Exploring the Monodic Fragment of First-Order Temporal Logic using Clausal Temporal Resolution

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Abstract. Until recently, first-order temporal logic has been little understood. While it is well known that the full logic has no finite axiomatisation, a more detailed analysis of fragments of the logic was not available. However, a breakthrough by Hodkinson et.al., identifying a finitely axiomatisable fragment, termed the *monodic* fragment, has led to improved understanding of first-order temporal logic. Following this result, the clausal temporal resolution technique, originally developed for propositional temporal logics, has been extended to such monodic fragments. While this is useful as a practical proof technique for certain monodic classes, in this paper we show that the use of clausal temporal resolution provides us with increased understanding of the monodic fragment. In particular, several features of monodic first-order temporal logic are established as corollaries of the completeness result for the clausal temporal resolution method. These include definitions of new decidable monodic classes, simplification of existing monodic classes by reductions, an extension of finite axiomatisability beyond the monodic fragment.

1 Introduction

First order (discrete) linear temporal logic (FOLTL) is complex. It is known that even "small" fragments of FOLTL, such as the *two-variable monadic* fragment (all predicates are unary), are not recursively enumerable [15, 13]. However, the set of valid *monodic* formulae is known to be finitely axiomatisable [16] (an FOLTL-formula ϕ is called monodic if any subformulae of the forms $\mathcal{T}\psi$ or $\psi_1\mathcal{T}\psi_2$, where $\mathcal{T}$ is a temporal operator, contains at most one free variable). Moreover, a number of decidable subfragments of the monodic fragment has been defined [13, 16]; this highlights the importance of the monodic class to practical temporal verification. The addition of either equality or function symbols to monodic FOLTL leads to the loss of recursive enumerability [16, 4]. The only known decidable monodic fragment with equality (but without functional symbols), termed *monodic packed fragment*, was established in [12].

In previous work [2, 3], we have extended the clausal temporal resolution approach [8] to the monodic fragment of FOLTL. While this is useful as a practical proof technique for certain monodic classes, in this paper we show that the use of clausal temporal resolution provides us with increased understanding of the monodic

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fragment. In particular, several features of monodic first-order temporal logic are established as corollaries of the completeness result for the clausal temporal resolution method. These three aspects concern new decidable monodic classes, simplification of existing monodic classes by reductions, and an extension of finite axiomatisability beyond the monodic fragment.

Thus, following a a brief description of the background required in Section 2, we examine each of these aspects in turn.

In Section 3 we apply monodic temporal resolution to establish decidability of monodic fragments, especially those defined by imposing restrictions on quantifier alternation and interaction between quantifiers and temporal operators within formulae.

FOLTL can be considered as a combination of classical first-order logic and propositional temporal logic, in other words classical first order logic is enriched by temporal operators. We can classify forms of the combination depending on which variety of first-order formulae are placed under temporal operators. The simplest combination is the case when only *closed* first-order formulae under temporal operators are allowed. We call such temporal formulae *grounded*. The monodic fragment is the case when formulae with at most one free variable are allowed under temporal operators.

A more refined classification takes into account which temporal operator is considered and, depending on that, whether non-closed formulae are allowed to be placed under the operator. Such a classification based on a normal form representation is introduced in Section 4, and satisfiability preserving reductions of one class into another are considered using the completeness of the monodic temporal resolution calculus. Interestingly, the classification of temporal problems considered in Section 4 exactly corresponds to the classification of monodic formulae obtained by translation from distinctive fragments of spatio-temporal logic [9, 10, 17]. (Consequently, we are going to apply monodic temporal resolution to checking the satisfiability of formulae in spatio-temporal logic.) The complexity of our reductions given in theorems 3, 4, and 5, within Section 4, agrees with the complexity bounds on corresponding spatio-temporal fragments from [10, 17].

In Section 5 we consider an extension of the monodic fragment which allows a 'local' *next-time* operator to be applied to formulae with more than one free variable. Such kind of extension is motivated by possible applications in transaction protocol verification and temporal databases where changes are often local, e.g. at the next moment of time. We show that temporal resolution can be used to check the satisfiability of formulae from this extension as well after translating them into monodic temporal problems with linear grows in size.

Finally, in Section 6, we provide concluding remarks and outline future work.

2 Preliminaries

2.1 First-Order Temporal Logic

First-Order (discrete) Linear Time Temporal Logic, FOLTL, is an extension of classical first-order logic with operators that deal with a linear and discrete model of time (isomorphic to $\mathbb{N}$, and the most commonly used model of time). The first-order temporal language is constructed in a standard way [6, 13] from: *predicate symbols* $P_0, P_1, \ldots$

each of which is of some fixed arity (null-ary predicate symbols are called *propositions*); *individual variables* $x_0, x_1, \ldots$; *individual constants* $c_0, c_1, \ldots$; *booleans operators* $\wedge$, $\neg$, $\vee$, $\Rightarrow$, $\equiv$, **true** ('true'), **false** ('false'); *quantifiers* $\forall$ and $\exists$; together with *temporal operators* $\Box$ ('always in the future'), $\Diamond$ ('sometime in the future'), $\bigcirc$ ('at the next moment')[3]. There are no function symbols or equality in this FOLTL language, but it does contain constants. For a given formula, ϕ, $const(\phi)$ denotes the set of constants occurring in ϕ. We write $\phi(x)$ to indicate that $\phi(x)$ *has at most one free variable* x (if not explicitly stated otherwise).

Formulae in FOLTL are interpreted in *first-order temporal structures* of the form $\mathfrak{M} = \langle D, I \rangle$, where D is a non-empty set, the *domain* of $\mathfrak{M}$, and I is a function associating with every moment of time, $n \in \mathbb{N}$, an interpretation of predicate and constant symbols over D. A *(variable) assignment* $\mathfrak{a}$ over D is a function from the set of individual variables to D. We require that the interpretation of variables and constants is *rigid*, e.g., neither assignments nor interpretations of constants depend on time. For every moment of time n, there is a corresponding *first-order* structure, $\mathfrak{M}_n = \langle D, I_n \rangle$, where $I_n = I(n)$. Intuitively, FOLTL formulae are interpreted in sequences of *worlds*, $\mathfrak{M}_0, \mathfrak{M}_1, \ldots$ with truth values in different worlds being connected by means of temporal operators.

