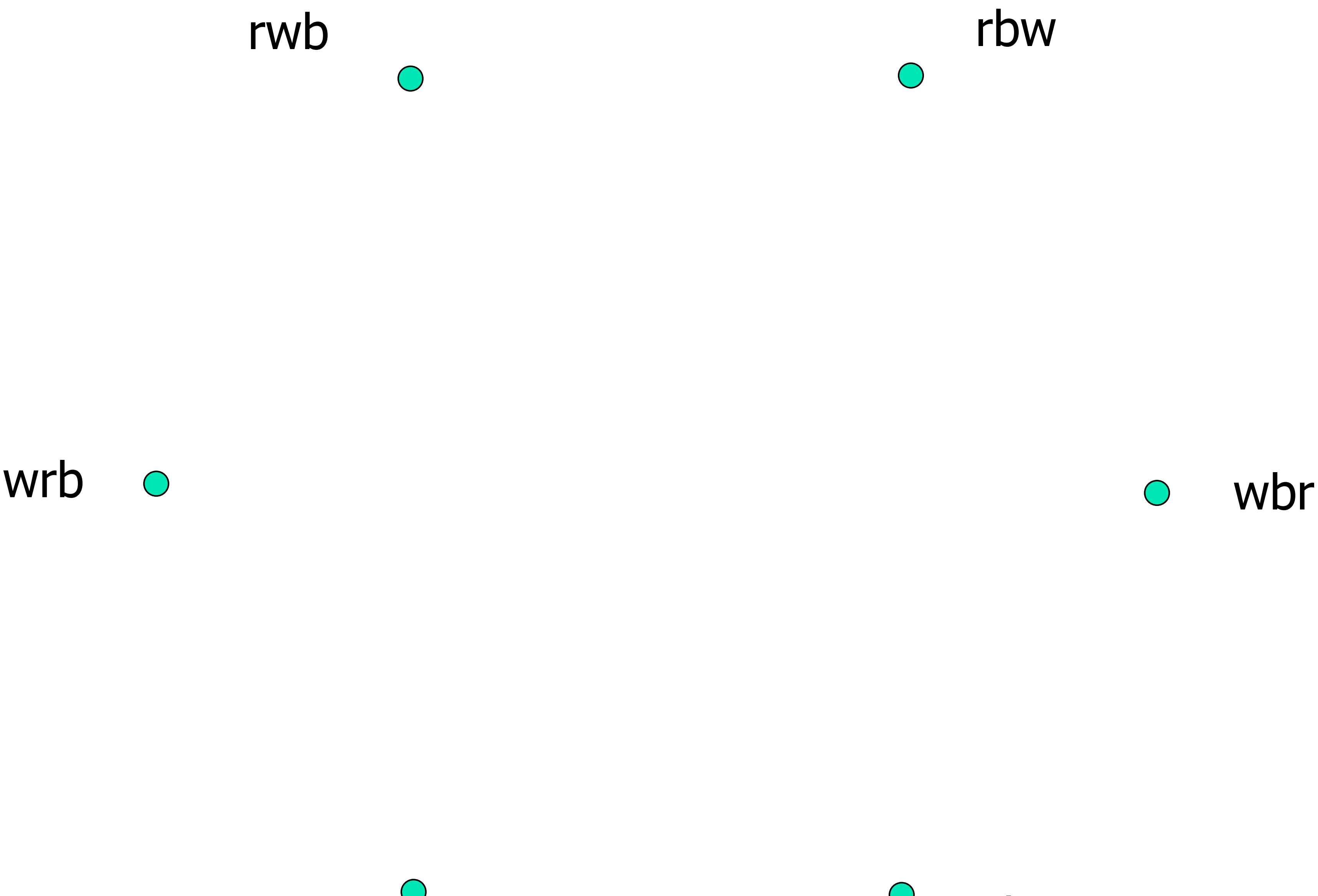
Card Games

- there are three cards: r, w and b
- three players: 1, 2 and 3
- every player sees its own card
- use names for worlds:
  - rwb for 1 has red, 2 white, 3 black, etc.
- use colors for accessibilities:
  - black for 1, green for 2 and red for 3

