Ontology Based Data Access

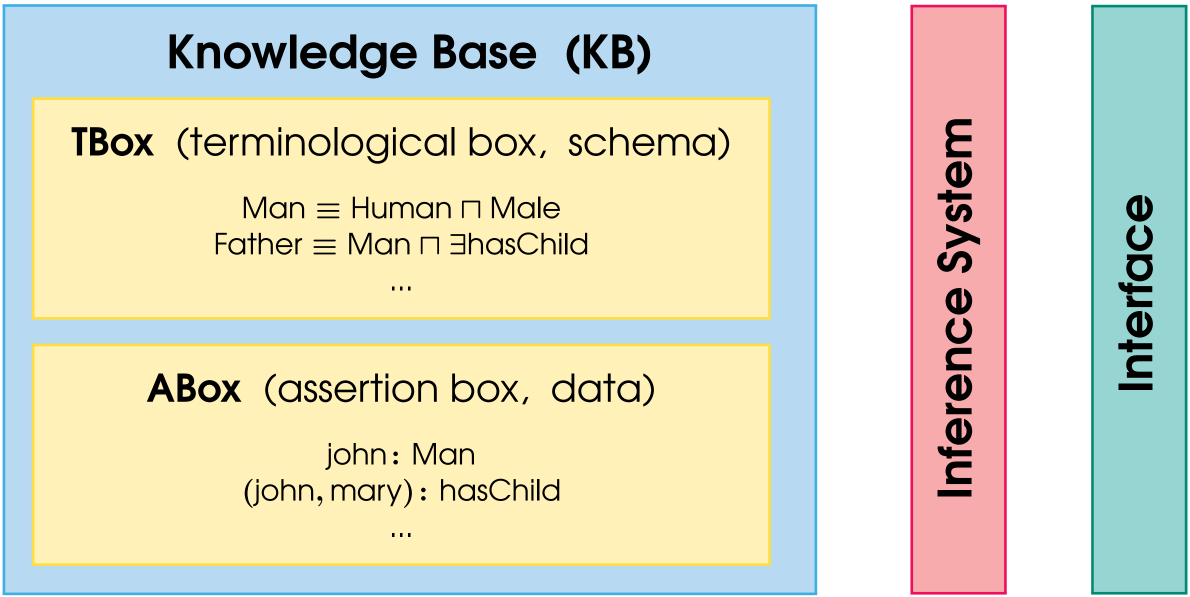
Vision: Ontologies at the Core of Information Systems

- Usage of all system resources (data and services) is done through a domain conceptualization.
- Cooperation between systems is done at the level of the conceptualizations.
- This implies:
  - Hide to the user where and how data and services are stored or implemented;
  - Present to the user a conceptual view of the data and services.

Ontology based Data Access

- An ontology provides meta-information about the data and the vocabulary used to query the data. It can impose constraints on the data.
- Actual data can be incomplete w.r.t. such meta-information and constraints. So data should be stored using open world semantics rather than closed world semantics: use ABoxes instead of relational database instances.
- During query answering, the system has to take into account the ontology.

We discuss ontology based data access in the framework of description logic knowledge bases.


Knowledge Base (KB)
TBox (terminological box, schema)
Man ≡ Human ⊓ Male
Father ≡ Man ⊓ ∃hasChild
...
ABox (assertion box, data)
john : Man
(john, mary) : hasChild
...
Inference System
Interface

Knowledge Base (= Ontology with database instance)

A **knowledge base** $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ consists of a TBox $\mathcal{T}$ and a simple ABox $\mathcal{A}$ (or, equivalently, a database instance).

We combine the open world semantics for TBoxes and ABoxes in the obvious manner, and obtain an **open world semantics** for knowledge bases.

An interpretation $\mathcal{I}$ **satisfies** a knowledge base $(\mathcal{T}, \mathcal{A})$, in symbols

$$\mathcal{I} \models (\mathcal{T}, \mathcal{A}),$$

if it satisfies both $\mathcal{T}$ and $\mathcal{A}$. In this case we also say that $\mathcal{I}$ is a **model** of $(\mathcal{T}, \mathcal{A})$. The set of models of $(\mathcal{T}, \mathcal{A})$ is denoted by $\mathbf{Mod}(\mathcal{T}, \mathcal{A})$.