The *truth* relation $\mathfrak{M}_n \models^{\mathfrak{a}} \phi$ in a structure $\mathfrak{M}$, for an assignment $\mathfrak{a}$, is defined inductively in the usual way under the following understanding of temporal operators:

$$
\begin{array}{l}
\mathfrak{M}_n \models^{\mathfrak{a}} \bigcirc\phi \text{ iff } \mathfrak{M}_{n+1} \models^{\mathfrak{a}} \phi; \\
\mathfrak{M}_n \models^{\mathfrak{a}} \Diamond\phi \text{ iff there exists } m \geq n \text{ such that } \mathfrak{M}_m \models^{\mathfrak{a}} \phi; \\
\mathfrak{M}_n \models^{\mathfrak{a}} \Box\phi \text{ iff for all } m \geq n,\ \mathfrak{M}_m \models^{\mathfrak{a}} \phi.
\end{array}
$$

$\mathfrak{M}$ is a *model* for a formula ϕ (or ϕ is *true* in $\mathfrak{M}$) if there exists an assignment $\mathfrak{a}$ such that $\mathfrak{M}_0 \models^{\mathfrak{a}} \phi$. A formula is *satisfiable* if it has a model. A formula is *valid* if it is satisfiable in any temporal structure under any assignment.

2.2 Monodic Temporal Resolution

In this section, we review the resolution-like procedure for the monodic fragment, previously introduced in [3]. As with usual first-order resolution, temporal resolution relies on a simple normal form and a small set of inference rules. We sketch the monodic temporal resolution system here to make the paper self-contained — results of the following sections are based on properties of the normal form and inference rules of the system.

Divided Separated Normal Form (DSNF) is a special representation of first-order temporal formulae devised for the temporal resolution calculus. DSNF simultaneously realises two aims: the separation of present-time and future-time formulae; and the division the temporal part of a formula from its (classical) first-order part in such way that the temporal part is as simple as possible.

Arbitrary monodic FOLTL-formulae can be transformed into DSNF in a satisfiability equivalence preserving way with at most a linear increase in the size of the formula using a renaming technique replacing non-atomic subformulae with new predicates and removing all occurrences of the U operator [6, 3]. The result is a quadruple $\langle \mathcal{I}, \mathcal{U}, \mathcal{S}, \mathcal{E} \rangle$

[3] With respect to satisfiability, other temporal operators, U (until) and W (weak until) can be represented using these operators [7, 2].

where I and $\mathcal{U}$ are an *initial part* and a universal part, respectively, given by sets of arbitrary closed first-order formulae; $\mathcal{S}$ is a *step part* given by a set of *step clauses* (or *step rules*),

$$\begin{array}{ll} \forall x(P(x) \Rightarrow \bigcirc M(x)) & \text{(a } \textit{non-ground} \text{ step clause)}, \\ p \Rightarrow \bigcirc m & \text{(a } \textit{ground} \text{ step clause)}, \end{array}$$

and $\mathcal{E}$ is an *eventuality part* given by a set of eventuality clauses,

$$\begin{array}{ll} \Diamond L(x) & \text{(a } \textit{non-ground} \text{ eventuality clause)}, \\ \Diamond l & \text{(a } \textit{ground} \text{ eventuality clause)}, \end{array}$$

where $P(x)$ is a unary predicate, $L(x)$ and $M(x)$ are unary literals, p is a propositional symbol, and l and m are propositional literals. We do not allow two different temporal step clauses with the same left-hand sides. Formulae in DSNF are called *temporal problems*. In what follows we will not distinguish between a finite set of formulae $\mathcal{X}$ and the conjunction $\bigwedge \mathcal{X}$ of formulae within it. With each temporal problem we associate the formula $I \wedge \Box\mathcal{U} \wedge \Box\forall x\mathcal{S} \wedge \Box\forall x\mathcal{E}$. Now, when we talk about particular properties of temporal problems (e.g., satisfiability, validity, logical consequences etc) we refer to properties of the associated formula.

Example 1. Let us consider the temporal formula $\exists x \Box\Diamond\forall y\forall z\exists u\, \Phi(x,y,z,u)$ where $\Phi(x,y,z,u)$ does not contain temporal operators and reduce it to DSNF. First, we rename the inmost temporal subformula by a new predicate,

$$\exists x \Box P_1(x) \wedge \Box\forall x[P_1(x) \Rightarrow \Diamond\forall y\forall z\exists u\, \Phi(x,y,z,u)].$$

Now, we rename the first '$\Box$'-formula and the subformula under the '$\Diamond$' operator,

$$\begin{aligned} \exists x P_3(x) \wedge \Box\forall x[P_1(x) \Rightarrow \Diamond P_2(x)] \;\wedge\; &\Box\forall x[P_2(x) \Rightarrow \forall y\forall z\exists u\, \Phi(x,y,z,u)] \\ \wedge\; &\Box\forall x[P_3(x) \Rightarrow \Box P_1(x)], \end{aligned}$$

"unroll" the '$\Box$' operator

$$\begin{aligned} \exists x P_3(x) \;\wedge\; &\Box\forall x[P_1(x) \Rightarrow \Diamond P_2(x)] \wedge \Box\forall x[P_2(x) \Rightarrow \forall y\forall z\exists u\, \Phi(x,y,z,u)] \\ \wedge\; &\Box\forall x[P_3(x) \Rightarrow P_4(x)] \wedge \Box\forall x[P_4(x) \Rightarrow \bigcirc P_4(x)] \wedge \Box\forall x[P_4(x) \Rightarrow P_1(x)], \end{aligned}$$

and, finally, reduce the conditional eventuality to an unconditional one.

$$\begin{aligned} \exists x P_3(x) \;\wedge\; &\Box\forall x[P_2(x) \Rightarrow \forall y\forall z\exists u\, \Phi(x,y,z,u)] \wedge \Box\forall x[P_3(x) \Rightarrow P_4(x)] \\ \wedge\; &\Box\forall x[P_4(x) \Rightarrow \bigcirc P_4(x)] \wedge \Box\forall x[P_4(x) \Rightarrow P_1(x)] \\ \wedge\; &\Box\forall x[(P_1(x) \wedge \neg P_2(x)) \Rightarrow \mathit{waitforP}_2(x)] \\ \wedge\; &\Box\forall x[(\mathit{waitforP}_2(x) \wedge \bigcirc\neg P_2(x)) \Rightarrow \bigcirc \mathit{waitforP}_2(x)] \\ \wedge\; &\Box\forall x\Diamond\neg \mathit{waitforP}_2(x). \end{aligned}$$

