A CORE ONTOLOGY FOR SPATIAL REASONING

Frans Coenen and Pepijn Visser

Department of Computer Science, The University of Liverpool

Liverpool, L69 ZF, UK, email: (frans,pepijn)@csc.liv.ac.uk

tel: +44 (0)151 794 3698, fax: +44 (0)151 794 3715

Abstract

In this paper we describe a core ontology for N-dimensional spatial reasoning. The ontology is intended to support both quantitative and qualitative approaches and is expressed using set notation. Using the ontology; spatial domains of discourse, spatial objects and their attributes, and the relationships that can link spatial objects can be expressed in terms of sets, and sets of sets. The core ontology has been emphparameterised to express a number of task dependent ontologies in application areas such as Geographic Information Systems (GIS), noise pollution monitoring, environmental impact assessment, shape fitting, timetabling and scheduling, and AI problems such as the N-queens problem. We illustrate this parameterisation by using the core ontology to express both Allen's interval calculus, and Egenhofer's "9-Intersection" approach to topological spatial reasoning. Although there is still much work to do, the directed task ontologies that have been investigated indicate that the core ontology described provides an excellent foundation for further research.

KEYWORDS: Core ontologies, Spatio-temporal reasoning.

1 INTRODUCTION

In this paper we describe a core ontology for spatial reasoning applications. In the context of this paper a core ontology is deemed to have those domain commitments that are considered to be common to all spatial reasoning domains. The aim is to produce a mechanism which will facilitate the high level conceptualisation of such spatial reasoning applications in a manner which is both convenient and as widely applicable as possible. As a consequence researchers and manufacturers of spatial reasoning systems will benefit from the advantages that core ontologies have stimulated in other fields, for example Knowledge Based System (KBS) development ([30]).

The core ontology described is expressed using well established set theoretic notation. This approach was adopted because spatio-temporal objects naturally lend themselves to definition in terms of sets of addresses (coordinates). The core ontology thus provides a mechanism whereby a spatial domain of discourse, spatial objects and their attributes, and the relationships that can link spatial objects can be expressed in terms of sets, and sets of sets. Application directed task ontologies can then be expressed by *parameterising* the core ontology.

A spatial reasoning system, the SPARTA (SPAtial Reasoning using Tesseral Addressing) system, which is founded on a constraint logic programming paradigm, has also been developed. The input to this system is a task ontology, expressed by parameterising the core ontology, which is then processed so as to produce appropriate results. Although the system is referred to within this paper its precise nature is not described (interested readers are directed to Coenen et al. [15, 16] for further information).

The rest of this paper is organised as follows. We commence by presenting an overview of ontologies (section 2) and the application area of spatio-temporal reasoning (section 3). The core ontology is presented using set notation. The advantages offered by this representation and some details with respect to syntax are presented in section 4. In section 5 a number of assumptions regarding the nature of multi-dimensional space and its definition with respect to the core ontology are established. This is followed in sections 6 and 7 with discussion concerning the definition of spatial entities and the attributes that may be associated with such entities using the core ontology. In the following two sections we illustrate how the core may be parameterised with respect to specific tasks. Firstly (section 8) we use the core ontology to describe Allen's interval calculus [3] and secondly (section 9) to express Egenhofer's 9-Intersection mechanism [20]. It should be noted that these two illustrations are of necessity brief and therefore only serve to give a flavour of the full power of the core ontology (in a paper of this form we cannot demonstrate all its features). Finally in section 10 we present some conclusions and consider directions for further research into the field of spatial ontologies.

2 ONTOLOGIES

Building knowledge systems involves the creation of a model of a particular domain (e.g., electronic circuits, legislation, plants). Such a model is necessarily an abstraction of the domain under consideration. This is what makes a model useful, it abstracts from irrelevant details and thereby allows us to focus on the aspects of the domain we are interested in. Building a model of a domain involves deciding upon what entities in the domain are to be distinguished, and what relations will be recognised between these entities. Moreover, it involves deciding upon what types of entities, and what types of relation exist. Often, the latter kind of decisions are straightforward and not always explicitly documented. For instance, in building a model of the blocks world we use predicates like $holds(B)$, $block(A)$ and $on(A, B)$ thereby implicitly assuming the domain to consist of blocks, tables, and hands, and that blocks, tables and hands have spatial relations. Making these assumptions requires the domain to be sub-divided into concepts. Alternatively stated, they reflect a conceptualisation of the domain under consideration. The conceptualisation tells us the types of entities and how they can relate. In the blocks world the conceptualisation tells us that there are blocks, tables, hands and that blocks and tables have spatial relations. It does not, necessarily, tell us what particular blocks, tables and hands there are, nor how they are spatially related.

It is important to note that there may be many "correct" conceptualisations of one particular domain. Creating an ontology is not an unequivocally defined process, the same blocks world can be conceptualised in several ways. Some entities in the world may not need to be conceptualised at all (for instance, it is not always necessary to conceptualise the hand in the blocks world), other entities could have been specified more abstractly (e.g., recognising only entities and relations) or just less abstractly (e.g., recognising cubes, and cylinders). We may state that making a conceptualisation is accompanied by a considerable amount of freedom.

To be able to compare conceptualisations of a domain, it is necessary to make them explicit. Thus, we need to have an explicit conceptualisation. This is what the word *ontology* is used for in AI research:

"an ontology is defined as an explicit conceptualisation of a domain" (Gruber [27]).

Typically, an ontology is a set of definitions of (hierarchically ordered) classes, objects, attributes, relations, and constraints (henceforth referred to simply as entities and relations). It's main function is to provide a vocabulary for the expression of domain knowledge. An ontology is a knowledge-level description (Newell [42]) in that it is independent of any representational formalism (Van Heijst [30]). Also, an ontology is considered to be a meta-level description because it is a specification of (the entities and relations in) a specification (viz. the model under construction); it abstracts from the particulars of the model under construction and specifies only its building blocks. We could say that an ontology describes the domain knowledge that remains invariant over various knowledge bases in a certain domain (cf. Guarino and Giaretta [29]). For instance, an ontology could specify that in all knowledge bases of the blocks world an empty block is a block with no block on top of it. It should be stressed here that different interpretations of the word ontology are used by different authors. This is nicely illustrated by Guarino and Giaretta, who discuss seven interpretations of the word.

Recently the result of gathering vocabularies and structuring domains has been recognised as a valuable effort in its own right, deserving of attention (Wiederhold, p.7 [59]). In general, we can say that ontologies may contribute to the following five areas.

1. **Domain-theory development:** Because an ontology explicitly states the building blocks of particular domains, it can be used for the analysis, comparison, and development of domain theories. Examples of this kind of ontology use can be found in Sim and Rennels [49], and Visser and Bench-Capon [58].

2. **Knowledge acquisition"** Ontologies describe and structure the entities and relations that need to be acquired for the domain under consideration. Examples of this kind of ontology use are CUE ([31], [30]), and MOBAL ([40]).

3. **Knowledge-system design:** Ontologies are reusable constructs in the design of knowledge systems because they can be used to represent the invariant assumptions underlying different knowledge bases in the same domain. As such, they can be considered as initial building blocks of the knowledge base under construction. An example of this kind of ontology use can be found in the GAMES methodology ([30]).

4. **System documentation:** Ontologies provide a meta-level view (vocabulary, structure) on their application domain which facilitates adequate system documentation for end-users. An example of this kind of ontology use is found in the Cyc project ([36]).