Certain Answers

Given a knowledge base $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ and an FOPL query $F(x_1, \ldots, x_k)$, we say that $(a_1, \ldots, a_k)$ is a **certain answer** to $F(x_1, \ldots, x_k)$ by $\mathcal{K}$, in symbols

$$\mathcal{K} \models F(a_1, \ldots, a_k),$$

if

- $a_1, \ldots, a_k$ are individual names in $\mathcal{A}$;
- for all interpretations $\mathcal{I}$:

$$\mathcal{I} \models \mathcal{K} \quad \Rightarrow \quad \mathcal{I} \models F(a_1, \ldots, a_k).$$

The set of certain answers given to F by $\mathcal{K}$ is defined as:

$$\textbf{certanswer}(F, \mathcal{K}) = \{(a_1, \ldots, a_k) \mid \mathcal{K} \models F(a_1, \ldots, a_k)\}$$

Boolean Queries

Let $\mathcal{K}$ be a knowledge base. For a query $\boldsymbol{F}$ without variables (Boolean query), we say that

- the certain answer given by $\mathcal{K}$ is "yes" if $\mathcal{I} \models \boldsymbol{F}$, for all interpretations $\mathcal{I}$ satisfying $\mathcal{K}$;
- the certain answer given by $\mathcal{K}$ is "no" if $\mathcal{I} \not\models \boldsymbol{F}$, for all interpretations $\mathcal{I}$ satisfying $\mathcal{K}$.
- Otherwise the certain answer is: "Don't know".

Example

Consider the TBox $\mathcal{T}_U$:

- $\textbf{BritishUniversity} \sqsubseteq \textbf{University}$;
- $\textbf{University} \sqcap \textbf{Student} \sqsubseteq \bot$;
- $\top \sqsubseteq \forall \textbf{registered\_at}.\textbf{University}$;
- $\top \sqsubseteq \forall \textbf{student\_at}.\textbf{University}$;
- $\exists \textbf{student\_at}.\top \sqsubseteq \textbf{Student}$;
- $\textbf{Student} \sqsubseteq \exists \textbf{student\_at}.\top$;
- $\textbf{NonBritishUni} \equiv \textbf{University} \sqcap \neg \textbf{BritishUniversity}$.

Example (continued)

and the simple ABox (equivalently, database instance) $\mathcal{A}$:

- $\mathsf{NonBritishUni}(\mathsf{CMU})$
- $\mathsf{Institution}(\mathsf{Harvard}), \quad \mathsf{Institution}(\mathsf{FUBerlin})$
- $\mathsf{BritishUniversity}(\mathsf{LU}), \quad \mathsf{BritishUniversity}(\mathsf{MU})$
- $\mathsf{Student}(\mathsf{Tim})$
- $\mathsf{registered}(\mathsf{Tim}, \mathsf{LU}), \quad \mathsf{registered}(\mathsf{Bob}, \mathsf{MU})$
- $\mathsf{student\_at}(\mathsf{Tom}, \mathsf{Harvard})$

Example (continued)

Denote by $\mathcal{I}_{\mathcal{A}}$ the interpretation corresponding to the database instance $\mathcal{A}$:

- $\Delta^{\mathcal{I}_{\mathcal{A}}} = \{\textbf{CMU}, \textbf{Harvard}, \textbf{FUBerlin}, \textbf{Tim}, \textbf{Tom}, \textbf{Bob}, \textbf{MU}, \textbf{LU}\}$;
- $\textbf{NonBritishUni}^{\mathcal{I}_{\mathcal{A}}} = \{\textbf{CMU}\}$;
- $\textbf{Institution}^{\mathcal{I}_{\mathcal{A}}} = \{\textbf{Harvard}, \textbf{FUBerlin}\}$;
- $\textbf{BritishUniversity}^{\mathcal{I}_{\mathcal{A}}} = \{\textbf{LU}, \textbf{MU}\}$;
- $\textbf{Student}^{\mathcal{I}_{\mathcal{A}}} = \{\textbf{Tim}\}$;
- $\textbf{registered\_at}^{\mathcal{I}_{\mathcal{A}}} = \{(\textbf{Tim}, \textbf{LU}), (\textbf{Bob}, \textbf{MU})\}$;
- $\textbf{student\_at}^{\mathcal{I}_{\mathcal{A}}} = \{(\textbf{Tom}, \textbf{Harvard})\}$.

(Certain) Answers

In the table below, we consider Boolean queries $C(a)$ (in description logic notation!) and give the (certain) answer to $C(a)$ of the database instance $\mathcal{I}_{\mathcal{A}}$, the ABox $\mathcal{A}$, and the knowledge base $\mathcal{K}_U = (\mathcal{T}_U, \mathcal{A})$.