The parts of this formula form the following monodic temporal problem (we also rename the complex $P_2(x) \vee \mathit{waitforP}_2(x)$ expression by $P_5(x)$):

$\mathcal{E} = \{\Box\forall x\Diamond\neg \mathit{waitforP}_2(x)\}$, $\qquad I = \{\exists x P_3(x)\}$,

$$\mathcal{U} = \left\{ \begin{array}{l} \forall x(P_2(x) \Rightarrow \forall y\forall z\exists u\, \Phi(x,y,z,u)), \forall x(P_3(x) \Rightarrow P_4(x)), \quad \forall x(P_4(x) \Rightarrow P_1(x)), \\ \forall x((P_1(x) \wedge \neg P_2(x)) \Rightarrow \mathit{waitforP}_2(x)), \quad \forall x(P_5(x) \Rightarrow P_2(x) \vee \mathit{waitforP}_2(x)) \end{array} \right\},$$

$\mathcal{S} = \{P_4(x) \Rightarrow \bigcirc P_4(x), \mathit{waitforP}_2(x) \Rightarrow \bigcirc P_5(x)\}$. □

Let P be a monodic temporal problem, $\mathsf{P}^c = \mathsf{P} \cup \{\Diamond L(c) \mid \Diamond L(x) \in \mathcal{E}, c \in const(\mathsf{P})\}$ is the *constant flooded form*[4] of P. Evidently, P^c is satisfiability equivalent to P.

Let P be a monodic temporal problem, and let

$$P_{i_1}(x) \Rightarrow \bigcirc M_{i_1}(x), \ldots, P_{i_k}(x) \Rightarrow \bigcirc M_{i_k}(x) \tag{1}$$

be a subset of the set of its original non-ground step clauses. Then formulae of the form

$$P_{i_j}(c) \Rightarrow \bigcirc M_{i_j}(c), \tag{2}$$

$$\exists x(P_{i_1}(x) \wedge \ldots \wedge P_{i_k}(x)) \Rightarrow \bigcirc \exists x(M_{i_1}(x) \wedge \ldots \wedge M_{i_k}(x)), \tag{3}$$

$$\forall x(P_{i_1}(x) \vee \ldots \vee P_{i_k}(x)) \Rightarrow \bigcirc \forall x(M_{i_1}(x) \vee \ldots \vee M_{i_k}(x)) \tag{4}$$

are called *derived* step clauses, where $c \in const(\mathsf{P})$. Note that formulae (2), (3), and (4) are logical consequences of (1).

Let $\{\Phi_1 \Rightarrow \bigcirc \Psi_1, \ldots, \Phi_n \Rightarrow \bigcirc \Psi_n\}$ be a set of derived step clauses or original *ground* step clauses. Then $\bigwedge_{i=1}^{n} \Phi_i \Rightarrow \bigcirc \bigwedge_{i=1}^{n} \Psi_i$ is called a *merged derived step clause*.

Let $\mathcal{A} \Rightarrow \bigcirc \mathcal{B}$ be a merged derived step clause, let $P_1(x) \Rightarrow \bigcirc M_1(x), \ldots, P_k(x) \Rightarrow \bigcirc M_k(x)$ be a subset of the original step clauses, and let $\mathcal{A}(x) \leftrightharpoons \mathcal{A} \wedge \bigwedge_{i=1}^{k} P_i(x)$, $\mathcal{B}(x) \leftrightharpoons \mathcal{B} \wedge \bigwedge_{i=1}^{k} M_i(x)$. Then $\forall x(\mathcal{A}(x) \Rightarrow \bigcirc \mathcal{B}(x))$ is called a *full merged step clause*.

Inference Rules.[5]

- *Step resolution rule w.r.t.* $\mathcal{U}$: $\dfrac{\mathcal{A} \Rightarrow \bigcirc \mathcal{B}}{\neg \mathcal{A}}\ (\bigcirc^{\mathcal{U}}_{res})$, where $\mathcal{U} \cup \{\mathcal{B}\} \vdash \bot$.
- *Initial termination rule w.r.t.* $\mathcal{U}$: The contradiction $\bot$ is derived and the derivation is (successfully) terminated if $\mathcal{U} \cup I \vdash \bot$.
- *Eventuality resolution rule w.r.t.* $\mathcal{U}$:

$$\frac{\forall x(\mathcal{A}_1(x) \Rightarrow \bigcirc \mathcal{B}_1(x)) \ldots \forall x(\mathcal{A}_n(x) \Rightarrow \bigcirc \mathcal{B}_n(x)) \qquad \Diamond L(x)}{\forall x \bigwedge_{i=1}^{n} \neg \mathcal{A}_i(x)}\ (\Diamond^{\mathcal{U}}_{res}),$$

  where $\forall x(\mathcal{A}_i(x) \Rightarrow \bigcirc \mathcal{B}_i(x))$ are full merged step clauses such that for all $i \in \{1, \ldots, n\}$, the *loop* side conditions $\forall x(\mathcal{U} \wedge \mathcal{B}_i(x) \Rightarrow \neg L(x))$ and $\forall x(\mathcal{U} \wedge \mathcal{B}_i(x) \Rightarrow \bigvee_{j=1}^{n} (\mathcal{A}_j(x)))$ are both valid
- *Eventuality termination rule w.r.t.* $\mathcal{U}$: The contradiction is derived and the derivation is (successfully) terminated if $\mathcal{U} \vdash \forall x \neg L(x)$, where $\Diamond L(x) \in \mathcal{E}$[6].

[4] Strictly speaking, P^c is not in DSNF: we have to rename ground eventualities by propositions.

[5] $\mathcal{A} \Rightarrow \bigcirc \mathcal{B}$ and $\mathcal{A}_i \Rightarrow \bigcirc \mathcal{B}_i$ denote here merged derived step clauses, $\forall x(\mathcal{A}(x) \Rightarrow \bigcirc(\mathcal{B}(x)))$ and $\forall x(\mathcal{A}_i(x) \Rightarrow \bigcirc(\mathcal{B}_i(x)))$ denote full merged step clauses, and $\mathcal{U}$ denotes the (current) universal part of the problem.

