

3. Let the knowledge base Γ be given by the following set of assertional and terminological sentences.

Male \doteq \neg Female	(liz, andy) : child
Mother \doteq Female \sqcap \exists child.Human	andy : Human
Father \doteq Male \sqcap \exists child.Human	liz : Female
Parent \doteq Mother \sqcup Father	will : Human

- (a) Give the expanded TBox of the knowledge base Γ . (5 marks)

Male \doteq \neg Female
Mother \doteq Female \sqcap \exists child.Human
Father \doteq \neg Female \sqcap \exists child.Human
Parent \doteq (Female \sqcap \exists child.Human) \sqcup (\neg Female \sqcap \exists child.Human)

One mark each is given if the terminological sentences for Male and Mother remain unchanged. One mark for the correct expansion of Father and two marks for the correct expansion of Parent.

- (b) Give a formal derivation of the negation normal form of \neg Mother with respect to the TBox of Γ . (4 marks)

$$\begin{aligned}
 \text{nnf}(\neg\text{Mother}) &= \text{nnf}(\neg(\text{Female} \sqcap \exists\text{child.Human})) \\
 &= \text{nnf}(\neg\text{Female}) \sqcup \text{nnf}(\neg\exists\text{child.Human}) \\
 &= (\neg\text{Female}) \sqcup \forall\text{child.nnf}(\neg\text{Human}) \\
 &= (\neg\text{Female}) \sqcup \forall\text{child.}\neg\text{Human}
 \end{aligned}$$

The derivation requires 4 steps. One mark is given for each step that is performed correctly.

- (c) State the completion rules for the operators \sqcap , \sqcup , and \forall . (6 marks)

- $\Delta \Rightarrow_{\sqcap} \Delta \cup \{a : C, a : D\}$
if $a : (C \sqcap D)$ is in Δ , $a : C$ and $a : D$ are not both in Δ .
- $\Delta \Rightarrow_{\sqcup} \Delta \cup \{a : E\}$
if $a : (C \sqcup D)$ is in Δ , neither $a : C$ nor $a : D$ is in Δ and $E = C$ or $E = D$.
- $\Delta \Rightarrow_{\forall} \Delta \cup \{b : C\}$
if $a : \forall R.C$ and $(a, b) : R$ are in Δ and $b : C$ is not in Δ .

Two marks are given for each completion rules that is stated correctly.

- (d) Give a formal derivation which determines whether liz is an element of the concept Parent. (10 marks)

Let Δ denote the set $\{(liz, andy) : \text{child}, andy : \text{Human}, liz : \text{Female}, will : \text{Human}\}$.

First we have to determine the negation normal form of \neg Parent with respect to the TBox:

$$\text{nnf}(\neg\text{Parent}) = ((\neg\text{Female}) \sqcup \forall\text{child.}\neg\text{Human}) \sqcap (\text{Female} \sqcup \forall\text{child.}\neg\text{Human})$$

The derivation consists of two sequences of applications of the completion rules stated in the answer to 1c plus the rule

- $\Delta \Rightarrow_{\perp} \Delta \cup \{a : \perp\}$, if $a : A$ and $a : \neg A$ are in Δ , where A is a concept symbol.

Both sequences end in the derivation of a contradiction:

$$\begin{aligned} & \{\text{liz} : ((\neg\text{Female}) \sqcup \forall\text{child}.\neg\text{Human}) \sqcap (\text{Female} \sqcup \forall\text{child}.\neg\text{Human})\} \cup \Delta \\ & \Rightarrow_{\sqcap} \{\text{liz} : (\neg\text{Female}) \sqcup \forall\text{child}.\neg\text{Human}, \text{liz} : \text{Female} \sqcup \forall\text{child}.\neg\text{Human}\} \cup \Delta \\ & \Rightarrow_{\sqcup} \{\text{liz} : \neg\text{Female}\} \cup \Delta \\ & \Rightarrow_{\perp} \{\text{liz} : \perp, \text{liz} : \neg\text{Female}\} \cup \Delta \end{aligned}$$

$$\begin{aligned} & \{\text{liz} : ((\neg\text{Female}) \sqcup \forall\text{child}.\neg\text{Human}) \sqcap (\text{Female} \sqcup \forall\text{child}.\neg\text{Human})\} \cup \Delta \\ & \Rightarrow_{\sqcap} \{\text{liz} : (\neg\text{Female}) \sqcup \forall\text{child}.\neg\text{Human}, \text{liz} : \text{Female} \sqcup \forall\text{child}.\neg\text{Human}\} \cup \Delta \\ & \Rightarrow_{\sqcup} \{\text{liz} : \forall\text{child}.\neg\text{Human}\} \cup \Delta \\ & \Rightarrow_{\forall} \{\text{andy} : \neg\text{Human}, \text{liz} : \forall\text{child}.\neg\text{Human}\} \cup \Delta \\ & \Rightarrow_{\perp} \{\text{andy} : \perp, \text{andy} : \neg\text{Human}, \text{liz} : \forall\text{child}.\neg\text{Human}\} \cup \Delta \end{aligned}$$

Consequently, liz is an element of the concept Parent.

Both sequences together require 7 steps. One mark is given for each step that is performed correctly. Plus 3 marks for the correct negation normal form of $\neg\text{Parent}$.