Boolean Query	$\mathcal{I}_{\mathcal{A}}$	Abox $\mathcal{A}$	KB $\mathcal{K}_U$
University(**CMU**)	No	Don't know	Yes
University(**Harvard**)	No	Don't know	Yes
NonBritishUni(**CMU**)	Yes	Yes	Yes
Student(**Tim**)	Yes	Yes	Yes
Student(**Tom**)	No	Don't know	Yes
∃**student_at**.⊤(**Tom**)	Yes	Yes	Yes
∃**student_at**.⊤(**Tim**)	No	Don't know	Yes
(**Student** ⊓ ¬**University**)(**Tim**)	Yes	Don't know	Yes
(**Institution** ⊓ ¬**University**)(**FUBerlin**)	Yes	Don't know	Don't know

Example

Let $\mathcal{S} = (\mathcal{O}, \mathcal{B})$ be a knowledge base with simple ABox $\mathcal{B}$ given by

$$\mathbf{Person(john)}, \quad \mathbf{Person(nick)}, \quad \mathbf{Person(toni)}$$

$$\mathbf{hasFather(john, nick)}, \quad \mathbf{hasFather(nick, toni)}$$

and TBox $\mathcal{O}$ defined as

$$\mathcal{O} = \{\mathbf{Person} \sqsubseteq \exists \mathbf{has\_Father.Person}\}$$

For the FOPL query

$$F(x, y) = \mathbf{hasFather}(x, y)$$

we obtain

$$\mathbf{certanswer}(F, \mathcal{S}) = \{(\mathbf{john}, \mathbf{nick}), (\mathbf{nick}, \mathbf{toni})\}.$$

Example

- For the query

$$F(x) = \exists y.\mathsf{hasFather}(x, y)$$

we obtain

$$\mathsf{certanswer}(F(x), \mathcal{S}) = \{\mathsf{john}, \mathsf{nick}, \mathsf{toni}\}$$

- For the query

$$F(x) = \exists y_1 \exists y_2 \exists y_3.(\mathsf{hasFather}(x, y_1) \wedge \mathsf{hasFather}(y_1, y_2) \wedge \mathsf{hasFather}(y_2, y_3))$$

we obtain

$$\mathsf{certanswer}(F(x), \mathcal{S}) = \{\mathsf{john}, \mathsf{nick}, \mathsf{toni}\}$$

- For the query

$$F(x, y_3) = \exists y_1 \exists y_2.(\mathsf{hasFather}(x, y_1) \wedge \mathsf{hasFather}(y_1, y_2) \wedge \mathsf{hasFather}(y_2, y_3))$$

we obtain

$$\mathsf{certanswer}(F(x, y_3), \mathcal{S}) = \emptyset$$

Complexity of querying $(\mathcal{T}, \mathcal{A})$

Consider, for simplicity, Boolean queries. There are two different ways of measuring the complexity of querying:

- Data complexity: Measures the time/space needed to evaluate a fixed query F for a fixed TBox $\mathcal{T}$ in $(\mathcal{T}, \mathcal{A})$ (i.e., check $\mathcal{T}, \mathcal{A}) \models F$). The only input variable is the size of $\mathcal{A}$.

- Combined complexity: Measure the time/space needed to evaluate a query in $(\mathcal{T}, \mathcal{A})$. The input variables are the size of the query, the size of $\mathcal{T}$, and the size of $\mathcal{A}$.

Data complexity is relevant if $\mathcal{T}$ and the query are very small compared to $\mathcal{A}$. This is the case in most applications.

Non-Tractability of Query answering in $\mathcal{ALC}$ in Data Complexity

A graph G is a pair (W, E) consisting of a set W and a symmetric relation E on W.

G is 3-colorable if there exist subsets **blue**, **red**, and **green** of W such that

- the sets **blue**, **green**, and **red** are mutually disjoint;
- **blue** $\cup$ **red** $\cup$ **green** $= W$;
- if $(a, b) \in E$, then a and b do not have the same color.

3-colorability of graphs is an NP-complete problem.

3-Colorability as a Query Answering Problem

Assume $G = (W, E)$ is given. Construct the ABox $\mathcal{A}_G$ by taking a role name r and setting

- $r(a, b) \in \mathcal{A}$ for all $a, b \in W$ with $(a, b) \in E$.

Construct the TBox $\mathcal{ALC}$ TBox $\mathcal{T}_C$ by taking concept names **Blue**, **Green**, and **Red** and taking the inclusions:

- $\top \sqsubseteq$ **Blue** $\sqcup$ **Green** $\sqcup$ **Red**
- **Blue** $\sqcap \exists r.$**Blue** $\sqsubseteq$ **Clash**
- **Red** $\sqcap \exists r.$**Red** $\sqsubseteq$ **Clash**
- **Green** $\sqcap \exists r.$**Green** $\sqsubseteq$ **Clash**

Let $F = \exists x$ **Clash**(x). Then $(\mathcal{T}_C, \mathcal{A}_G) \models F$ if, and only if, G is not 3-colorable.