[6] In the case $\mathcal{U} \vdash \forall x \neg L(x)$, the *degenerate clause*, $\mathbf{true} \Rightarrow \bigcirc \mathbf{true}$, can be considered as a premise of the eventuality resolution rule; the conclusion of the rule is then $\neg\mathbf{true}$ and the derivation successfully terminates.

– *Ground eventuality resolution rule w.r.t. $\mathcal{U}$:*

$$\frac{\mathcal{A}_1 \Rightarrow \bigcirc\mathcal{B}_1 \ldots \mathcal{A}_n \Rightarrow \bigcirc\mathcal{B}_n \qquad \Diamond l}{\bigwedge_{i=1}^{n} \neg\mathcal{A}_i}\ (\Diamond_{res}^{\mathcal{U}}),$$

where $\mathcal{A}_i \Rightarrow \bigcirc\mathcal{B}_i$ are merged derived step clauses such that for all $i \in \{1,\ldots,n\}$, the *loop* side conditions $\mathcal{U} \wedge \mathcal{B}_i \Rightarrow \neg l$ and $\mathcal{U} \wedge \mathcal{B}_i \Rightarrow \bigvee_{j=1}^{n} \mathcal{A}_j$ are both valid.

– *Ground eventuality termination rule w.r.t. $\mathcal{U}$:* The contradiction is derived and the derivation is (successfully) terminated if $\mathcal{U} \vdash \neg l$, where $\Diamond l \in \mathcal{E}$.

A *derivation* in the described inference system is a sequence of universal parts, extended, step by step, by the conclusions of the above inference rules. Successful termination means that the given problem is unsatisfiable. The $\mathcal{I}$, $\mathcal{S}$ and $\mathcal{E}$ parts of the temporal problem are not changed during any derivation.

Theorem 1. *The rules of temporal resolution preserve satisfiability. If a monodic temporal problem P is unsatisfiable, then there exists a successfully terminating derivation from P^c.*

A saturation-based proof procedure based on the described calculus is a decision procedure when the side condition checks are decidable, and is a semi-decision procedure otherwise.

3 Decidable Classes

Temporal resolution provides a decision procedure for a class of monodic temporal formulae provided that there exists a first-order decision procedure for side conditions of all inference rules. Direct examination of the side conditions shows that we are interested in the satisfiability of the union of the (current) universal part and sets of monadic formulae built from predicate symbols which occur in the temporal part. At the same time the current universal part of a derivation is obtained by extending the initially given universal part by monadic formulae from the conclusions of the inference rules. So, after imposing appropriate restrictions on the form of the universal part of a given temporal problem, we can guarantee its decidability (adding monadic formulae usually does not affect decidability).

To reflect our "rename and unwind" transformation to the normal form, we define decidable fragments in terms of *surrogates* [13]. Let us reserve, for every formula ϕ whose main connective is a temporal operator, a unary predicate $P_\phi(x)$, and for every sentence ψ, whose main connective is a temporal operator, a propositional variable p_ψ. $P_\phi(x)$ and p_ψ are called surrogates. Given a monodic temporal formula ϕ, we denote by $\overline{\phi}$ the formula that results from ϕ by replacing all of its subformulae whose main connective is a temporal operator and which is not within a scope of another temporal operator with their surrogates.

Such an approach allows us to define decidable monodic classes based on the properties of surrogates analogously to the classical first-order decision problem [1]. Note however, that it is necessary to take into consideration occurrences of temporal operators as the following example shows.

Example 2. The first-order formula $\exists x \forall y \forall z \exists u \Phi(x,y,z,u)$, where Φ is quantifier free, belongs to the classical decidable fragment $\exists^* \forall^2 \exists^*$. Let us consider the temporal formula $\exists x \Box \Diamond \forall y \forall z \exists u \Phi(x,y,z,u)$ with the same Φ. It is not hard to see that after the translation into DSNF (see Example 1), the first formula from $\mathcal{U}$ does not belong to $\exists^* \forall^2 \exists^*$ any more. (Formally, it belongs to the undecidable Surányi class $\forall^3 \exists$.) $\square$

The following definition takes into account the considerations above.

Definition 1 (Temporalisation by renaming). *Let $\mathfrak{C}$ be a class of first-order formulae. Let ϕ be a monodic temporal formula in the Negation Normal Form (i.e. the only boolean connectives are conjunction, disjunction and negation, and negations are only applied to atoms). We say that ϕ belongs to the class $\mathcal{T}_{ren}\mathfrak{C}$ if*

1. *$\overline{\phi}$ belongs to $\mathfrak{C}$ and*
2. *for every subformula of the form $\mathcal{T}\psi$, where $\mathcal{T}$ is a temporal operator (or of the form $\psi_1 \mathcal{T} \psi_2$ if $\mathcal{T}$ is binary), either $\overline{\psi}$ is a closed formula belonging to $\mathfrak{C}$ or the formula $\forall x(P(x) \Rightarrow \overline{\psi})$, where P is a new unary predicate symbol, belongs to $\mathfrak{C}$ (analogous conditions for ψ_1, ψ_2).*

Note that the formulae indicated in the first and second items of the definition exactly match the shape of the formulae contributing to $\mathcal{U}$ when we reduce a temporal formula to the normal form by renaming the complex expressions and replacing temporal operators by their fixpoint definitions.

Theorem 2. *Let $\mathfrak{C}$ be a decidable class of first-order formulae which does not contain equality and functional symbols, but possibly containing constants, such that*

- *$\mathfrak{C}$ is closed under conjunction;*
- *$\mathfrak{C}$ contains monadic formulae.*

Then $\mathcal{T}_{ren}\mathfrak{C}$ is decidable.

Proof After reduction to DSNF, all formulae from $\mathcal{U}$ belong to $\mathfrak{C}$. The (monadic) formulae from side conditions and the (monadic) formulae generated by temporal resolution rules belong to $\mathfrak{C}$. Theorem 1 gives the decision procedure. ❑

Theorem 2 provides the possibility of using temporal resolution to confirm decidability of all temporal monodic classes listed in [13, 16]: *monadic, two-variable, fluted, guarded and loosely guarded*. Moreover, combining the constructions from [12] and the saturation-based decision procedure for the guarded fragment with equality [11], it is possible to build a temporal resolution decision procedure for the monodic guarded and loosely guarded fragments with equality.

In addition, using the above theorem, we also obtain decidability of some monodic *prefix-like* classes.