5. **Knowledge exchange:** Ontologies can be used to define assumptions that enable knowledge exchange between different systems. Examples of this kind of ontology use can be found in SHADE ([33]), and Wiederhold ([59]).

An ontology describes a domain from a particular viewpoint. The assumptions about the domain that define such a viewpoint are referred to as ontological commitments ([27]). Because different ontologies express different viewpoints they have different ontological commitments. The type of ontological commitments can be used to classify ontologies. If we can distinguish different types of ontological commitments it will enable us to distinguish between different types of ontologies, and hence, it will enable us to classify ontologies. As a starting point to distinguish commitments we adopt the commonly made distinction in AI literature between task, methods, and domains (cf. Wielinga et al. [60]). Hence, we arrive at three different types of ontological commitments (1) task commitments, (2) method commitments, and (3) domain commitments.

1. **Task commitments:** An ontology has task commitments if it defines entities and relations that express a task-specific perspective on the domain knowledge (by a task we mean a specification of a goal together with some input and required output, see also Visser [57]). Typical task commitments are found in, for instance, an ontology for a diagnosis task, which contains entities, such as observations, causes, and hypotheses.

2. **Method commitments:** An ontology has method commitments if it defines entities and relations that express a method-specific perspective on the domain knowledge (by a method we mean a specification of *how* a task can be performed, see also Visser [57]). Typical method commitments are found in, for instance, an ontology for the propose-and-revise method (within a design task), which contains entities, such as proposed solution, constraints, and value-assessment.

3. **Domain commitments:** An ontology has domain commitments if it defines entities and relations that relate to a particular domain (by domain we here refer to the commonly distinguished fragments of the real world which is being modelled, such as medical, legal, mathematical, financial, or social domains). In this article, we address ontologies which make domain commitments to the spatial reasoning domain.

Ontologies can be classified according to the type of commitments they make. Using the three types of commitments mentioned above, we could say that a task ontology is an ontology that makes (substantial) commitments towards a certain (group of) task(s), a method ontology is an ontology that makes (substantial) commitments towards a particular (group of) method(s), and a domain ontology is an ontology that makes (substantial) commitments towards a particular (group of) domain(s). We note that the amount of commitments (of a certain type) made in an ontology may vary. It can be a small amount or a large amount of commitments. Hence, we can define a core domain ontology, an ontology that makes commitments towards a particular domain but is generic in that it can be refined with respect to several subdomains, methods and tasks. It should be noted that ontologies may have a combination of all three types of commitments. It could be argued, for instance, that the distinction of the hand in the blocks world is a commitment towards a certain method (namely: allowing methods that use "intermediate" states), but it obviously commits to the domain of the blocks world.

Although ontologies are now well established in the Knowledge Based Systems (KBS) community ([30]) very little work has been undertaken in the domain of spatial ontologies. Some work has been done on the language of spatial reasoning, i.e. "what is meant by the term near by?" Etc., but this is more concerned with natural language semantics than ontologies. In addition mereology has been taken up as a theory of part-whole relations in formal philosophical ontology ([50]), the theory of parts and boundaries ([51]), and as the foundation for a number of spatial axiomisations ([17, 18]). Some related work has also been undertaken with respect to ontologies for Geographic Information systems ([52]). The core ontology described here seeks to lay the foundation for an "all-purpose" abstract ontology to support spatio-temporal reasoning applications so that the researchers and builders of such systems can gain similar advantages to those gained by the KBS community through the use of ontologies.

3 SPATIAL REASONING

Spatial reasoning can be defined, very broadly, as the automated manipulation of data objects belonging to spatial domains so as to arrive at application dependent conclusions. A spatial domain, in this context, is considered to imply any N-dimensional space (real or imaginary, including 1-D temporal spaces) in which objects of interest may be contained. Automated manipulation then implies the computer simulation of some higher mental processes (not just the simple response to some stimulus or the mechanical performance of an algorithm). We are therefore not concerned with (say) the automated retrieval of spatially referenced data contained in some database format. At its simplest, spatial reasoning can be considered to revolve round the identification of (previously unknown) relationships that exist between spatial objects according to the relationships that are known or desired to exists between such objects. In more complex systems the identified relations are then used as the foundation whereby further reasoning can take place, and consequently additional conclusions drawn.

4 SYNTAX

A number of languages/tools are available in which ontological conceptualisations may be expressed. The most well known example is Ontolingua [27] which is directed at ontologies for the conceptualisation of expert systems and developed as part of the KSE project ([28]). Another example is KRSL (Knowledge Representation Specification Language) which is directed at a similar domain (Leher [35]).

The core ontology described here is presented using set notation. The principal reason for this is that space is generally conceptualised as a comprising groups of “cells” — the Cartesian coordinate system is the most widely used mechanism for referencing space, whether it be: (say) X-Y-Z coordinates; Eastings, Northings and Height; or Latitude and Longitude. It is thus a natural step to consider such groups of cells in terms of set. Further reasons for using set notation are that the theory is long established and is consequently well understood, and secondly that there is a recognised notation for expressing sets and the operations that can be performed on them. Moreover it suffices for present purposes as a more comprehensive language is not required.

The proposed core ontology is defined in terms of a "singleton" set, $CoreOntology$, which comprises a 3-tuple each element of which, in turn, is a set:

$$CoreOntology = \{\langle Space, Objects, Relations\rangle\}$$

The set *Space* describes the nature of the domain of discourse. The set *Objects* defines the set of spatial entities of interest, and the attributes that may be associated with those entities. The set *Relations* then defines the relationships that can exist between pairs of spatial entities. Note that a distinction is made between attributes that may be associated with an object in isolation, and attributes that serve to link pairs of objects. The more precise nature of each of these sets is described in the following sections.

5 SPACE

For quantitative spatial reasoning we must have some reference framework. For qualitative reasoning we will still require a reference framework if we wish to express relations such as "before" or "inFrontOf". In a purely topological qualitative approach no reference framework is required, but this is not to say that we cannot define this approach in terms of such a framework.

In the case of the reference framework for the core ontology it is assumed that:

- N-dimensional space comprises a set of N-dimensional isohedral cells each of which is identified by a unique *address* (the SPARTA system uses a tesseral addressing system — see Diaz and Bell [19] for an overview of this technique).

- The addressing systems has the effect of linearising space so that an ordering can be imposed on sets of addresses (regardless of the number of dimensions under consideration).

- The addressing system has an origin to which all other addresses can be referenced.
- The system has an arithmetic superimposed that facilitates translation through the space (and rotation).

Given this representation we can identify a domain of discourse U which may be defined as follows:

$$U = P \amalg N$$

where $\amalg$ is the "disjoint union" operator, and:

$$P = \{p \mid p : \{the\ set\ of\ all\ referencesrepresenting\ positive\ space\}\}$$

$$N = \{n \mid n : \{the\ set\ of\ all\ referencesrepresenting\ negative\ space\}\}$$

The reasons for this distinction between positive and negative space will become apparent when we go on to consider the attributes that can be associated with spatial objects in section 6.