Restricting the Description Logic and the Query Language

- FOPL is too expressive as a query language for knowledge bases. The combined complexity of querying even DL-Lite or $\mathcal{EL}$ knowledge bases with FOPL queries is undecidable.

- For $\mathcal{ALC}$ knowledge bases and basic Boolean queries of the form $\exists x A(x)$, (A a concept name) query answering is still non-tractable. The best algorithms for query answering in this case are extensions of the $\mathcal{ALC}$ tableaux algorithms discussed above.

- We consider
  - knowledge bases in $\mathcal{EL}$, restricted Schema.org, and DL-Lite only;
  - queries in $\mathcal{EL}$ and conjunctive queries only.

Answering $\mathcal{EL}$-Queries in $\mathcal{EL}$ Knowledge Bases

$\mathcal{EL}$ Concept Queries

An **$\mathcal{EL}$ concept query** is a Boolean query of the form

$$C(a)$$

where C is an $\mathcal{EL}$-concept and a an individual name. We develop a method for answering $\mathcal{EL}$ concept queries in knowledge bases

$$(\mathcal{T}, \mathcal{A}),$$

where $\mathcal{T}$ is a $\mathcal{EL}$-TBox and $\mathcal{A}$ a simple ABox.

Note: Then we also have a method for computing

$$\textbf{certanswer}(C(x), (\mathcal{T}, \mathcal{A})) = \{a \mid (\mathcal{T}, \mathcal{A}) \models C(a)\}$$

Fundamental Idea: reduce knowledge base querying to relational database querying

To answer the question whether

$$(\mathcal{T}, \mathcal{A}) \models C(a)$$

we construct from $(\mathcal{T}, \mathcal{A})$ a finite interpretation $\mathcal{I}_{\mathcal{T},\mathcal{A}}$ such that

$$(\mathcal{T}, \mathcal{A}) \models C(a) \quad \Leftrightarrow \quad \mathcal{I}_{\mathcal{T},\mathcal{A}} \models C(a).$$

Thus, we reduce ontology based reasoning to database querying. After this construction database technology can be used to process queries.

Note: Such a reduction works only for a very limited number of ontology and query languages!

From $(\mathcal{T}, \mathcal{A})$ to $\mathcal{I}_{\mathcal{T},\mathcal{A}}$

The algorithm constructing $\mathcal{I}_{\mathcal{T},\mathcal{A}}$ is a rather simple extension of the algorithm deciding concept subsumption $A \sqsubseteq_{\mathcal{T}} B$ for $\mathcal{EL}$.

Firstly, we assume again that $\mathcal{T}$ is in normal form: it consists of inclusions of the form

- $A \sqsubseteq B$, where A and B are concept names;
- $A_1 \sqcap A_2 \sqsubseteq B$, where A_1, A_2, B are concept names;
- $A \sqsubseteq \exists r.B$, where A, B are concept names;
- $\exists r.A \sqsubseteq B$, where A, B are concept names.

General Description

The domain $\Delta^{\mathcal{I}_{\mathcal{T},\mathcal{A}}}$ of $\mathcal{I}_{\mathcal{T},\mathcal{A}}$ consists of

- all individual names a that occur in $\mathcal{A}$;
- objects d_A, for every concept name A in $\mathcal{T}$. (In the description of the subsumption algorithm d_A is denoted by A!)

It remains to compute

- $r^{\mathcal{I}_{\mathcal{T},\mathcal{A}}}$, for all role names r;
- $A^{\mathcal{I}_{\mathcal{T},\mathcal{A}}}$, for all concept names A.

This is done by computing functions S and R that are very similar to the functions introduced in the subsumption algorithm.

Algorithm Computing $\mathcal{I}_{\mathcal{T},\mathcal{A}}$

Given $\mathcal{T}$ in normal form and ABox $\mathcal{A}$, we compute functions S and R:

- S maps every $d \in \Delta^{\mathcal{I}_{\mathcal{T},\mathcal{A}}}$ to a set $S(d)$ of concept names.

  We then set $d \in A^{\mathcal{I}_{\mathcal{T},\mathcal{A}}}$ if $A \in S(d)$;

- R maps every role name r to a set $R(r)$ of pairs (d_1, d_2) in $\Delta^{\mathcal{I}_{\mathcal{T},\mathcal{A}}}$.

  We then set $(d_1, d_2) \in r^{\mathcal{I}_{\mathcal{T},\mathcal{A}}}$ if $(d_1, d_2) \in R(r)$.