Corollary 1 ($\mathcal{T}_{ren}\exists^*\forall^2\exists^*$, temporalised Gödel class). *The class $\mathcal{T}_{ren}\exists^*\forall^2\exists^*$ is decidable*

Proof Every monadic formula can be reduced, in a satisfiability equivalence preserving way, to a conjunction of formulae of the form $\forall x(l_1 \vee \ldots \vee l_p \vee L_1(x) \vee \ldots \vee L_q(x))$, $p, q \geq 0$ or $\exists x(L_1(x) \wedge \ldots \wedge L_r(x))$, $r \geq 0$, where l_j are ground literals and $L_j(x)$ are non-ground literals. Obviously, every conjunct is in $\exists^*\forall^2\exists^*$. Satisfiability of a conjunction of formulae belonging to $\exists^*\forall^2\exists^*$ is decidable, e.g. by the resolution-based technique (see clause set class $\mathcal{S}^+$ in [5]). ❏

Corollary 2 **($\mathcal{T}_{ren}K$, temporalised Maslov class).** *The class $\mathcal{T}_{ren}K$ is decidable (where K is the Maslov class).*

Proof Again, monadic formulae can be rewritten as a conjunction of Maslov formulae; satisfiability of a conjunction of Maslov formulae is decidable as shown in [14]. ❏

4 Grounding Temporal Problems

We now turn to an analysis of temporal problems with partial groundedness.

Definition 2 **(Groundedness).** *A temporal problem P is called* grounded *if all the step clauses and the eventuality clauses of P are ground. Correspondingly, a temporal monodic formula is called grounded if it is translated to a grounded temporal problem. A temporal problem P is called a* ground eventuality *problem if all the eventualities of P are ground. A temporal problem P is called a* ground next-time *problem if all the step clauses of P are ground.*

If P is a ground eventuality problem then only the ground versions of the eventuality resolution and eventuality termination rules are needed. Note that the ground eventuality resolution rule, step resolution rule, and initial termination rule operate on merged derived step clauses. So, if instead of original step clauses we consider step clauses given by formulae (2), (3), and (4) (and strictly speaking, rename by propositions closed first-order formulae in the right- and left-hand sides), we obtain a satisfiability equivalent grounded temporal problem.

Theorem 3 **(reducing a ground eventuality problem).** *Every ground eventuality monodic temporal problem can be reduced to a satisfiability equivalent grounded monodic problem with at an exponential growth in size of the given problem.*

Example 3. Consider an unsatisfiable formula $\Diamond\exists x(P(x) \wedge \bigcirc\neg P(x)) \wedge \Box(P(x) \Rightarrow \bigcirc P(x))$. In DSNF we have (note that I is empty throughout),

$$\mathcal{S} = \left\{ \begin{array}{l} P(x) \Rightarrow \bigcirc P(x) \\ Q(x) \Rightarrow \bigcirc\neg P(x) \end{array} \right\}, \qquad \begin{array}{l} \mathcal{U} = \emptyset, \\ \mathcal{E} = \{\Diamond\exists x(P(x) \wedge Q(x))\}. \end{array}$$

According to our reduction, this problem is satisfiability equivalent to the following

$$\mathcal{S} = \left\{ \begin{array}{l} \exists x P(x) \Rightarrow \bigcirc\exists x P(x) \\ \forall x P(x) \Rightarrow \bigcirc\forall x P(x) \\ \exists x Q(x) \Rightarrow \bigcirc\exists x \neg P(x) \\ \forall x Q(x) \Rightarrow \bigcirc\forall x \neg P(x) \\ \exists x(P(x) \wedge Q(x)) \Rightarrow \bigcirc\exists x(P(x) \wedge \neg P(x)) \\ \forall x(P(x) \vee Q(x)) \Rightarrow \bigcirc\forall x(P(x) \vee \neg P(x)) \end{array} \right\}, \qquad \begin{array}{l} \mathcal{U} = \emptyset, \\ \mathcal{E} = \{\Diamond\exists x(P(x) \wedge Q(x))\}. \end{array}$$

The last step clause is a tautology which can be eliminated immediately, the next to last can be moved to the universal part by an application of step resolution. .

$$\mathcal{S} = \left\{ \begin{array}{l} \exists x P(x) \Rightarrow \bigcirc \exists x P(x) \\ \forall x P(x) \Rightarrow \bigcirc \forall x P(x) \\ \exists x Q(x) \Rightarrow \bigcirc \exists x \neg P(x) \\ \forall x Q(x) \Rightarrow \bigcirc \forall x \neg P(x) \end{array} \right\}, \qquad \begin{array}{l} \mathcal{U} = \{\forall x(\neg P(x) \vee \neg Q(x))\}, \\ \mathcal{E} = \{\Diamond \exists x (P(x) \wedge Q(x))\}. \end{array}$$

Now the ground eventuality termination rule can be applied. □

Together with Theorem 3 the following theorem shows that for any problem P, if either all the step clauses are ground or all the eventuality clauses are ground, then it can be reduced to a grounded problem.

Theorem 4 (reducing a ground next-time problem).
Let $\mathsf{P}^c = \langle I, \mathcal{U}, \mathcal{S}, \mathcal{E} \rangle$ be a temporal problem such that all step rules of P are ground. Let $\mathcal{E}^{\exists}$ be obtained from $\mathcal{E}$ as follows: every eventuality clause of the form $\Diamond L(x)$ (in the meaning of $\forall x \Diamond L(x)$) is replaced with its ground *consequence $\exists x \Diamond L(x)$ (equivalent to $\Diamond \exists x L(x)$). Let $\mathsf{P}' = \langle I, \mathcal{U}, \mathcal{S}, \mathcal{E}^{\exists} \cup \{\Diamond L(c) \mid \Diamond L(x) \in \mathcal{E},\ c \in const(\mathsf{P})\} \rangle$. Then P is satisfiable if and only if P' is satisfiable.*

Proof (Sketch) Evidently, if P' is unsatisfiable, then P is unsatisfiable. Suppose now P is unsatisfiable, then there exists a successfully terminating temporal resolution derivation from P^c. Note that the added eventualities of the form $\Diamond L(c)$ exactly correspond to the eventualities added by reduction to constant-flooded form.