An example 2-D space is given in Figure 1. Note that the maximum dimensions are denoted by the labels max_1 and max_2. The nature of these dimensions will be domain dependent. Note also that the $globalOrigin$ of the space, the cell 0, is located at the intersection of the two axes. For the purposes of this document it is necessary to identify a number of further addresses with respect to the $globalOrigin$. These are shown in the inset in Figure 1 which assumes a linearisation the flows from the bottom-left cell up to the top-right cell. The addresses -1 and $+1$ indicate the addresses immediately before and after the $globalOrigin$ as defined by the linearisation. The addresses sw and ne indicate the addresses which are "corner adjacent" to the $globalOrigin$ but furthest away (in the negative and positive directions) with respect to the linearisation. The values that may be associated with the variables sw and ne will be implementation dependent.

The domain of discourse of any task is then either equivalent to U (the universe of discourse) or is some subset of U — provided that the origin is included. In this paper we will indicate any

*** Figure in here ***

Figure 1: Example 2D space

subset of U which defines a task domain of discourse task using the notation U_D which comprises the disjoint intersection of the negative and positive component spaces P_D and N_D.

Given a particular application the set *Space* (see section 4) will be defined as follows:

$$Space = \{max_1, max_2, ...max_N\}$$

where N is an integer representing the number of dimensions under consideration. Thus, in the context of a particular task ontology, a 1-D space would be defined as $\{max_1\}$, a 2-D space as $\{max_1, max_2\}$, and so on. This in turn will define the nature of the sets U_D, P_D and N_D. Once the domain of discourse has been defined we can go on to define sub-spaces within this domain, in terms of subsets of U_D, as dictated by the nature of the application.

5.1 Local Origins

We have seen that any set U_D ($U_D \subseteq U$) has a $globalOrigin$. Similarly any subset of U_D will have a $localOrigin$ which is defined as the reference nearest to the $globalOrigin$ as dictated by the nature of the linearisation. Thus, given a subset X of the set U_D we define a function $f_{localOrigin}$ that returns the reference for the local origin of the set X. This function is defined as follows:

$$Dom(f_{localOrigin}) = Pow(U_D)$$

$$Cod(f_{localOrigin}) = U_D$$

$$Gr(f_{localOrigin}) = \{\langle X, a\rangle \mid a : X \wedge a \textit{ nearest the global origin}$$

where Dom, Cod and Gr are the domain, codomain and graph of the indicated function, and Pow is a function that returns the power set as its argument (which must be a set).

5.2 Linearisation and sequences of addresses

The linear representation assumed offers many computational advantages. In particular it allows for the unambiguous identification of local origins and the definition of sequences of addresses. With respect to the latter we use the infix operator ".." to define such sequences. The prefix operand for this operator then represents the start of the sequence (the address nearest to the origin) and the postfix operand the end of the sequence (the address furthest away from the origin).

6 OBJECTS

Any spatial application will comprise a number of spatial entities. These may describe points, intervals, events, areas, volumes and so on. Using the core ontology spatial entities of interest are described in terms of a set of sets, $Objects$, which is defined as follows:

$$Objects = \{Obj_1, Obj_2, ..., Obj_N\}$$

where the elements Obj_1 through to Obj_N are themselves set of sets whose elements describe attributes that may be associated with particular spatial entities. Spatial entities may have many attributes, however, a number are common to all such entities and are therefore supported by the core ontology as follows:

- Identifier
- Location
- Shape
- Rotation
- Size (actual, maximum, minimum)
- Connectivity (a spatial entity does not have to be comprised of a continuous set of cells)

- Class (type)

Each of the above attributes is discussed further in the following subsections.

6.1 Identifier

To do anything with a spatial entity we need to be able to identify it in some manner. The simplest approach is to give the entity in question a unique *name* selected from a set $Names$:

$$Names = the\ set\ of\ all\ possible\ names$$

Thus one (and only one) element of any set Obj_N is always a member of the set $Names$.

6.2 Location

Other than an identifier the second most important attribute that may be associated with a spatial entity is its location. There are two possibilities, either an entity's location is precisely known (i.e. it is fixed) or we know of some *location space* in which the entity can be said to exist. What ever the case the location or location space is described in terms of some subset ($Location$ or $LocationSpace$) of P_D, i.e. in terms of a positive set of cells contained within the domain. In the absence of any other information the sets $Location$ or $LocationSpace$ are assumed to be equal to P_D. Thus:

$$\begin{aligned} Location &= \{L \mid L \subseteq P_D\} \\ LocationSpace &= \{L \mid L \subseteq P_D\} \end{aligned}$$

the definitions are the same, the interpretations are different. Note also that, given a spatial entity whose location is known (i.e. there exists a set $Location$ as opposed to a set $LocationSpace$), further attributes such as shape, size, connectivity etc., are defined by default. We will refer to such an object as a *fixed object*.

6.3 Shape

Shape is probably the third most significant attribute that a spatial entity has. We have already seen that in the case of a fixed object this is defined by default. However, in the absence of a fixed location, shape is often significant. Shape is defined, in a similar manner to the *Location* and *LocationSpace* sets, as follow:

$$Shape = \{S \mid S \subseteq U_D \wedge globalOrigin \in S\}$$

Note that any shape definition must include the $globalOrigin$. This is required because if we wish to do anything useful with the shape it must have some reference with respect to which it can be manipulated.

We will refer to spatial entities that have a known shape, but no definite location other than some location space within which they are considered to exist, as *free objects* (in the sense that they are not fixed). Given a free object, the shape definition also defines further attributes such as size and contiguity. Note also that a free object must have some location space associated with it — somewhere within which it is known to exist. From subsection 6.2 this may be either the entire space P_D or some subset of P_D as defined in the previous sub-section.

6.4 Rotation

A further important attribute associated with free objects is orientation — given a particular object this may be either fixed or we may be free to rotate the object. Whether a shape/object can be rotated or not is defined in terms of a singleton set *Rotation*:

$$Rotation = \{r \mid r \in Bool\}$$

where *Bool* is the Boolean set comprised of the elements $\{true, false\}$.

6.5 Size

In the absence of location and shape information we may have knowledge of size. The size of a spatial object is expressed in terms of the number of members of a set describing its shape, i.e. the *cardinality* of this set. In the case of a fixed object this will be the set $Location$, in the case of a free object this will be the set $Shape$. Where the precise definition of these sets is not known we can still express the required number of elements for this set. The minimum size of any object (that can be physically realised) is 1 indicating that it is represented by a single cell. The maximum size of an object is equivalent to $f_{cardinality}(P_D)$ (where $f_{cardinality}$ is a function that returns the "size" of its argument which must be a set), otherwise the object exceeds the application domain. In addition the nature of the size of an object can be expressed in terms of (i) a minimum size, (ii) a maximum size, or (iii) an actual size — the set of operators $\{<, =, >\}$. Thus we can define a set $Size$, comprising a single 2-tuple, as follows:

$$Size = \{\langle q, z\rangle \mid q \in \{<, =, >\} \wedge z \in \{1 \,..\, cardinality(P_D)\}\}$$

6.6 Connectivity

Finally we may know something about whether an object's location/shape is represented by a contiguous set of addresses or not. In this context continuity is defined as the situation where each element of a *Location* or *shape* set of addresses associated with any object is adjacent to at least one other element of this set (by adjacent we mean either edge or corner adjacency). We define the nature of the connectivity attribute in terms of a singleton set $Contiguity$ as follows:

$$Contiguity = \{c \mid c \in Bool\}$$

6.7 Class

From the above we can conclude that (so far) the attributes of any object are taken from the set of all possible attributes (*Attributes*) defined as follows:

$$Attributes = \{identifiers, locations, locationspaces,$$
$$shapes, rotation, size, connectivity\}$$

Given any spatial object only a sub-set of this possible set of attributes will be associated with it. The nature of this subset is, in turn, described by the *classes* of an object. This is defined by a singleton set $Class$:

$$Class = \{c \mid c \in \{fixed, free, shapeless\}\}$$

Thus the complete *Attribute* set should more properly be expressed as follows:

$$Attributes = \{identifiers, \mathbf{classes}, locations, locationspaces,$$
$$shapes, rotation, size, connectivity\}$$

The nature of the attributes that can be associated with each class has been discussed in the foregoing, however, a summary is presented below.