We initialise S and R as follows:

- $S(a) = \{B \mid B(a) \in \mathcal{A}\}$;
- $S(d_A) = \{A\}$ (as in the subumption algorithm, where we had $d_A = A$!)
- $R(r) = \{(a, b) \mid r(a, b) \in \mathcal{A}\}$.

Algorithm

Apply the following four rules to S and R exhaustively:

(simpleR) If $A \in S(d)$ and $A \sqsubseteq B \in \mathcal{T}$ and $B \notin S(d)$, then

$$S(d) := S(d) \cup \{B\}.$$

(conjR) If $A_1, A_2 \in S(d)$ and $A_1 \sqcap A_2 \sqsubseteq B \in \mathcal{T}$ and $B \notin S(d)$, then

$$S(d) := S(d) \cup \{B\}.$$

(rightR) If $A \in S(d)$ and $A \sqsubseteq \exists r.B \in \mathcal{T}$ and $(d, d_B) \notin R(r)$, then

$$R(r) := R(r) \cup \{(d, d_B)\}.$$

(leftR) If $(d_1, d_2) \in R(r)$ and $B \in S(d_2)$ and $\exists r.B \sqsubseteq A \in \mathcal{T}$ and $A \notin S(d_1)$, then

$$S(d_1) := S(d_1) \cup \{A\}.$$

Example

Let $\mathcal{T}$ be defined as:

$$
\begin{array}{rcl}
\mathbf{BasketballClub} & \sqsubseteq & \mathbf{Club} \\
\mathbf{BasketballPlayer} & \sqsubseteq & \exists \mathbf{plays\_for.BasketballClub} \\
\exists \mathbf{plays\_for.Club} & \sqsubseteq & \mathbf{Player} \\
\mathbf{Player} & \sqsubseteq & \mathbf{Human\_being}
\end{array}
$$

Let $\mathcal{A}$ be defined as:

$$
\mathbf{Basketballplayer(bob)}, \quad \mathbf{Player(jim)}
$$

$$
\mathbf{Basketballclub(tigers)}, \quad \mathbf{Club(lions)}
$$

$$
\mathbf{plays\_for(rob, tigers)}, \quad \mathbf{plays\_for(bob, lions)}
$$

Construction of $\mathcal{I}_{\mathcal{T},\mathcal{A}}$

The initial assignment (with obvious abbreviations) is given by

$$
\begin{aligned}
S(d_{\textsf{Basketclub}}) &= \{\textsf{Basketclub}\} \\
S(d_{\textsf{Basketplayer}}) &= \{\textsf{Basketplayer}\} \\
S(d_{\textsf{Club}}) &= \{\textsf{Club}\} \\
S(d_{\textsf{Player}}) &= \{\textsf{Player}\} \\
S(d_{\textsf{Human}}) &= \{\textsf{Human}\} \\
R(\textsf{plays\_for}) &= \{(\textsf{rob}, \textsf{tigers}), (\textsf{bob}, \textsf{lion})\} \\
S(\textsf{bob}) &= \{\textsf{Baskplayer}\} \\
S(\textsf{jim}) &= \{\textsf{Player}\} \\
S(\textsf{tigers}) &= \{\textsf{Baskclub}\} \\
S(\textsf{lions}) &= \{\textsf{Club}\} \\
S(\textsf{rob}) &= \emptyset
\end{aligned}
$$

Rule Applications

Now applications of (simpleR), (rightR), (leftR) are step-by-step as follows:

- Update S using (simpleR):

$$S(d_{\textsf{BaskClub}}) = \{\textsf{BaskClub}, \textsf{Club}\}.$$

- Update R using (rightR):

$$R(\textsf{plays\_for}) = \{(d_{\textsf{Baskplayer}}, d_{\textsf{BaskClub}})\}.$$

- Update S using (simpleR):

$$S(d_{\textsf{Player}}) = \{\textsf{Player}, \textsf{Human}\}.$$

- Update S using (leftR):

$$S(d_{\textsf{Baskplayer}}) = \{\textsf{Baskplayer}, \textsf{Player}\}.$$

- Update S using (simpleR):

$$S(d_{\textsf{Baskplayer}}) = \{\textsf{Baskplayer}, \textsf{Player}, \textsf{Human}\}.$$