Suppose the eventuality resolution rule is applied to a non-ground eventuality $\forall x \Diamond L(x)$. The validity of the side conditions implies the validity of the formula

$$\Box \forall x (\mathcal{U} \wedge \bigvee_{j=1}^{n} \mathcal{A}_i \Rightarrow \bigcirc \Box \neg L(x)) \tag{5}$$

for a set $\{\mathcal{A}_i \Rightarrow \bigcirc \mathcal{B}_i, 1 \leq i \leq\}$ of *ground* merged derived step rules. (5) is resolved with the formula $\Box \forall x \Diamond L(x)$ giving the conclusion $(\bigwedge_{i=1}^{n} \neg \mathcal{A}_i)$. However (5) or, equivalent to (5),

$$\Box (\mathcal{U} \wedge \bigvee_{j=1}^{n} \mathcal{A}_i \Rightarrow \bigcirc \Box \forall x \neg L(x)) \tag{6}$$

can be resolved with a 'weaker' formula $\Box \exists x \Diamond L(x)$ giving the same result.

If the eventuality termination rule is applied to $\forall x \Diamond L(x)$, its side condition, $\mathcal{U} \vdash \forall x \neg L(x)$, equally contradicts to the ground eventuality $\exists x \Diamond L(x)$. So, we can conclude that replacing non-ground eventualities of the form $\forall x \Diamond L(x)$ with ground eventualities $\exists x \Diamond L(x)$ (equivalent to $\Diamond \exists x L(x)$) does not affect (un)satisfiability. ❏

Example 4.

$$\begin{array}{ll} I = \{l\}, & \mathcal{U} = \{\forall x (l \Rightarrow Q(x))\}, \\ \mathcal{S} = \{l \Rightarrow \bigcirc l\}, & \mathcal{E} = \{\Diamond \neg Q(x)\}. \end{array}$$

Evidently, the initial, universal, and step parts imply $\Box \forall x Q(x)$ which also contradicts to $\Box \forall x \Diamond \neg Q(x)$ and $\Box \exists x \Diamond \neg Q(x)$. □

We conclude our consideration by one more case: monodic temporal problems over *finite domains* (see e.g. [13]).

Theorem 5 (grounding eventualities over finite domains). *Every monodic temporal problem over finite domains can be reduced to a satisfiability equivalent ground-eventuality monodic problem with a linear growth in size of the given problem.*

The reduction is an immediate consequence of the two following lemmas. We use the (past-time) operator S whose semantics is defined as follows

$$\begin{aligned}\mathfrak{M}_n \models^{\mathfrak{a}} \phi \mathsf{S} \psi &\text{ iff there exists a } k \in \mathbb{N}, \text{ such that } 0 \leqslant k < n \text{ and } \mathfrak{M}_k \models^{\mathfrak{a}} \psi \\ &\text{and, for all } j \in \mathbb{N}, \text{ if } k \leqslant j < n \text{ then } \mathfrak{M}_j \models^{\mathfrak{a}} \phi\end{aligned}$$

and which is then eliminated by Lemma 2, following the approach in [7].

Lemma 1. *The formula $\Phi \wedge \Box \forall x \Diamond L(x)$, where Φ is an arbitrary temporal formula, is satisfiability equivalent* over finite domains *to*

$$\Phi \wedge l \wedge \Box(l \Rightarrow \Diamond \bigcirc (\forall x(\neg l \mathsf{S} L(x)) \wedge l)),$$

where l is a new propositional symbol.

Proof Let us reformulate the given problem in a two-sorted temporal language with variables over $\mathbb{N}$ for the temporal sort, i.e. $\Phi \wedge \forall x \forall n \exists m(n \leq m \wedge L(m,x))$, meaning that for all x, $L(x)$ is satisfied infinitely often. This problem is satisfiability equivalent *over finite domains* to the following (this can be easily checked by considering possible models):

$$\Phi \wedge \forall n \exists m(m > n \wedge \forall x \exists k(n \leq k < m \wedge L(k,x))) \tag{7}$$

which states, informally, that for each moment of time, n, there is a moment $m > n$, such that for all x the eventuality $\Diamond L(x)$ is satisfied "after n and before m" (there are only finitely many elements in a domain).

We prove that given a model for (7) it is possible to find a model for

$$\Phi \wedge l \wedge \Box(l \Rightarrow \Diamond \bigcirc (\forall x(\neg l \mathsf{S} L(x)) \wedge l)), \tag{8}$$

and vice versa. First, consider a model $\mathfrak{M} = \langle D, I \rangle$ for (7). We construct a model $\mathfrak{M}' = \langle D, I' \rangle$ for (8) by extending $\mathfrak{M}$ with a new proposition l and defining its value as follows. Formula (7) states that for each moment of time n there exists a future moment m when a certain property holds, defining thus a moment m for each n. Let us construct a sequence of times defined by (7) starting from 0, i.e. $m_0 = 0$, $m_1 = m(0), \ldots, m_{i+1} = m(m_i)$; and let us define l in $\mathfrak{M}'$ as **true** at those times and as **false** everywhere else. Note that for every element d of the domain D and for all $i \geq 0$, there exists a moment $k : m_i \leq k < m_{i+1}$ such that $L(d,k)$. Therefore, $\mathfrak{M}_{m_{i+1}} \models \forall x(\neg l \mathsf{S} L(x))$; hence, $\mathfrak{M}_{m_i} \models \Diamond \bigcirc (\forall x(\neg l \mathsf{S} L(x)) \wedge l)$, making (8) **true** in $\mathfrak{M}'$.

We show that any model for (8), $\mathfrak{M}$, is also a model for (7). It is enough to show that for infinitely many n's there exists an m such that $m > n \wedge \forall x \exists k(n \leq k < m \wedge L(x,k))$ holds. Let m_i be the sequence of all moments when l is **true** (there are infinitely many of these). We show that for all $i \geq 0$, $n = m_i$, and $m = m_{i+1}$, the formula $\forall x \exists k(n \leq k < m \wedge L(x,k))$ is true in $\mathfrak{M}$. Indeed, $\mathfrak{M}_n \models l$, $\mathfrak{M}_m \models l$; by semantics of the operator "S", $\mathfrak{M}_m \models \forall x(\neg l \mathsf{S} L(x))$ means that $\mathfrak{M}_m \models \forall x \exists k(n \leq k < m \wedge L(x,k))$. ❑