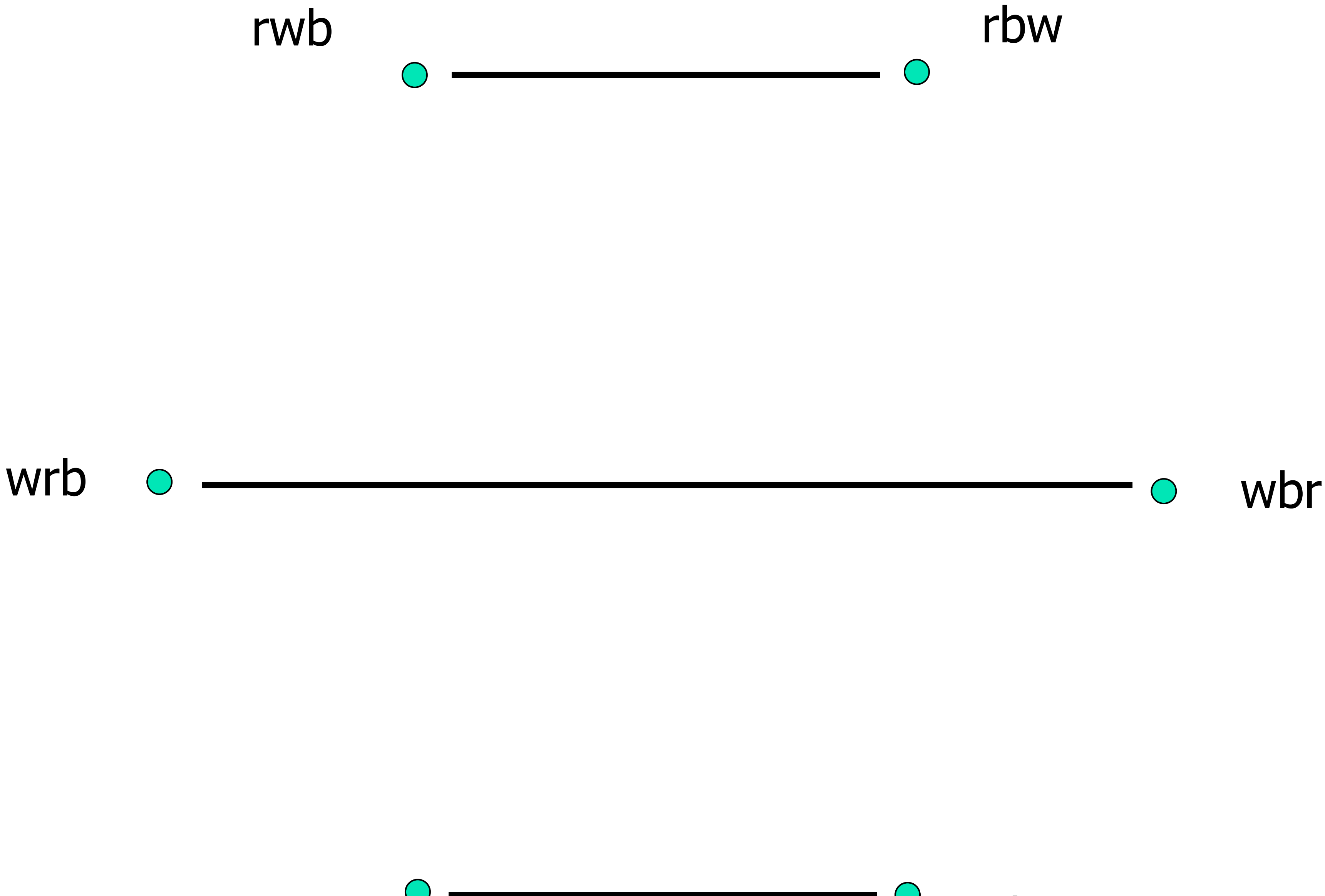
Card Games (ctd)

- draw the appropriate $S5_{(3)}$ Kripke model
- show that in rwb it holds that:
  - $K_1 r_1$
  - $K_1(K_2 \neg r_2) \wedge K_1(\neg K_2 r_1 \wedge \neg K_2 r_3)$