- **Fixed objects**: Objects whose locations are "fixed". Consequently they have only three attributes: an identifier, a class and a *Location* set (all other attributes are implied by the location set). Thus given a particular fixed object this will be defined as follows:

$$FixedObject = \{name, class, Location\}$$

- **Free objects**: Objects whose location is not known, but who have a fixed shape. Objects belonging to this class have five attributes: an identifier, a class, *LocationSpace* and *Shape* sets, and whether rotation is permitted or not (the size and connectivity attributes are implied by the shape attribute and therefore do not need to be explicitly included). Thus given a specific free object this would be defined thus:

$$FreeObject = \{name, class, LocationSpace, Shape, rotation\}$$

- **Shapeless objects**: Objects whose location and shape are both not known. These may be objects for which we know nothing (other than they exist), or we may know details about size and/or connectivity. Their definition will also require a description of the location space in which they are known to be contained. Thus:

$$\begin{aligned} ShapelessObject = {} & \{name, class, LocationSpace\} \vee \\ & \{name, class, LocationSpace, Size\} \vee \\ & \{name, class, LocationSpace, Contiguity\} \vee \\ & \{Name, Class, LocationSpace, Size, Contiguity\} \end{aligned}$$

7 RELATIONS

In addition to the above attributes, applicable to individual objects, there are a further set of attributes that serve to link pairs of objects or objects and locations. We will refer to such attributes as relationships. For example, given two objects we might say that one is contained within the other. The relation "contained" as used here is a predicate in the sense that the relation is either true or false. More precisely, with respect to the nature of the object attributes described above, we are saying that the location (or locationSpace) of one of the objects is such that it is contained wholly within the location (or location space) of the other. We have seen that in the core ontology all locations and location spaces are expressed as sets. Therefore the most natural way of describing relations (in the context of the core ontology) is in terms of the standard set relations expressed as a Boolean functions:

$$f_{filterEquals}(A,B) \Rightarrow true\ if\ A = B$$

$$f_{filterIntersects}(A,B) \Rightarrow true\ if\ A \cap B$$

$$f_{filterSubset}(A,B) \Rightarrow true\ if\ A \subset B$$

$$f_{filterSubsetOrEquals}(A,B) \Rightarrow true\ if\ A \subseteq B$$

$$f_{filterSuperset}(A,B) \Rightarrow true\ if\ A \supset B$$

$$f_{filterSupersetOrEquals}(A,B) \Rightarrow true\ if\ A \supseteq B$$

where A and B are sets of addresses. We use the term filter here to indicate that these relations are, in practice, used to express constraints that are desired to hold between objects. If we add the negation of these predicates we have 12 possible relations.

Alternatively we can also conceive of relations as non-Boolean functions which can be used to refine the location space associated with shapeless objects (as defined above). Thus given such an object we can get some idea of its nature using relations. We can identify two such relations:

$$f_{mapIntersects}(A,B) \Rightarrow A_n = A \cap B$$

$$f_{mapComplement}(A,B) \Rightarrow A_n = B \setminus A$$

A union relation is not included here because this would have the effect of enlarging location space sets, which in turn will necessitate an ordering of constraints which is seen as to be both undesirable and unnecessary). We refer to the above relations as mappings in the sense that we "map" some set operation (intersection, union, complement) on to the first argument (A) with respect to the second argument (B) so as the produce a revised set (A_n). Given a set of such relations we can continuously revise the location space for a shapeless object with respect to fixed or free objects until we have a precise knowledge of the form of location for the shapeless object.

The total possible set of relations is then as follows:

$$\begin{aligned}
Relations = \{&equals, intersects, subset, subsetEqual, superset, supersetEqual,\\
&notEquals, notIntersects, notSubset, notSubsetEqual, notSuperset,\\
&notSupersetEqual, intersection, union, complement\}
\end{aligned}$$

In the core ontology a relation, at its simplest, is expressed as a set of three elements

$$Relation \quad = \{\{a, r, b\} \mid a, b \in Names \wedge r \in Relations\}$$

Note that locations/location spaces are referenced using object names.

7.1 Offsets

To increase the expressiveness of these relations we can apply an *offset* to the location of an object. An offset is a set of addresses defined as some subset of U_D which is applied either to (a) all the elements describing a location (or location space) or (b) the local origin for this set. The effect in the first case is to translate or expand (or both) the space. In the second case this allows us to define other "locations" with respect to the given location (or location space).

Although the offset principle allows us to expand objects the only way in which objects can be "shrunk" is to define a space with respect to the local origin of the object in question so that it appears that the object has been shrunk. This is often a cumbersome process, thus a facility is also provided to "shrink" by a given number of addresses.

What ever the case offsets are defined by a 2-tuple as follows:

$$Offset \quad = \quad \{\langle t, F\rangle | t \in \{ref, all, shrink\} \wedge F \subseteq U_D\}$$

*** Figure in here ***

Figure 2: Allen's 13 interval relations

Thus a relation can, more broadly, be defined as follows:

$$
\begin{aligned}
Relation = \{a, r, b\} \vee \\
\{a, \langle t_a, F_a \rangle, r, b\} \vee \\
\{a, r, b, \langle t_b, F_b \rangle\} \vee \\
\{a, \langle t_a, F_a \rangle, r, b, \langle t_b, F_b \rangle\}
\end{aligned}
$$

7.2 Quantifying the intersection relations

The expressiveness of the *intersects* and *intersection* relations (either in addition to or as an an alternative to the use of offsets) may be further expanded by quantifying the size of the intersection. In the core ontology we achieve this by allowing two optional arguments for the intersection predicate — an operator and a desired cardinality. The set of possible operators is as follows:

$$<, \leq, =, \geq, >$$

The cardinality is expressed as a positive integer in the range of 1 to $f_{cardinality}(P_D)$. Thus we can (say) insist that an intersection is equal to a cardinality of 4 as follows:

$$\{a, intersection(=, 4), b\}$$

8 PARAMETERISATION I: ALLEN'S INTERVAL CALCULUS

The most influential modern work carried out in the field of temporal reasoning can be considered to be James Allen's Interval Calculus. Although Allen's work cannot be said to be definitive, its importance lies in that it is the basis on which many subsequent temporal (and spatial) reasoning systems have been built, or if not the basis at least the catalyst for such work. For example Freksa [24], Hernandez [32] and Mukerjee and Joe [41] have all transferred the approach to the spatial domain, while Ligozat [38] has generalised the interval concept for reasoning with chains of events. Good and available accounts of Allen's original work can be found in [1, 2] (see also Allen and Koomen [7]) and a more general overview in [3]. The work was later extended to incorporate points as well as intervals (see Allen and Hayes [4, 5, 6]). A good critic of Allen's work can be found in Galton [26], while Vilain et al. [55, 56] and Nokel [43] studied the computational complexity of Allen's reasoning scheme and some of its variants.