Rule applications continued

- Update S using (simpleR):
$$S(\textbf{tigers}) = \{\textbf{BaskClub}, \textbf{Club}\}.$$
- Update S using (simpleR):
$$S(\textbf{jim}) = \{\textbf{Player}, \textbf{Human}\}.$$
- Update R using (rightR):
$$R(\textbf{plays\_for}) = \{(d_{\textbf{Baskplayer}}, d_{\textbf{BaskClub}}), (\textbf{bob}, d_{\textbf{BaskClub}})\}.$$
- Since $S(\textbf{bob})$ contains **Baskplayer**, we obtain using rules:
$$S(\textbf{bob}) = \{\textbf{Baskplayer}, \textbf{Player}, \textbf{Human}\}.$$
- Update S using (leftR):
$$S(\textbf{rob}) = \{\textbf{Player}\}.$$
- Update S using (leftR):
$$S(\textbf{rob}) = \{\textbf{Player}, \textbf{Human}\}.$$

The final assignment

$$
\begin{aligned}
S(d_{\mathsf{Baskclub}}) &= \{\mathsf{Baskclub}, \mathsf{Club}\} \\
S(d_{\mathsf{Baskplayer}}) &= \{\mathsf{Baskplayer}, \mathsf{Player}, \mathsf{Human}\} \\
S(d_{\mathsf{Club}}) &= \{\mathsf{Club}\} \\
S(d_{\mathsf{Player}}) &= \{\mathsf{Player}, \mathsf{Human}\} \\
S(d_{\mathsf{Human}}) &= \{\mathsf{Human}\} \\
R(\mathsf{plays\_for}) &= \{(d_{\mathsf{Baskplayer}}, d_{\mathsf{BaskClub}}), (\mathsf{rob}, \mathsf{tigers}), (\mathsf{bob}, \mathsf{lion}), (\mathsf{bob}, d_{\mathsf{BaskClub}})\} \\
S(\mathsf{bob}) &= \{\mathsf{Baskplayer}, \mathsf{Player}, \mathsf{Human}\} \\
S(\mathsf{jim}) &= \{\mathsf{Player}\} \\
S(\mathsf{tigers}) &= \{\mathsf{Baskclub}\} \\
S(\mathsf{lions}) &= \{\mathsf{Club}\} \\
S(\mathsf{rob}) &= \{\mathsf{Player}, \mathsf{Human}\}
\end{aligned}
$$

The interpretation $\mathcal{I}_{\mathcal{T},\mathcal{A}}$

- $\Delta^{\mathcal{I}}_{\mathcal{T},\mathcal{A}} = \{d_{\text{Baskclub}}, d_{\text{Baskplayer}}, d_{\text{Club}}, d_{\text{Player}}, d_{\text{Human}}, \textbf{bob}, \textbf{jim}, \textbf{tigers}, \textbf{lions}, \textbf{rob}\}$;
- $\textbf{Baskclub}^{\mathcal{I}_{\mathcal{T},\mathcal{A}}} = \{d_{\text{Baskclub}}, \textbf{tigers}\}$;
- $\textbf{Club}^{\mathcal{I}_{\mathcal{T},\mathcal{A}}} = \{d_{\text{Club}}, d_{\text{Baskclub}}, \textbf{tigers}\}$;
- $\textbf{Baskplayer}^{\mathcal{I}_{\mathcal{T},\mathcal{A}}} = \{d_{\text{Baskplayer}}, \textbf{bob}\}$;
- $\textbf{Player}^{\mathcal{I}_{\mathcal{T},\mathcal{A}}} = \{d_{\text{Player}}, d_{\text{Baskplayer}}, \textbf{bob}, \textbf{jim}, \textbf{rob}\}$;
- $\textbf{Human}^{\mathcal{I}_{\mathcal{T},\mathcal{A}}} = \{d_{\text{Human}}, d_{\text{Player}}, d_{\text{Baskplayer}}, \textbf{bob}, \textbf{jim}, \textbf{rob}\}$;
- $\textbf{plays\_for}^{\mathcal{I}_{\mathcal{T},\mathcal{A}}} = \{(d_{\text{Baskplayer}}, d_{\text{BaskClub}}), (\textbf{rob}, \textbf{tigers}), (\textbf{bob}, \textbf{lion}), (\textbf{bob}, d_{\text{BaskClub}})\}$.