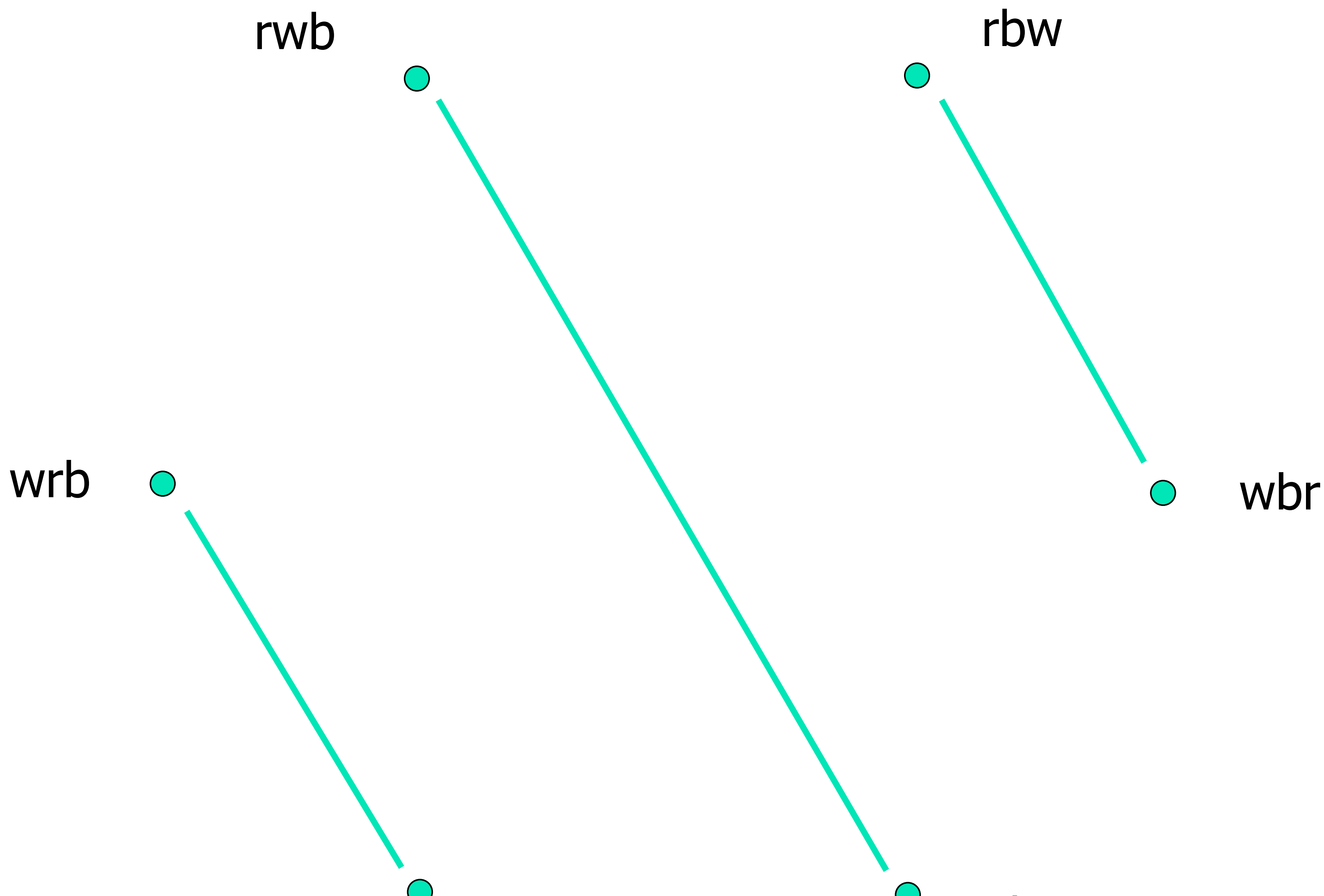
Card Games: HEXA



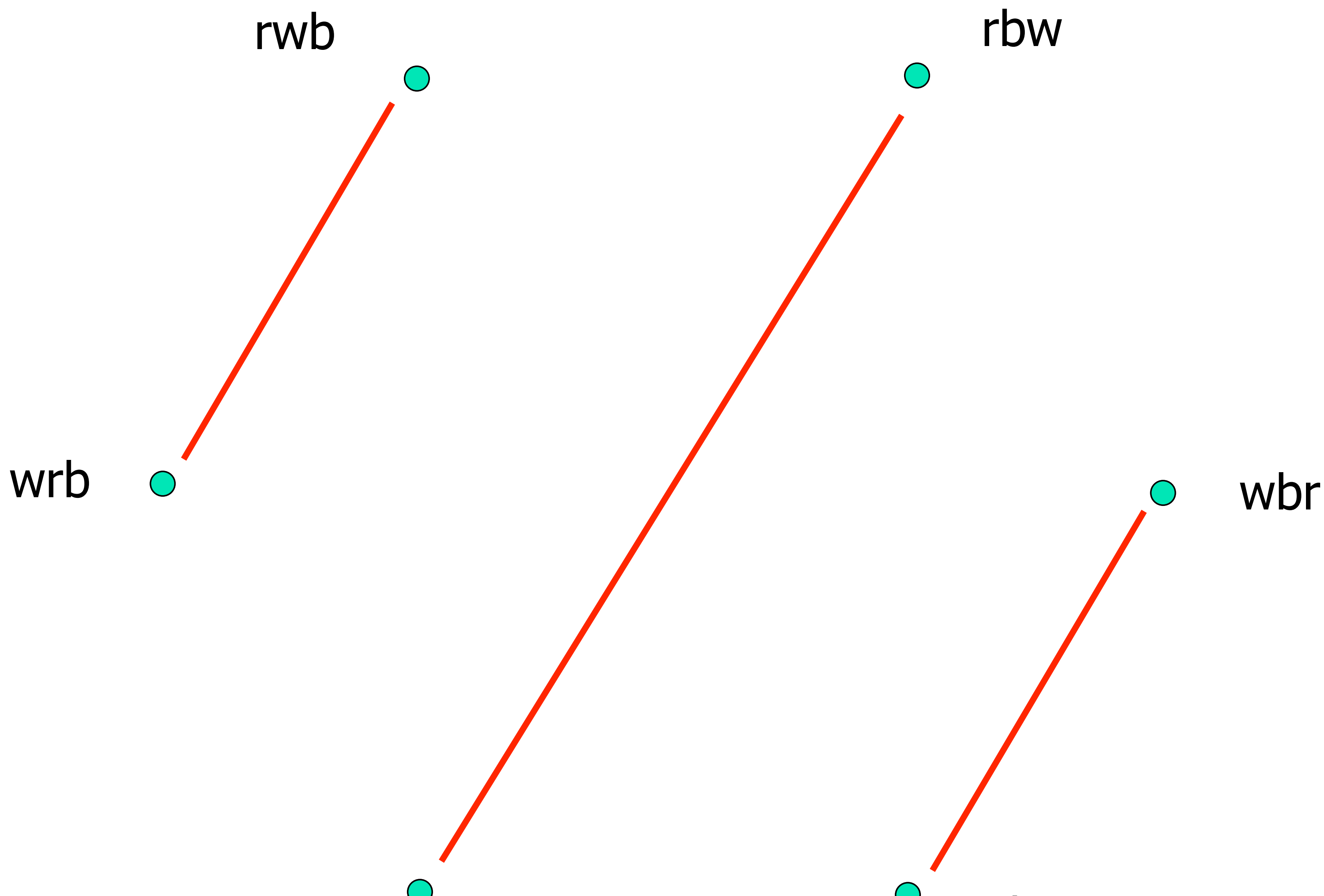
Card Games: HEXA