Allen, in his interval calculus, identifies 13 "interval" relations which are used to link shapeless one dimensional objects (Figure 2). The objects are stored in a network where each arc represents one or more relations. The network is typically expressed as a set of tuples of the form:

$$< node1, relationList, node2 >$$

By adding further tuples to the network we can check the continuing consistency of the network, remove possible relations which are no longer applicable (provided that one relation always remains linking a node pair), or deduce further relations not previously included in the network. Allen uses a *transitivity* table technique to achieve these revisions. Allen's 13 interval relations can be expressed in the core ontology as follows (core ontology relations given to the right):

$$A\ before\ B \Leftrightarrow \{a, subset, b, \langle ref, \{-max_1\ ..\ -1\}\rangle\}$$

$$A\ after\ B \Leftrightarrow \{a, subset, b, \langle ref, \{(length+1)\ ..\ max_1\}\rangle\}$$

$$A\ during\ B \Leftrightarrow \{a, subset, b\}$$

$$A\ contains\ B \Leftrightarrow \{a, superset, b\}$$

$$A\ overlaps\ B \Leftrightarrow \{a, intersects, b\} \wedge \{a, \langle ref, \{0\}\rangle, notSubset, b\}$$

$$A\ overlappedBy\ B \Leftrightarrow \{a, intersects, b\} \wedge \{a, notSubset, b, \langle ref, \{length\}\rangle\}$$

$$A\ meets\ B \Leftrightarrow \{a, \langle ref, \{length\}\rangle, equals, b, \langle ref, \{-1\}\rangle\}$$

$$A\ metBy\ B \Leftrightarrow \{a, \langle ref, \{-1\}\rangle\ equals, b, \langle ref, \{length\}\rangle\}$$

$$A\ starts(s)\ B \Leftrightarrow \{a, \langle ref, \{0\}\rangle, equalsb, \langle ref, \{0\}\rangle\} \wedge \{a, subset, b\}$$

$$A\ startedBy(si)\ B \Leftrightarrow \{a, \langle ref, \{0\}\rangle, equalsb, \langle ref, \{0\}\rangle\} \wedge \{a, superset, b\}$$

$$A\ finishes(f)\ B \Leftrightarrow \{a, \langle ref, \{length\}\rangle, equals, b, \langle ref, \{length\}\rangle\} \wedge \{a, superset, b\}$$

$$A\ finishedBy(fi)\ B \Leftrightarrow \{a, \langle ref, \{length\}\rangle, equals, b, \langle ref, \{length\}\rangle\} \wedge \{a, subset, b\}$$

$$A\ equals(=)\ B \Leftrightarrow \{a, equals, b\}$$

The above requires some further explanation. Allen uses upper case letters to represent temporal objects, we use lower case letters to indicate that each label is an element of the set $Names$. The identifiers max_1 and -1 were defined in section 5, and the sequence ".." notation is sub-section 5.2. The identifier $length$ indicates the "duration" of the interval in question. Note also that the application of an offset $\langle ref, \{0\}\rangle$ has the effect of isolating the local origin of the set referred to.

Thus, given the relations A *startedBy* B and B *overlappedBy* C, using Allen's transitivity table technique, we deduce C *overlappedBy* A. We can express this using the core ontology as follows:

$\{\langle$

$\{max_1\},$

$\{$

$\{a, free, S_a\},$

$\{b, free, S_b\},$

$\{c, free, S_c\}$

$\},$

$\{$

$\{a, \langle ref, \{0\}\rangle, equals, b, \langle ref, \{0\}\rangle\},$

$\{a, superset, b\},$

$\{b, intersects, c\},$

$\{b, notSubset, b, \langle ref, \{length\}\rangle\}$

$\}$

$\rangle\}$

Note that because of the nature of the spatial representation used by the core ontology we have dimensioned the scenario. Thus we have assumed a 1-D application domain comprising max_1 addresses and defined each interval as a free object with some shape S. If this task ontology is then passed to a system such as the SPARTA system all possible configurations for the set of objects will be returned. In the above case each configuration will also illustrate a possible relation between the objects a and c, i.e. c overlapped by a.

9 PARAMETERISATION II: TOPOLOGICAL APPROACHES TO SPATIAL REASONING

The most obvious approach to spatial reasoning is to extend established temporal reasoning techniques so as to encompass more than one dimension. Thus we consider that the relations that may exist in one dimensional space exist along all axes of interest in an N-dimensional space. Individual relations are then defined by the intersection of any two one dimensional relations. This approach has been considered by many authors such as Freksa [24], Hernandez [32] and Puller and Egenhofer [45]. There are some 26 1-D relations (including Allen's 13 interval relations) that can exist between points, intervals, and points and intervals in a 1-D space. Extending these relations to N dimensions results in an exponential explosion of the number of relations, as the number of dimensions increases. This is the principal reason why one dimensional (mostly temporal) reasoning techniques do not lend themselves to easy translation to higher dimensions. This has been recognised by many authors (e.g. Hernandez [32], Chang et al. [10] and Frank et al. [23] amongst many others), and much work has been done on techniques to address these concerns. One example is the work of Malik and Binford [39] who represent information about individual dimensions independently and reason about each dimension in isolation. Alternatively Hernandez differentiates between projection and orientation relations. However, although these techniques all serve to reduce the severity of the problem, the basic difficulty — that the complexity of the problem increases exponentially with the number of dimensions under consideration — has not been removed.

An alternative approach is to consider only *topological* relations. Egenhofer defines such relations as follows:

"those spatial relations that are invariant under topological transformations and, therefore, preserved if the objects are translated, rotated or scaled." ([21])

Egenhofer uses a *9-Intersection* model (Egenhofer [21], Papadias et al. [44]) founded on an earlier *4-Intersection* model (Egenhofer [20]) to identify and manipulate such relations. In this model spatial objects are considered to have three parts (a) an interior, (b) a boundary and (c) an exterior. The topological relation between two "point sets" (sets of addresses), A and B, is then described by the nine intersections of the interior, boundary and exterior of A with those of B as

intersection (interior(A), interior(B))	intersection (interior(A), boundary(B))	intersection (interior(A), exterior(B))
intersection (boundary(A), interior(B))	intersection (boundary(A), boundary(B))	intersection (boundary(A), exterior(B))
intersection (exterior(A), interior(B))	intersection (exterior(A), boundary(B))	intersection (exterior(A), exterior(B))

Table 1: Egenhofer's 9-Intersection Table

*** Figure in here ***

Figure 3: The 8 "contiguous object" topological relations

demonstrated in Table 1.

Various topological invariants can be used to evaluate and characterise Egenhofer's 9-intersection model. However it can be shown that there are 2^9 (512) possible combinations (Egenhofer and Franzosa [22]) of which only a small subset can be physically realised as follows:

- 8 between objects without holes (contiguous region objects, i.e. homogeneous, 2-D connected object with connected boundaries).
- 18 between objects with holes.
- 33 between simple lines.
- 19 between an abject without a hole and a line.

Considering only the relationships between contiguous region objects (objects without holes) these can be labelled as shown in Figure 3. The labelling in the figure is applied to 2-D examples, however it is equally applicable to any number of dimensions.