Now

$$(\mathcal{T}, \mathcal{A}) \models C(a) \quad \Leftrightarrow \quad \mathcal{I}_{\mathcal{T},\mathcal{A}} \models C(a)$$

for all $\mathcal{EL}$ concepts C and a in $\mathcal{A}$. For example,

$$\mathcal{I}_{\mathcal{T},\mathcal{A}} \models \exists \textbf{plays\_for}.\textbf{Baskclub}(\textbf{bob}), \quad \mathcal{I}_{\mathcal{T},\mathcal{A}} \models \textbf{Human}(\textbf{rob})$$

Another Example

We consider the knowledge base $\mathcal{S} = (\mathcal{O}, \mathcal{B})$ given by the ABox $\mathcal{B}$ consisting of

$$\mathbf{Person(john)}, \quad \mathbf{Person(nick)}, \quad \mathbf{Person(toni)}$$

$$\mathbf{hasFather(john, nick)}, \quad \mathbf{hasFather(nick, toni)}$$

and the TBox $\mathcal{O}$ given by

$$\mathcal{O} = \{\mathbf{Person} \sqsubseteq \exists \mathbf{has\_Father.Person}\}.$$

We construct $\mathcal{I}_{\mathcal{S}}$.

Constructing $\mathcal{I}_{\mathcal{S}}$

The initial assignment is given by

$$\begin{aligned} S(d_{\textbf{Person}}) &= \{\textbf{Person}\} \\ S(\textbf{john}) &= \{\textbf{Person}\} \\ S(\textbf{nick}) &= \{\textbf{Person}\} \\ S(\textbf{toni}) &= \{\textbf{Person}\} \\ R(\textbf{hasFather}) &= \{(\textbf{john}, \textbf{nick}), (\textbf{nick}, \textbf{toni})\} \end{aligned}$$

Four applications of the rule (rightR) add

$$\{(\textbf{john}, d_{\textbf{Person}}), (\textbf{nick}, d_{\textbf{Person}}), (\textbf{toni}, d_{\textbf{Person}}), (d_{\textbf{Person}}, d_{\textbf{Person}})\}$$

to the original $R(\textbf{hasFather})$. After that, no rule is applicable.

The interpretation $\mathcal{I}_\mathcal{S}$

We obtain the interpretation $\mathcal{I}_\mathcal{S}$ defined as

$$\begin{aligned}
\Delta^{\mathcal{I}_\mathcal{S}} &= \{d_{\mathsf{Person}}, \mathsf{john}, \mathsf{nick}, \mathsf{toni}\} \\
\mathsf{Person}^{\mathcal{I}_\mathcal{S}} &= \{d_{\mathsf{Person}}, \mathsf{john}, \mathsf{nick}, \mathsf{toni}\} \\
\mathsf{hasFather}^{\mathcal{I}_\mathcal{S}} &= \{(\mathsf{john}, \mathsf{nick}), (\mathsf{nick}, \mathsf{toni}), (\mathsf{john}, d_{\mathsf{Person}}), \\
&\quad (\mathsf{nick}, d_{\mathsf{Person}}), (\mathsf{toni}, d_{\mathsf{Person}}), (d_{\mathsf{Person}}, d_{\mathsf{Person}})\}
\end{aligned}$$

We have

$$\mathcal{S} \models C(a) \quad \Leftrightarrow \quad \mathcal{I}_\mathcal{S} \models C(a)$$

for all $\mathcal{EL}$ concepts C and a from $\mathcal{B}$. For example

$$\mathcal{I}_\mathcal{S} \models \exists \mathsf{hasFather}.\exists \mathsf{hasFather}.\mathsf{Person}(\mathsf{toni})$$

Answering Conjunctive Queries by Rewriting in DL-Lite

Conjunctive Queries

A FOPL query $F(x_1, \ldots, x_k)$ is a **conjunctive query** if it is constructed from atomic formulas $P(y_1, \ldots, y_n)$ using $\wedge$ and $\exists$ only.

In SQL, conjunctive queries correspond to

"Select-from-where queries",

where the "where-conditions" use only conjunctions of "$=$-conditions".

Examples

The queries

- $F(x) = \textbf{Person}(x)$;
- $F(x) = \exists y.\textbf{hasFather}(x, y)$;
- $F(x) = \exists y_1 \exists y_2 \exists y_3.(\textbf{hasFather}(x, y_1) \wedge \textbf{hasFather}(y_1, y_2); \wedge \textbf{hasFather}(y_2, y_3))$,
- $F(x, y_3) = \exists y_1 \exists y_2.(\textbf{hasFather}(x, y_1) \wedge \textbf{hasFather}(y_1, y_2) \wedge \textbf{hasFather}(y_2, y_3))$.

are conjunctive queries.