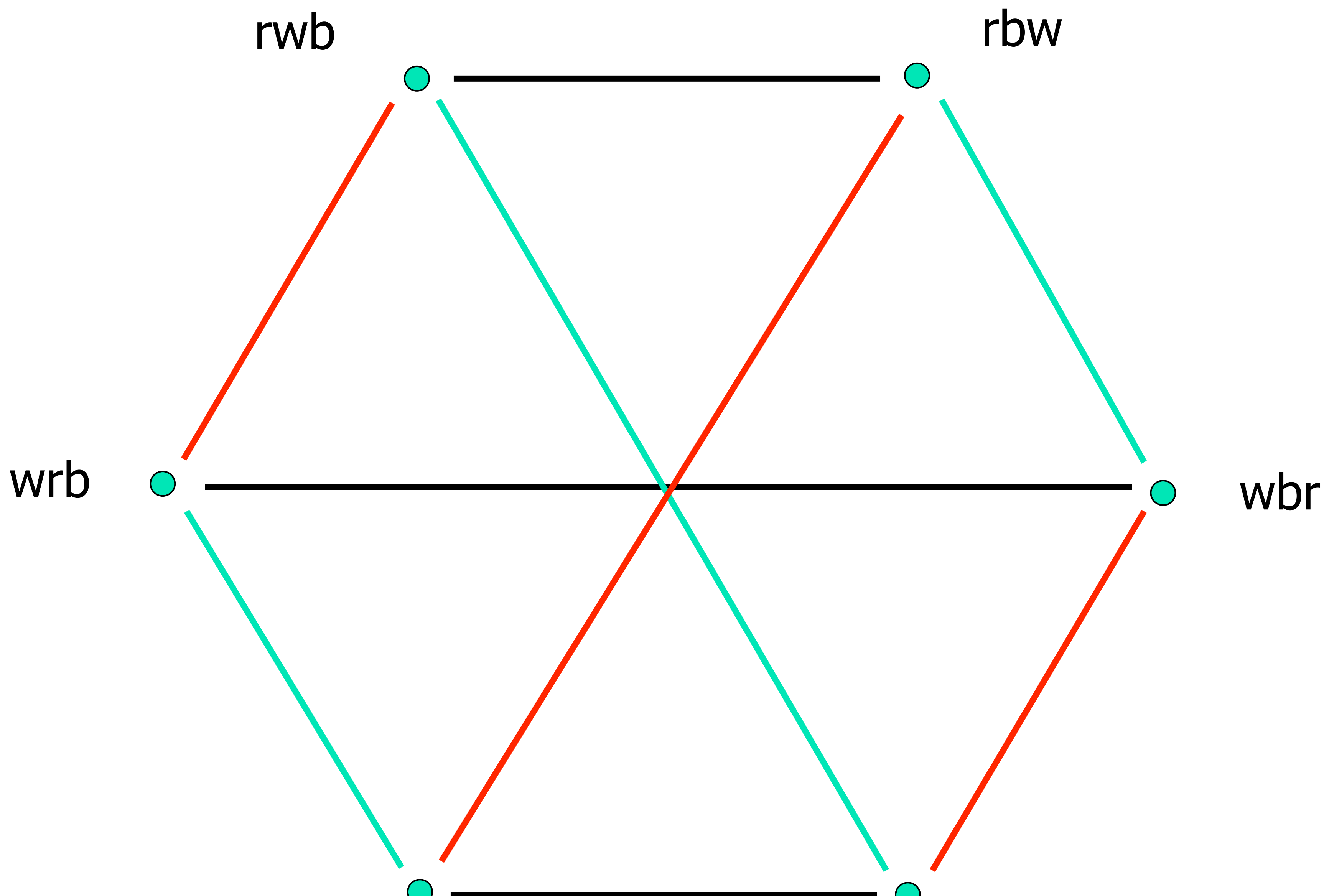
Card Games: HEXA



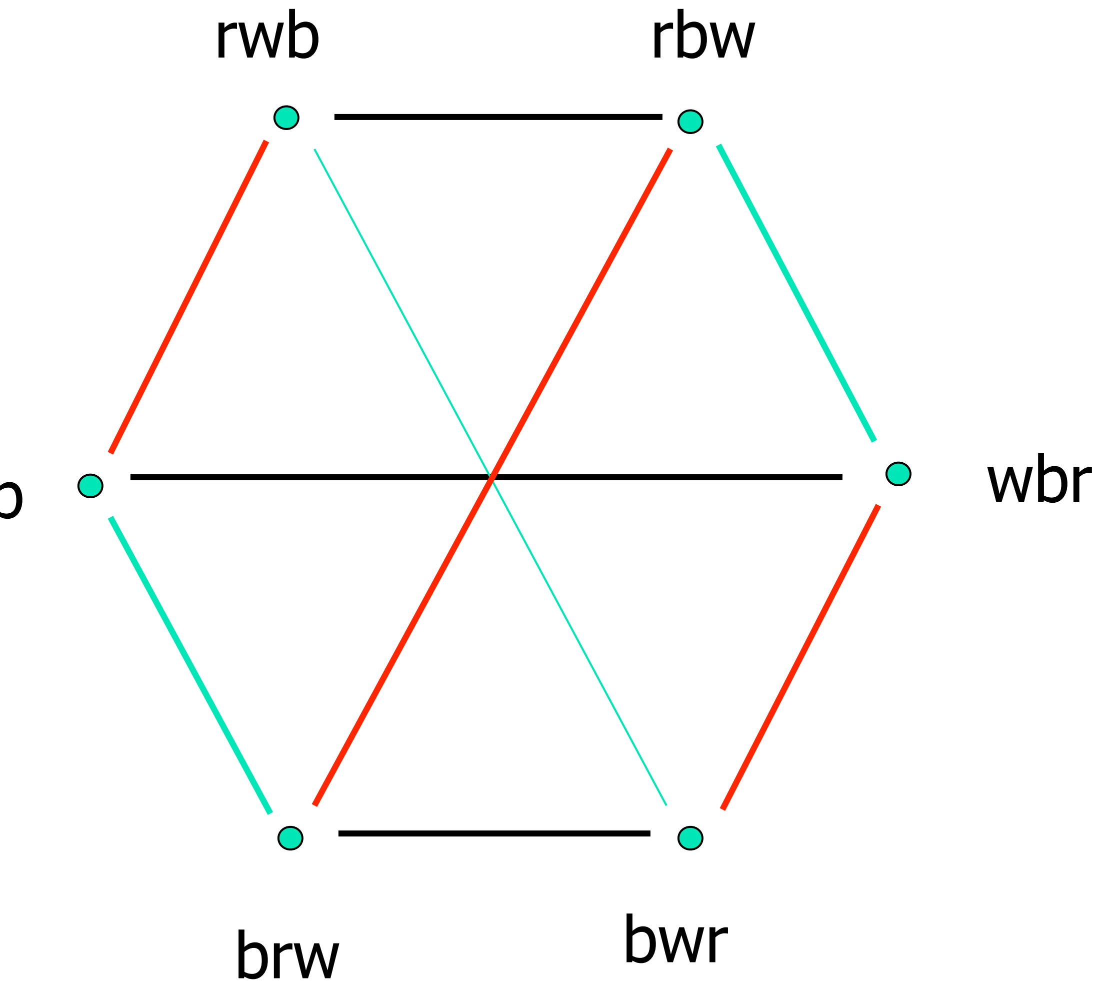
Card Games: HEXA



Card Games: HEXA