The 8 topological relations presented in Figure 3 can be expressed in the core ontology as follows:

$$A\ disjoint\ B \Leftrightarrow \{a, \langle all, sw..se\rangle, notIntersects, b\}$$

$$A\ meet\ B \Leftrightarrow \{a, \langle all, sw..se\rangle, intersects, b\} \wedge \{a, notIntersects, b\}$$

$$A\ overlap\ B \Leftrightarrow \{a, intersects, b\} \wedge \{a, notSubset, b\}$$

$$A\ coveredBy\ B \Leftrightarrow \{a, subset, b\} \wedge \{a, notSubset, b, \langle shrink, \{1\}\rangle\}$$

$$A\ covers\ B \Leftrightarrow \{a, superset, b\} \wedge \{a, \langle shrink, \{1\}\rangle, notSuperset, b, \}$$

$$A\ inside\ B \Leftrightarrow \{a, subset, b, \langle shrink, \{1\}\rangle\}$$

$$A\ contains\ B \Leftrightarrow \{a, \langle shrink, \{1\}\rangle, superset, b\}$$

$$A\ equals\ B \Leftrightarrow \{a, equals, b\}$$

Note that the application of the offset $\langle all, sw..se\rangle$ has the effect of uniformally expanding a given set of addresses by a factor of one cell in all directions. The application of the offset $\langle shrink, \{1\}\rangle$ will have the opposite effect.

Given two 9-intersections, representing two topological relations between two pairs of objects A and B, and B and C, the 9-intersection for the relation linking A to C can be determined by deriving a combined 9-intersection from the two given 9-intersections. To this end Egenhofer specifies eight rules, in terms of a set of inference rules, to describe the dependencies of the intersections so that given 9-intersections linking A to B and B to C, the 9-intersection linking A to C can be produced. The significance here is that the relationships can be produced without recourse to a transitivity table, although the possible ways in which the above 8 relationships can be combined can be expressed in the form of such a table.

Thus given the relations *A covers B* and *B overlap C*, using Egenhofer's inference rules we can deduce that either, *A overlaps C* or *A covers C* or *A contains B*. We can express this using the core ontology as follows:

$$\{\langle\{max_1..max_N\},$$

$$\{$$

$$\{a, free, S_a\},$$

$$\{b, free, S_b\},$$

$$\{c, free, S_c\}$$

$$\},$$

$$\{$$

$$\{a, superset, b\},$$

$$\{a, \langle shrink, 1\rangle, notSuperset, b\},$$

$$\{b, intersects, c\},$$

$$\{b, notSubset, c\}$$

$$\}\rangle\}$$

Note that because the core ontology is essentially directed at quantitative applications we have again dimensioned the scenario - an N-dimensional space is assumed. Each of the three regions (objects) is then defined as a free object with shape S. If this task ontology were then to be passed to a system such as the SPARTA system all possible configurations for the objects will be returned, each configuration illustrating a possible relation between the objects a and c, i.e. that a overlaps, covers or contains c in this case.

The concept of topological relations forms an important part of spatio-temporal theory and has been adopted by many researchers working in the field. For example the spatial axiomisation of Clark [11, 12], Randell and Cohn [17, 46, 47, 48] (see also Cui et al. [18]), and Vieu [54] are all directed at a topological conceptualisation of spatial relations. It is interesting to note that

Clark's work is founded on the theory of mereology, first introduced by Lesniewski [37] early in this century as a component of a system proposed as a alternative to set theory. Since then, mereology has repeatedly been taken up as a theory of part-whole relation in philosophical formal ontology (for example Simons [50]).

10 Conclusions

In this paper we have describe a core ontology to support the development of spatio-temporal reasoning applications. The ontology has been used to define a significant number of applications — Geographic Information Systems (GIS) [14], noise pollution monitoring [9], environmental impact assessment [8], shape fitting [16], scheduling and timetabling [13], and AI problems such as the N-queens problem — however, the ontology is not yet complete. There are still many application areas, such as map and chart interaction, spatial simulation, environmental planning; which require further mechanisms to increase the expressiveness of the relations. With respect to the attributes that can be associated with spatial entities there are still a number of variations on the given list that require further investigation. For example we can conceive of partially shaped objects in that we may know a minimum shape for the object. There is thus still much work to do. However, the work to date, has resulted in a core ontology which is already widely applicable. Further, in its present from, it is both concise and convenient (as illustrated in sections 9 and 8). The core ontology described in this paper thus provides an excellent foundation for the required further work.

References

[1] J.F. Allen, Maintaining knowledge about temporal intervals, *Communications of the ACM* 26(11) (1983) 832-843.

[2] J.F. Allen, Towards a general theory of action and time, *Artificial Intelligence* 23 (1984) 123-154.

[3] J.F. Allen, Time and time again: the many ways to represent time, *International Journal of Intelligent Systems* 6 (1991) 341-355.

[4] J.F. Allen and P.J. Hayesm A Common Sense Theory of Time, *Proceedings IJCAI'85* (Morgan Kaufman, Los Altos 1985) 528-531.

[5] J.F. Allen and P.J. Hayes, Short time periods, *Proceedings IJCAI'87* (Morgan Kaufman, Los Altos 1987) 981-983.

[6] J.F. Allen and P.J. Hayes, Moments and points in an interval-based temporal logic. *Computat. Intell. (Canada)* 5 (1989) 225-238.

[7] . J.F. Allen and J.A. Koomen, Planning using a temporal world model, *Proceedings IJCAI'83* 2 (Morgan Kaufman, Los Altos 1983) 741-747.

[8] B. Beattie, F.P. Coenen, A. Hough, T.J.M. Bench-Capon, B.M. Diaz and M.J.R. Shave, Spatial Reasoning for environmental impact assessment, *Proceedings 3rd International Conference on GIS and Environmental Modelling* (National Centre for Geographic Information and Analysis, WWW and CD 1996).

[9] A.G.P. Brown, F.P. Coenen, M.J.R. Shave and M.W. Knight, An AI approach to noise prediction, To appear in: *Building Acoustics* 4(2) (1998).

[10] S.K. Chang, Q.Y. Shi, and C.W. Yan, Iconic indexing by 2-D strings, *IEEE Transactions on Pattern Analysis and Machine Intelligence* PAM1-9(3) (1987) 413-428.

[11] B.L. Clarke , A Calculus of individuals based on connection, *Notre Dame Journal of Formal Logic* 2(3), (1981).

[12] B.L. Clarke (1985), Individuals and points, *Notre Dame Journal of Formal Logic*, 26(1), (1985).

[13] F.P. Coenen, B. Beattie, T.J.M. Bench-Capon, M.J.R Shave and B.M. Diaz, Spatial reasoning for timetabling: the TIMETABLER system, *Proceedings of the 1st International Conference on the Practice and Theory of Automated Timetabling - ICPTAT'95*, (Napier University, Edinburgh, 1995) 57-68.

[14] F.P. Coenen, B. Beattie, T.J.M. Bench-Capon, B.M. Diaz and M.J.R. Shave, Spatial reasoning for geographic information systems, *Proceedings 1st International Conference on GeoComputation* 1 (School of Geography, University of Leeds 1996) 121-131.

[15] F.P. Coenen, B. Beattie, T.J.M. Bench-Capon, B.M. Diaz and M.J.R. Shave, A tesseral approach to N-dimensional spatial reasoning, in: A. Hameurlain and A.M. Tjoa eds., *Database and Expert Systems Applications, (Proceedings DEXA'97), Lecture Notes in Computer Science 1308* (Springer Verlag 1997) 633-642.