Lemma 2. *Formula* $\Box\forall x(A(x) \Rightarrow B(x)\,\mathsf{S}\,C(x))$ *is satisfiability equivalent to*

$$\forall x \neg P(x) \wedge \Box\forall x(A(x) \Rightarrow P(x)) \wedge \Box\forall x(C(x) \vee (B(x) \wedge P(x)) \equiv \bigcirc P(x)),$$

where P *is a new predicate symbol.*

Proof Follows straightforwardly from consideration of possible models [7]. ❑

5 Extension of the Monodic Fragment

Let us extend the notion of a monodic temporal problem with an additional *extended part* $\mathcal{X}$ given by a set of arbitrary first-order temporal formulae in the language without function symbols and with the *only* temporal operator being '$\bigcirc$'. Since this temporal operator can be "moved inside", we can assume w.l.o.g. that $\mathcal{X}$ is given by a set of first-order formulae constructed from *temporal atoms* of the form $\bigcirc^i P(t_1, t_2, \ldots, t_n)$, where $P(t_1, t_2, \ldots, t_n)$ is a first-order atom.[7]

We are going to show that an extended monodic temporal problem can be translated (*with linear growth in size*) into a monodic temporal problem while preserving satisfiability. Essentially, we encode a few initial states of a temporal model as a first-order formula and ensure that this encoding is consistent with the rest of the model.

Reduction. Let $\mathsf{XP} = \mathsf{P} \cup \mathcal{X}$ be an extended monodic temporal problem. Let $\mathsf{P} = \langle \mathcal{I}, \mathcal{U}, \mathcal{S}, \mathcal{E} \rangle$. Let k be the maximal number of nested applications of $\bigcirc$ in $\mathcal{X}$, i.e. the maximal i such that $\bigcirc^i P(t_1, t_2, \ldots, t_n)$ occurs in $\mathcal{X}$ for some predicate symbol P. For every predicate, P, occurring in XP, we introduce $k+1$ new predicates $P^0, P^1, \ldots, P^k$ of the same arity. Let ϕ be a first-order formula in the language of XP. We denote by $[\phi]^i$, $0 \le i \le k$, the result of substitution of all occurrences of predicates in ϕ with their i-th counterparts; (e.g., $P(x_1, x_2)$ is replaced with $P^i(x_1, x_2)$).

We define the monodic problem $\mathsf{P}' = \langle \mathcal{I}', \mathcal{U}', \mathcal{S}', \mathcal{E}' \rangle$ as follows. Let $\mathcal{U}' = \mathcal{U}$, $\mathcal{S}' = \mathcal{S}$, $\mathcal{E}' = \mathcal{E}$. As for $\mathcal{I}'$, we take the following set of formulae.

1. For every $\phi \in \mathcal{I}$, the formula $[\phi]^0$ is in $\mathcal{I}'$.
2. For every $\phi \in \mathcal{U}$, the formula $\bigwedge_{i=0}^{k} [\phi]^i$ is in $\mathcal{I}'$.
3. For every $P(x) \Rightarrow \bigcirc Q(x) \in \mathcal{S}$, the formula $\bigwedge_{i=0}^{k-1} \big(\forall x(P^i(x) \Rightarrow Q^{i+1}(x)\big)$ is in $\mathcal{I}'$.
4. For every $\psi \in \mathcal{X}$, the formula ψ', the result of replacing all occurrences of temporal atoms $\bigcirc^i P(\bar{t})$, $i \ge 0$, in ψ with $P^i(\bar{t})$, is in $\mathcal{I}'$.
5. For every n-ary predicate P in the language of XP, the formula $\forall x_1, \ldots x_n (P(x_1, \ldots, x_n) \equiv P^k(x_1, \ldots, x_n))$ is in $\mathcal{I}'$.
6. No other formulae are in $\mathcal{I}'$.

Theorem 6. *XP is satisfiable if, and only if, P' is satisfiable.*

[7] Decidability of this extension of the monodic fragment was suggested in a private communication by M. Zakharyaschev.

Proof We prove that given a model for XP it is possible to find a model for P' and vice versa. First, consider a model $\mathfrak{M} = \langle D, I\rangle$ for XP and construct a model $\mathfrak{M}' = \langle D, I'\rangle$ as follows. The interpretation of constants in the language of XP in $\mathfrak{M}'$ is the same as in $\mathfrak{M}$ (recall that constants are *rigid*).

For every n-ary predicate P in the language of XP (in the initial signature), every n-tuple $(d_1, \ldots, d_n) \in D$, and every $i \geq 0$, we define

$$(\mathfrak{M}', i) \models P(d_1, \ldots, d_n) \text{ iff } (\mathfrak{M}, i+k) \models P(d_1, \ldots, d_n).$$

For every n-ary predicate P^i, $0 \leq i \leq k$, in the extension of the initial language (i.e. in the language of P' but not in the language of XP) we define

$$(\mathfrak{M}', 0) \models P^i(d_1, \ldots, d_n) \text{ iff } (\mathfrak{M}, i) \models P(d_1, \ldots, d_n),$$

and P^i is false in $\mathfrak{M}'$ for all other tuples and moments of time[8]. This definition is consistent with formulae from part 5 of I'; therefore $\mathfrak{M}'$ is defined correctly.

Since truth values of all predicates from P are not changed but "shifted", clearly, $\mathfrak{M}' \models \mathcal{U}$ and $\mathfrak{M}' \models \mathcal{S}$. Since all our eventualities are unconditional, i.e., are of the form $\Box\Diamond l$ and $\Box\forall x \Diamond L(x)$, the truth value of $L(x)$ in the first $k+1$ states of $\mathfrak{M}$ does not affect the truth value of $\mathcal{E}'$ in $\mathfrak{M}'$; so $\mathfrak{M}' \models \mathcal{E}'$. The fact that $\mathfrak{M}' \models I'$ can be established by considering step by step the definition of I'. Indeed:

1. Let a formula $[\phi]^0$ is in I' where $\phi \in I$. Then $(\mathfrak{M}', 0) \models [\phi]^0$ because for every predicates P and P^0
$$(\mathfrak{M}, 0) \models P(d_1, \ldots, d_n) \text{ iff } (\mathfrak{M}', 0) \models P^0(d_1, \ldots, d_n)$$
holds and $(\mathfrak{M}, 0) \models \phi$.
2. Let a formula $[\phi]^i$, $0 \leq i \leq k$, is in I' where $\phi \in \mathcal{U}$. Then $(\mathfrak{M}', 0) \models [\phi]^i$ because for all predicates P and P^i
$$(\mathfrak{M}, i) \models P(d_1, \ldots, d_n) \text{ iff } (\mathfrak{M}', 0) \models P^i(d_1, \ldots, d_n)$$
holds and $(\mathfrak{M}, i) \models \phi$.
3. Let $(\mathfrak{M}', 0) \models P^i(d_1, \ldots, d_n)$, $0 \leq i \leq k$. Then $(\mathfrak{M}, i) \models P(d_1, \ldots, d_n)$, and because of $P(x) \Rightarrow \bigcirc Q(x) \in \mathcal{S}$, then $(\mathfrak{M}, i+1) \models Q(d_1, \ldots, d_n)$ holds. It follows $(\mathfrak{M}', 0) \models Q^{i+1}(d_1, \ldots, d_n)$.
4. Let $\psi \in X$, i.e. $(\mathfrak{M}, 0) \models \psi$. For every subformula $\bigcirc^i P(d_1, \ldots, d_n)$ of ψ, then $(\mathfrak{M}, 0) \models \bigcirc^i P(d_1, \ldots, d_n)$ holds if, and only if, $(\mathfrak{M}', 0) \models P^i(d_1, \ldots, d_n)$. So, $(\mathfrak{M}', 0) \models \psi'$.
5. In accordance with the definition of $\mathfrak{M}'$, then $(\mathfrak{M}', 0) \models P(d_1, \ldots, d_n)$ holds if, and only if, $(\mathfrak{M}, k) \models P(d_1, \ldots, d_n)$ holds if, and only if, $(\mathfrak{M}', 0) \models P^k(d_1, \ldots, d_n)$.

Let $\mathfrak{M}'$ be a model for P'. We construct a model $\mathfrak{M}$ for XP. The interpretation of constants in the language of XP in $\mathfrak{M}$ is the same as in $\mathfrak{M}'$. For every n-ary predicate P in the language of XP and every n-tuple $(d_1, \ldots, d_n) \in D$ we define for every $i \geq k$

$$(\mathfrak{M}, i) \models P(d_1, \ldots, d_n) \text{ iff } (\mathfrak{M}', i-k) \models P(d_1, \ldots, d_n),$$

[8] Note that all new predicates occur only in I'.

and for every i such that $0 \leq i \leq k$

$$(\mathfrak{M}, i) \models P(d_1, \ldots, d_n) \text{ iff } (\mathfrak{M}', 0) \models P^i(d_1, \ldots, d_n).$$

Note that $(\mathfrak{M}', 0) \models I'$ and, in particular, formulae from part 5 of I'; therefore, $\mathfrak{M}$ is defined correctly. Indeed, in the case $i = k$ we obtain

$$(\mathfrak{M}', 0) \models P(d_1, \ldots, d_n) \text{ iff } (\mathfrak{M}', 0) \models P^k(d_1, \ldots, d_n).$$

Evidently, for $i \geq k$, $(\mathfrak{M}, i) \models \mathcal{U}$ and $(\mathfrak{M}, i) \models \mathcal{S}$. Again, since our eventualities are unconditional, evaluation of $\mathcal{E}$ does not depend on a finite number of initial states, and $\mathfrak{M} \models \mathcal{E}$. It is enough to show that $(\mathfrak{M}, i) \models \mathcal{U}$ and $(\mathfrak{M}, i) \models \mathcal{S}$ for $i \in [0, (k-1)]$, and $(\mathfrak{M}, 0) \models I$. Again, this can be done by analysing the definition of I'.

The first claim, $(\mathfrak{M}, i) \models \mathcal{U}$, follows from item 2 of the definition of I', from the relation

$$(\mathfrak{M}, i) \models P(d_1, \ldots, d_n) \text{ iff } (\mathfrak{M}', 0) \models P^i(d_1, \ldots, d_n)$$

and the fact that $(\mathfrak{M}', 0) \models [\phi]^i$ for every $\phi \in \mathcal{U}$, $0 \leq i \leq k$.

The second claim, $(\mathfrak{M}, i) \models \mathcal{S}$, follows from item 3 of the definition of I' and from the relation

$$(\mathfrak{M}, i) \models P(d_1, \ldots, d_n) \text{ iff } (\mathfrak{M}', 0) \models P^i(d_1, \ldots, d_n).$$

The last claim, $(\mathfrak{M}, 0) \models I$, follows immediately from item 1 of the definition of I' and from the relation

$$(\mathfrak{M}, 0) \models P(d_1, \ldots, d_n) \text{ iff } (\mathfrak{M}', 0) \models P^0(d_1, \ldots, d_n)$$

given above. ❑

Note 1. We can reduce extended monodic temporal problems to monodic temporal problems in such a straightforward way because of the divided separated normal form that we use. Note further that this simple reduction would not work if we have had *conditional eventualities* as the following example shows. Let

$$\begin{array}{ll} \mathcal{X} = \{\forall x \forall y (P(x,y) \Rightarrow (\bigcirc Q(x,y) \wedge \bigcirc\bigcirc P(x,y))\}, & I = \{\exists x \exists y P(x,y)\}, \quad \mathcal{S} = \emptyset, \\ \mathcal{U} = \{\forall x \forall y (Q(x,y) \Rightarrow R(x)), \forall x \neg T(x)\}, & \mathcal{E} = \{\Box \forall x (R(x) \Rightarrow \Diamond T(x))\}. \end{array}$$

This problem is unsatisfiable, but becomes satisfiable after the described reduction since $R(x)$ is forced only in the second moment of time.

6 Conclusions and future work

In this paper we have shown the advantages of using the clausal resolution approach to FOLTL, not in terms of it's practicality as a decision procedure, but in terms of it's utility in analysing the fine structure of monodic problems. In particular, we have shown how the clausal resolution approach allows us to easily describe new decidable monodic classes, simplifications of existing monodic classes by reductions, and extensions of finite axiomatisability beyond the monodic fragment.

As part of our future work, we are interested in finding expressive yet complete/decidable classes of FOLTL based on principles different to monodicity (e.g. the ground-negative fragment of FOLTL). In addition, following the work in Section 5, we intend to investigate extending the monodic fragment further (e.g. allowing application of a 'local' *eventually* operator to formulae with more than one free variable). Finally, concerning Section 4, we are interested in the development of a temporal resolution procedure for establishing *finite* satisfiability of (ground eventuality) monodic problems.

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