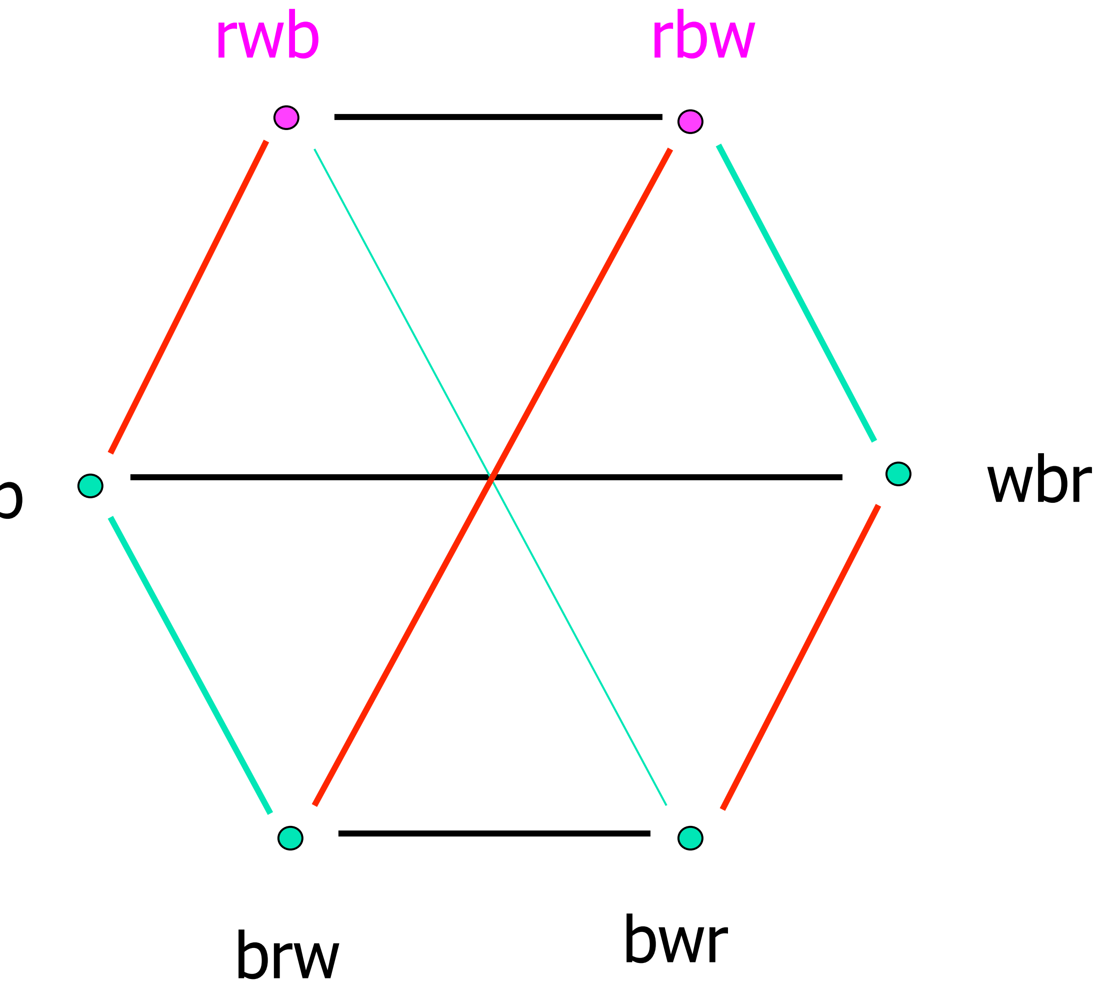


Properties of HEXA



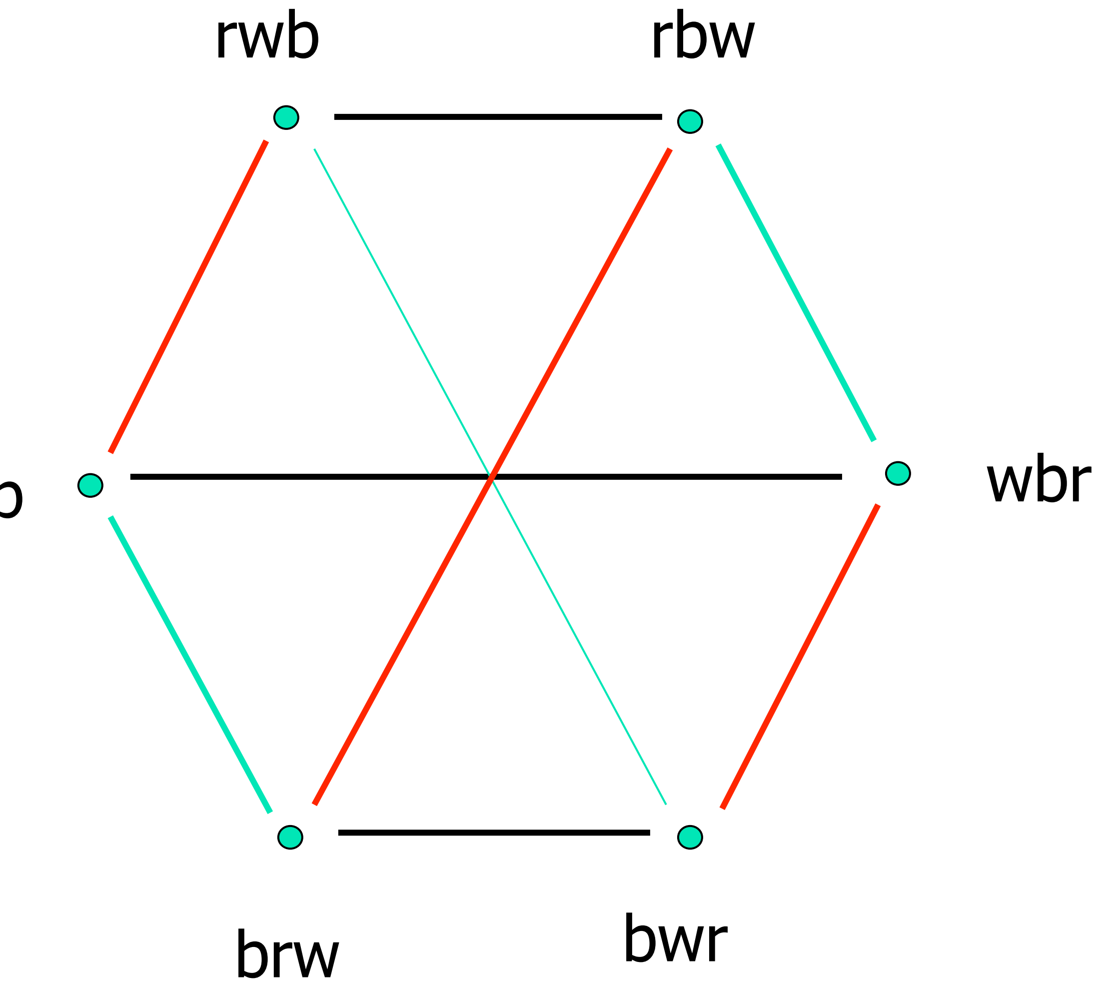
- $M, rwb \models K_1 r_1$

Properties of HEXA