[16] F.P. Coenen, B. Beattie, T.J.M. Bench-Capon, B.M. Diaz and M.J.R. Shave, Spatio-temporal reasoning using a multi-dimensional tesseral representation, *Proceedings ECAI'98* (John Wiley & Sons 1998) 140-144.

[17] A.G. Cohn, A more expressive formulation of many sorted logic, *Jo of Automation and Reasoning* 3(2) (1987) 113-200.

[18] Z. Cui, A.G. Cohn and D.A. Randell. Qualitative simulation based on a logic of space and time, *Proceedings AISB Workshop "Qualitative and causal Reasoning" (QCR'93)* (University of Birmingham 1993).

[19] B.M. Diaz and A.B.M. Bell, *Spatial data processing using tesseral methods* (Natural Environment Research Council publications, Swindon, England 1987).

[20] M.J. Egenhofer, A formal definition of binary topological relationships, in: W. Litwin and H-J. Scheck eds., *Proceedings third International Conference on Foundations of Data Organisation and Algorithms (FODO), Lecture Notes in Computer Science 367* (Springer-Verlag, New-York, 1989) 457-472.

[21] M.J. Egenhofer, Deriving the composition of binary topological relations, *Journal of Visual Languages and Computing* 5 (1994) 133-149.

[22] M.J. Egenhofer and R. Franzosa, Point-set topological spatial relationships, *International Journal of Geographic Information Systems* 5 (1991) 161-174.

[23] A. Frank, R. Barrera and K.K. Al-taha, Temporal relations in geographic information systems, *ACM SIGMOS Record* 20(3) (1991) 85-91.

[24] C. Freksa, Qualitative spatial reasoning, in: D.M. Mark and A.U. Frank eds., *Cognitive and linguistic aspects of geographic space* (Kluwer, Dordrecht, Netherlands, 1991) 361-372.

[25] E.C. Freuder A Sufficient condition for backtrack-free search, *Journal of the ACM* 29(1) (1982) 24-32.

[26] A. Galton, A critical examination of Allen's theory of action and time. *Artificial Intelligence*, 42 (1990) 159-188.

[27] T. Gruber, *Ontolingua: A mechanism to support portable ontologies*, (Knowledge Systems Laboratory, Standford University, Stanford, USA 1992).

[28] T. Gruber, Toward principles for the design of ontologies used for knowledge sharing. Technical report KSL-93-4, (Knowledge Systems Laboratory, Stanford University, Stanford, USA 1993).

[29] N. Guarino and P. Giaretta, Ontologies and knowledge bases: towards a terminological clarification, in: N,J.I. Mars ed., *Towards Very Large Knowledge Bases* (IOS Press 1995).

[30] G. van Heijst, The role of ontologies in knowledge engineering, (Doctoral Thesis, University of Amsterdam, Amsterdam, The Netherlands 1995).

[31] G. van Heijst and G. Schreiber, CUE: ontology-based knowledge acquisition, in: L. Steels, A.Th. Schreiber and W. Van de Velde, W. eds., *A Future for Knowledge Acquisition, Proceedings of the 8th European Knowledge Acquisition Workshop EKAW'94, Vol. 867 of Lecture Notes in Artificial Intelligence* (Springer-Verlag, Berlin/Heidelberg, Germany 1994) 178-199.

[32] D. Hernandez, D. (1991). Relative representation of spatial knowledge: the 2-D case, in: D.M. Mark and A.U. Frank eds., *Cognitive and linguistic aspects of geographic space* (Kluwer, Dordrecht, Netherlands, 1991) 832-843.

[33] D.R. Kuokka, J. McGuire, J.C. Weber, J.M. Tenenbaum, T.R. Gruber, and G.R. Olsen, SHADE: Knowledge-Based Technology for the Re-Engineering Problem, *Annual Report 1993.*

[34] P.B. Ladkin, Models of axioms for time intervals, *Proceedings AAAI'87* (1987) 234-239.

[35] N, Leher, Knowledge representations specification language, (Technical Report, DARPA/Rome Laboratory Planning and Scheduling Initiative, Reference manual 1990)

[36] D.B. Lenat and R.V. Guha, Building large knowledge-based systems: representation and reasoning in the Cyc project, (Addison-Wesley, Reading, Massachusetts, United States 1990).

[37] S. Lesniewski, O Podstawach Matematyki (on the foundation of Mathematics), *Przeglad Filozoficzny (Philosophical Review)* 30-34 (1927-1931).

[38] G. Ligozat, Weak representations of interval algebras, *Proceedings AAAI'90* (1990) 715-720.

[39] J. Malik and T.O. Binford, Reasoning in time and space, *Proceedings IJCAI'87* (Morgan Kaufman, Los Altos 1983) 343-345.

[40] K. Morik, S. Wrobel, J-U. Kietz and W. Emde, Knowledge acquisition and machine learning: theory, methods and applications, *Knowledge-Based Systems* (Academic Press Limited, London, United Kingdom 1993).

[41] A. Mukerjee and G. Joe, A qualitative model of space, *Proceedings AAAI'90* (1990) 721-727.

[42] A. Newell, The knowledge level, *Artificial Intelligence* 18 (1982) 87-127.

[43] K. Nokel, Convex relations between time intervals, *Proceedings 5th Osterreichische Artificial-Intelligence-Tagung* (Springer, Berlin, 1989) 298-302.

[44] D. Papadias, Y. Theodoridis, T. Sellis and M.J. Egenhofer, Topological relations in the world of minimum bounding rectangles: a study with R-trees, *SIGMOD'95* (San Jose, CA, USA, 1995) 92-103.

[45] D. Puller and M.J. Egenhofer (1988). Towards formal definitions of topological relations among spatial objects, in: D. Marble ed., *Proceedings Third International Symposium on Spatial Data Handling* (1988) 225-242.

[46] A.D. Randell, D.A. (1991). Analysing the familiar: reasoning about space and time in the everyday world, PhD. Thesis, University of Warwick, UK.

[47] A.D. Randell, D.A., Z. Cui, and A.G. Cohn, An interval logic for space based on "connections". *Proceedings of ECAI'92* (1992).

[48] A.D. Randell, Z. Cui and A.G. Cohn, A spatial logic based on regions and connection. *Proceedings 3rd International Conference on Principals of Knowledge Representation and Reasoning* (Morgan Kaufmann 1992).

[49] I. Sim and G. Rennels, Developing a clinical trial ontology: comments on domain modelling and ontological Reuse, Knowledge Systems Laboratory Medical Computer Science, KSL-95-60, June 1995.

[50] P. Simons, *Parts — a study in ontology*, (Clarendon Press, Oxford 1987).

[51] B. Smith, Mereotopology: a theory of parts and boundaries, *Data and Knowledge Engineering* 20 (1996) 287-303.

[52] B. Smith and D. Mark, Ontology and geographic kinds, *Proceedings, International Symposium on Spatial Data Handling (SDH'98), Vancouver, Canada* (Proceedings in Press 1998).

[53] M. Uschold and M. Gruninger, Ontologies: principles, methods and applications, *Knowledge Engineering Review* 11(2) (1996).

[54] L. Vieu, A logical framework for reasoning about space, in: A.U. Frank and I. Campari, eds., *Spatial Information theory: A Theoretical Basis for GIS, proceedings COSIT'93* (Springer-Verlag, Berlin, 1993) 25-35.

