are bitonic,
 - every element in the top half is at least as small as every element of the bottom half, and
 - at least one half is clear.

Proof:

The bitonic sorter

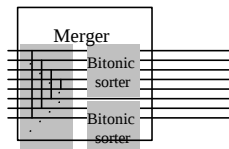
- By recursively combining half-cleaners we can build a bitonic sorter, which is a network that sorts bitonic sequences.
- The first stage of **bitonic-sorter[n]** is **half-cleaner[n]**, which produces two bitonic sequences.
- Thus we can complete the sort by using two copies of **bitonic-sorter[n/2]**



A Merging Network

- Our sorting network will be constructed from merging networks, which are networks that can merge two sorted sequences into one sorted output sequence.
- We modify the first half cleaner in the Bitonic-sorter[n]

A network that merges two sorted input sequences into one sorted output sequence



Sorting network