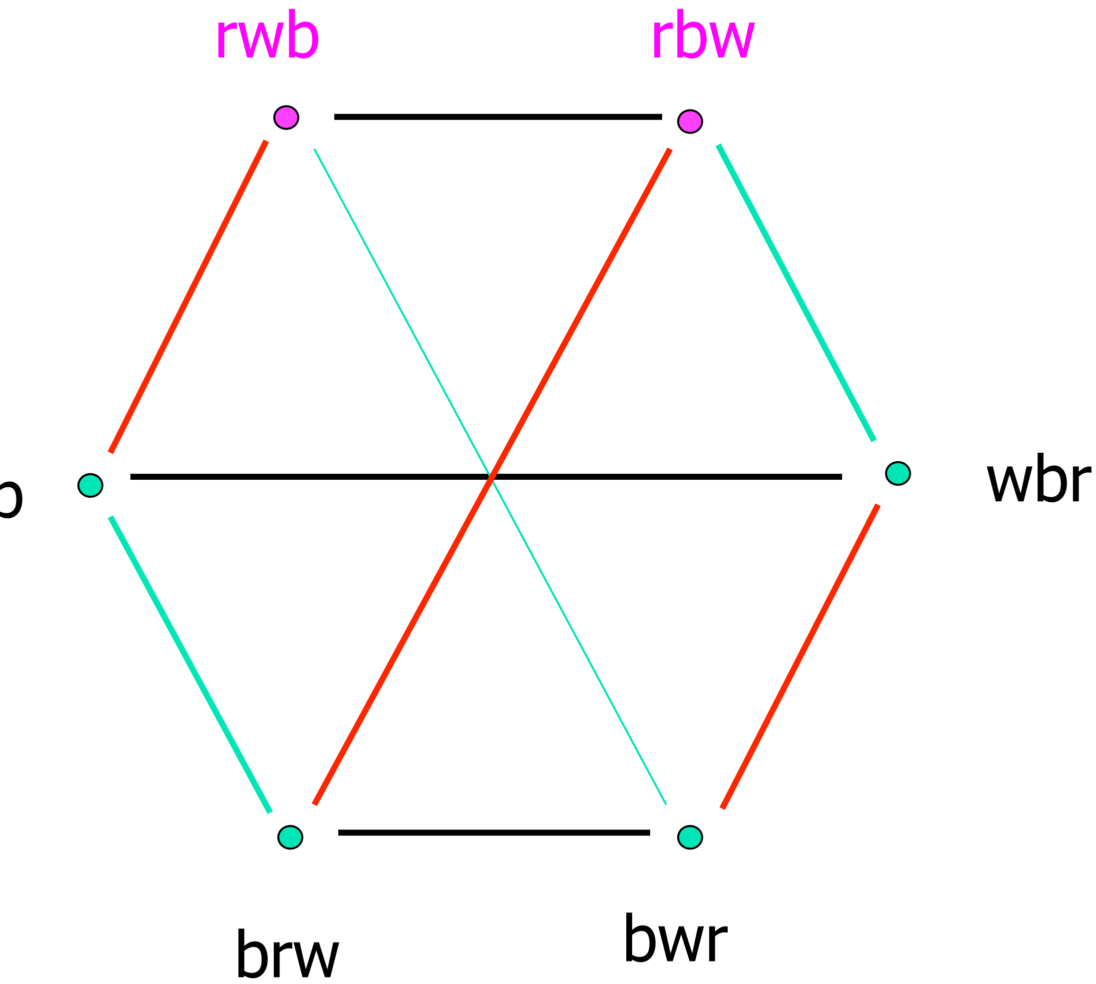
- M, rwb $\models K_1 r_1$ since
  - M, rwb $\models r_1$ and
  - M, rbw $\models r_1$