[55] M.B. Vilain and H. Kautz, Constraint propagation algorithms for temporal reasoning, *Proceedings AAAI'86* (Morgan Kaufmann, Los Altos, CA, 1986) 377-382.

[56] M.B. Vilain, H. Kautz and P.G. van Beek, Constraint propagation algorithms for temporal reasoning: a revised report, in: A.D. Weld and J. de Kleer, eds., *Readings in Qualitative Reasoning about Physical Systems* (Morgan Kaufmann, San Mateo, CA., 1989) 373-381.

[57] P.R.S. Visser, Knowledge Specification for Multiple Legal Tasks; A Case Study of the Interaction Problem in the Legal Domain, *Computer/Law Series* (Kluwer Law International, Dordrecht, The Netherlands, 1995).

[58] P.R.S. Visser and T.J.M. Bench-Capon, A comparison of four ontologies for the design of legal knowledge systems, *Artificial Intelligence and Law* (to appear 1998).

[59] G. Wiederhold, Interoperation, mediation, and ontologies, *Proceedings International Symposium on Fifth Generation Computer Systems (FGCS94), Workshop on Heterogeneous Cooperative Knowledge-Bases* W3, (ICOT, Tokyo, Japan 1994), 33-48.

[60] B. Wielinga, G. Schreiber, W. Jansweijer, A. Anjewierden and F. van Harmelen, Framework for expressing ontologies, DO1b.1-1-Framework-1.1-UvA-BW+GS+WJ+AA, University of Amsterdam, 1994.

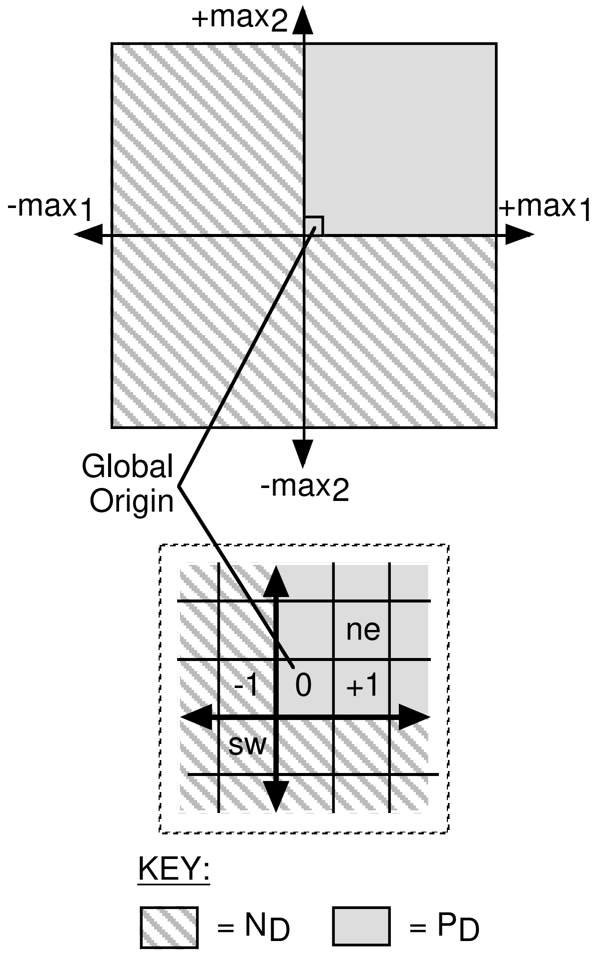

Figure 1: Example 2D space

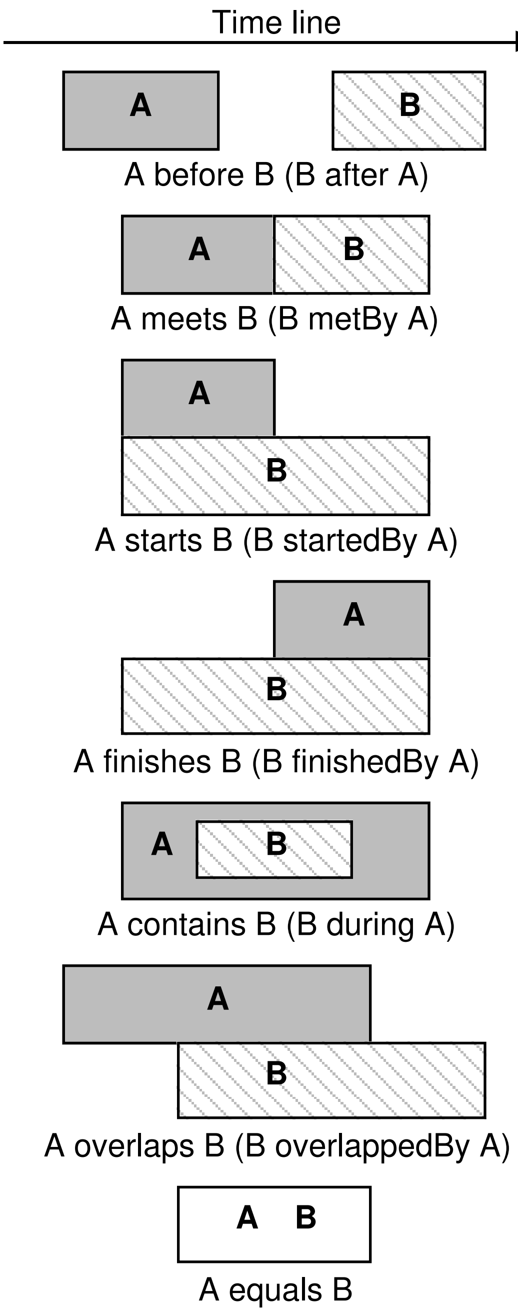

Figure 2: Allen's 13 interval relations

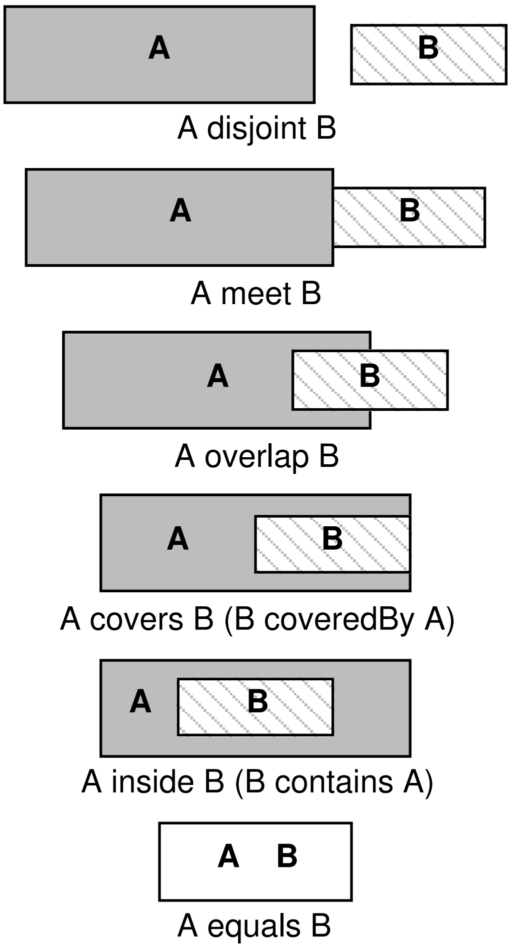

Figure 3: The 8 “contiguous object” topological relations