Query Rewriting for DL-Lite

Given a DL-Lite TBox $\mathcal{T}$ and a conjunctive query $F(x_1, \ldots, x_n)$ one can compute a FOPL query

$$F_{\mathcal{T}}(x_1, \ldots, x_n)$$

such that for every simple ABox $\mathcal{A}$, the database instance $\mathcal{I}_{\mathcal{A}}$ corresponding to $\mathcal{A}$, and any $a_1, \ldots, a_n$ in $\mathsf{Ind}(\mathcal{A})$ the following holds:

$$(\mathcal{T}, \mathcal{A}) \models F(a_1, \ldots, a_n) \quad \Leftrightarrow \quad \mathcal{I}_{\mathcal{A}} \models F_{\mathcal{T}}(a_1, \ldots, a_n).$$

Checking $\mathcal{I}_{\mathcal{A}} \models F_{\mathcal{T}}(a_1, \ldots, a_n)$ is again a standard database evaluation problem.

We first illustrate the construction of $F_{\mathcal{T}}(x_1, \ldots, x_n)$ using an example.

Example: Rewriting

For the TBox

$$\mathcal{T} = \{\textsf{Basketballplayer} \sqsubseteq \textsf{Player}, \textsf{Footballplayer} \sqsubseteq \textsf{Player}, \textsf{Handballplayer} \sqsubseteq \textsf{Player}\}$$

and the query

$$F(x) = \textsf{Player}(x)$$

one can take

$$F_{\mathcal{T}}(x) = \textsf{Basketballplayer}(x) \vee \textsf{Footballplayer}(x) \vee \textsf{Handballplayer}(x) \vee \textsf{Player}(x)$$

Rewriting Algorithm for Fragment DL-Lite$_{\text{tiny}}$

We give the rewriting algorithm for a small fragment DL-Lite$_{\text{tiny}}$ of DL-Lite (and Schema.org) consisting of inclusions of the form

- $A \sqsubseteq B$, where A and B are concept names;
- domain restrictions $\exists r.\top \sqsubseteq A$, where r is a role name and A a concept name;
- range restrictions $\exists r^-.\top \sqsubseteq A$, where r is a role name and A a concept name.

Rewriting Algorithm for Fragment DL-Lite$_{\text{tiny}}$

The rewriting algorithm computes for any

- query of the form $F(x) = A(x)$ with A a concept name and
- DL-Lite$_{\text{tiny}}$ TBox $\mathcal{T}$

a FOPL query $F_{\mathcal{T}}(x)$ such that for every simple ABox $\mathcal{A}$ and $a \in \mathbf{Ind}(\mathcal{A})$:

$$(\mathcal{T}, \mathcal{A}) \models A(a) \quad \Leftrightarrow \quad \mathcal{I}_{\mathcal{A}} \models F_{\mathcal{T}}(a)$$

The Algorithm

Assume $\mathcal{T}$ and $F(x) = A(x)$ are given. We compute sets $I(A)$, $I_R(A)$, and $I_{R^-}(A)$ which together provide 'all possible reasons for $A(a)$':

- Compute $I(A) = \{B \mid \mathcal{T} \models B \sqsubseteq A\}$ as follows: Initialise $I(A) = \{A\}$. Now apply exhaustively the following rule: if $B' \in I(A)$ and $B \sqsubseteq B' \in \mathcal{T}$ and $B \notin I(A)$, then update

$$I(A) := I(A) \cup \{B\}$$

- We obtain $I_R(A) = \{\exists r.\top \mid \mathcal{T} \models \exists r.\top \sqsubseteq A\}$ as

$$I_R(A) = \{\exists r.\top \mid \exists r.\top \sqsubseteq B \in \mathcal{T}, B \in I(A)\}$$

- We obtain $I_{R^-}(A) = \{\exists r^-.\top \mid \mathcal{T} \models \exists r^-.\top \sqsubseteq A\}$ as

$$I_{R^-}(A) = \{\exists r^-.\top \mid \exists r^-.\top \sqsubseteq B \in \mathcal{T}, B \in I(A)\}$$

The Algorithm

Then set

$$F_{\mathcal{T}}(x) = \bigvee_{B \in I(A)} B(x) \vee \bigvee_{\exists r.\top \in I_R(A)} \exists y r(x,y) \vee \bigvee_{\exists r.\top \in I_{R^-}(A)} \exists y r(y,x)$$

Consider $\mathcal{T}$ defined as

$$\exists \textbf{student\_at}.\top \sqsubseteq \textbf{Student}, \quad \exists \textbf{student\_at}^{-}.\top \sqsubseteq \textbf{University}$$

$$\textbf{Student} \sqsubseteq \textbf{Person}, \quad \textbf{University} \sqsubseteq \textbf{Institution}$$

For $F(x) = \textbf{Person}(x)$ we obtain

$$F_{\mathcal{T}}(x) = \textbf{Person}(x) \vee \textbf{Student}(x) \vee \exists y \textbf{student\_at}(x,y)$$