Properties of HEXA



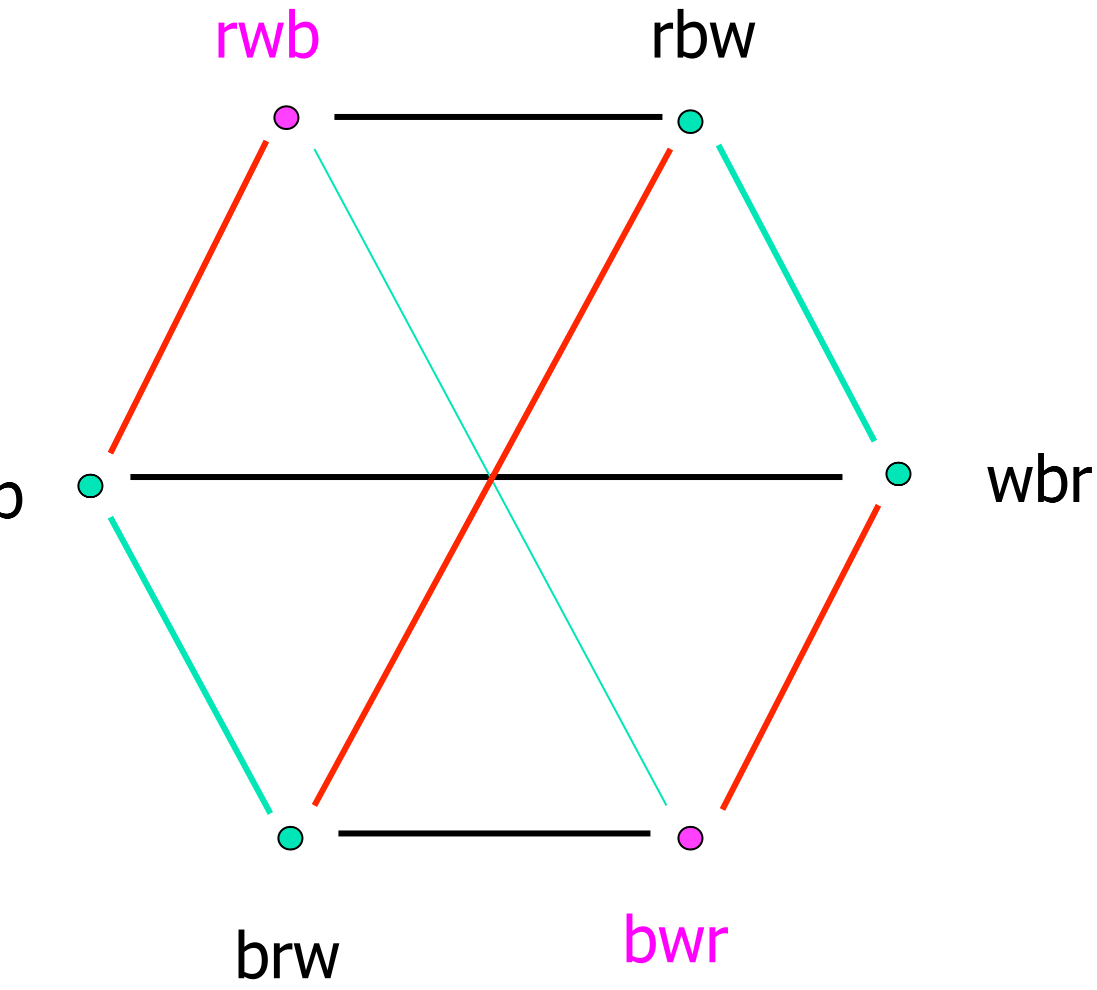
- $K_1(K_2\neg r_2) \wedge K_1(\neg K_2 r_1 \wedge \neg K_2 r_3)$
- $M, rwb \models K_1 K_2 \neg r_2$

Properties of HEXA



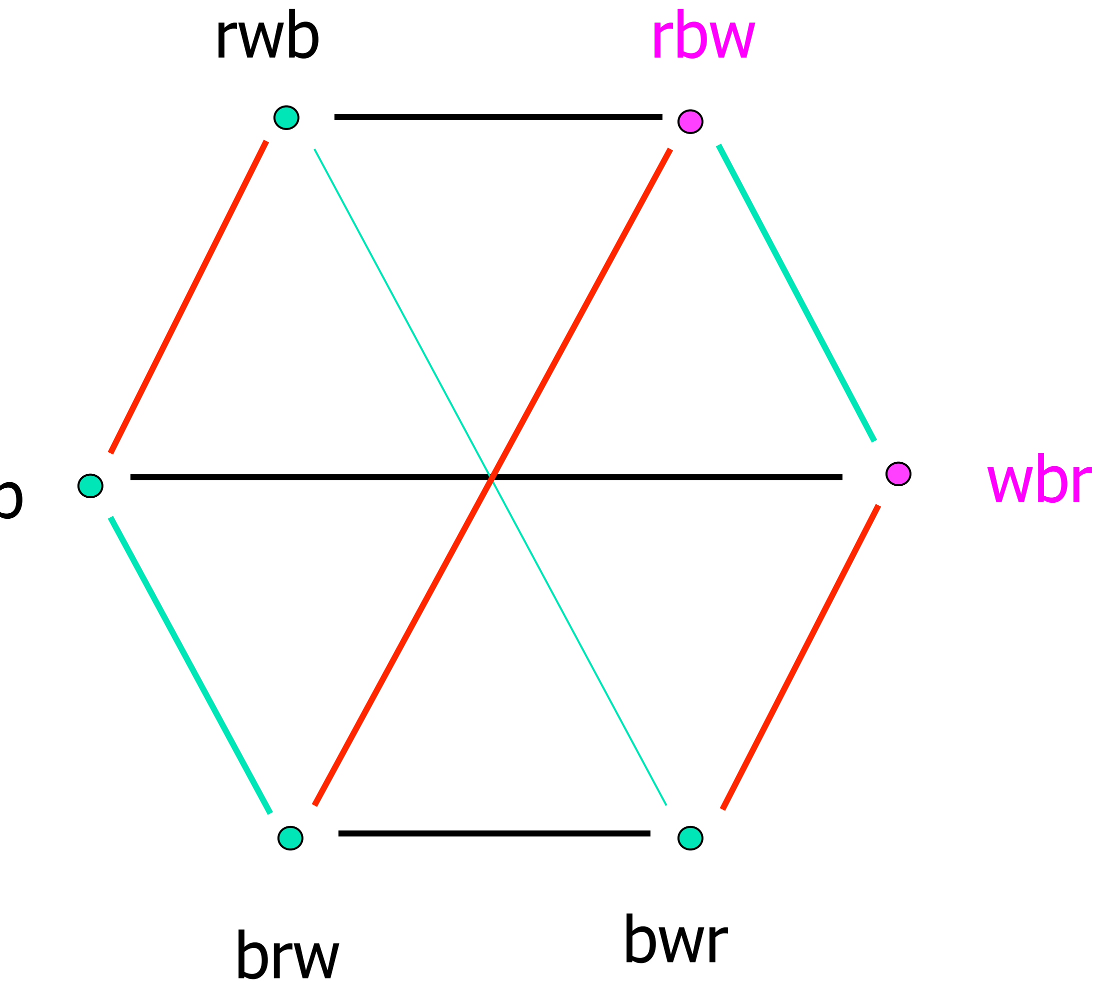
- $K_1(K_2\neg r_2) \wedge K_1(\neg K_2 r_1 \wedge \neg K_2 r_3)$
- $M, rwb \models K_1 K_2 \neg r_2$, since
  - $M, rwb \models K_2 \neg r_2$, and
  - $M, rbw \models K_2 \neg r_2$

Properties of HEXA



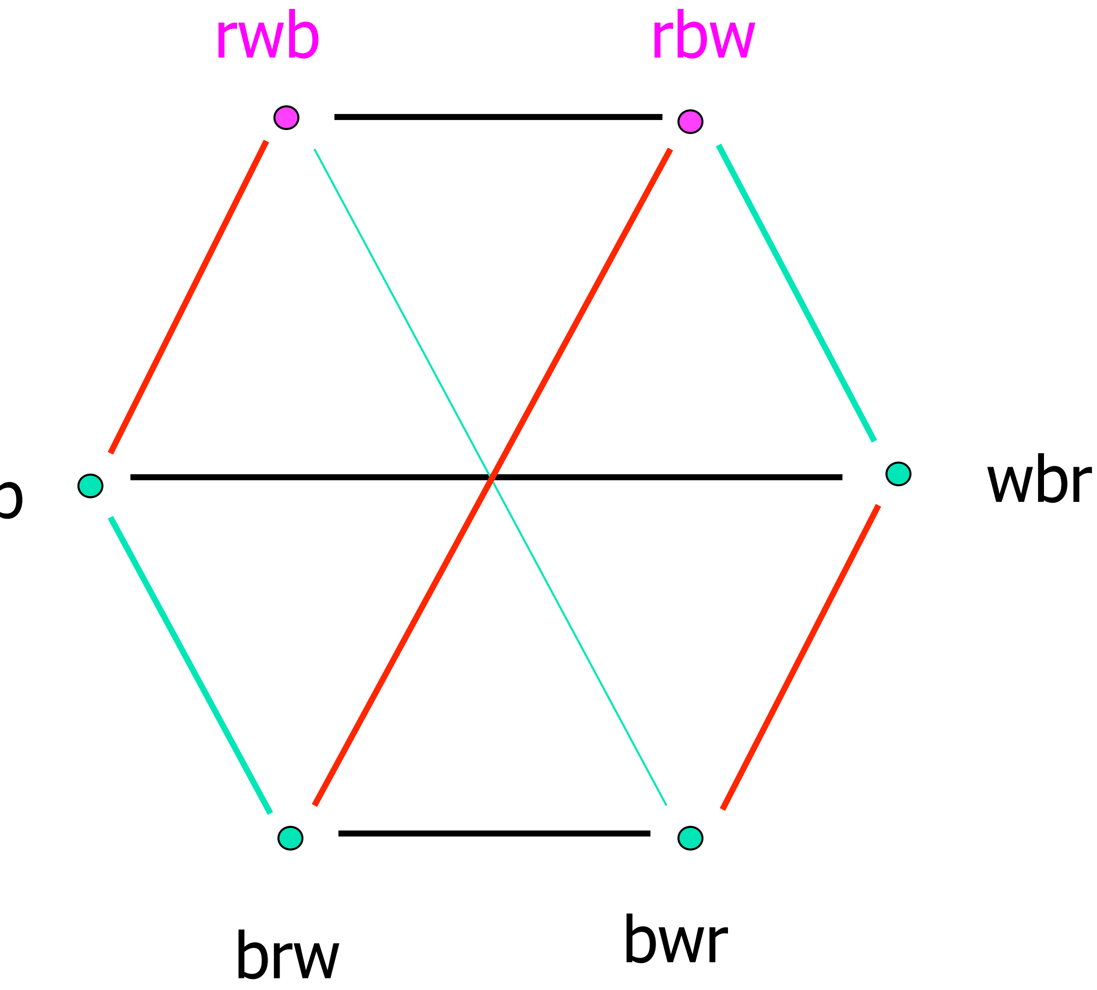
- $K_1(K_2\neg r_2) \wedge K_1(\neg K_2 r_1 \wedge \neg K_2 r_3)$
- $M, rwb \models K_1 K_2 \neg r_2$, since
  - $M, rwb \models K_2 \neg r_2$, since
    - $M, rwb \models \neg r_2$ and
    - $M, bwr \models \neg r_2$
  - $M, rbw \models K_2 \neg r_2$, since

Properties of HEXA



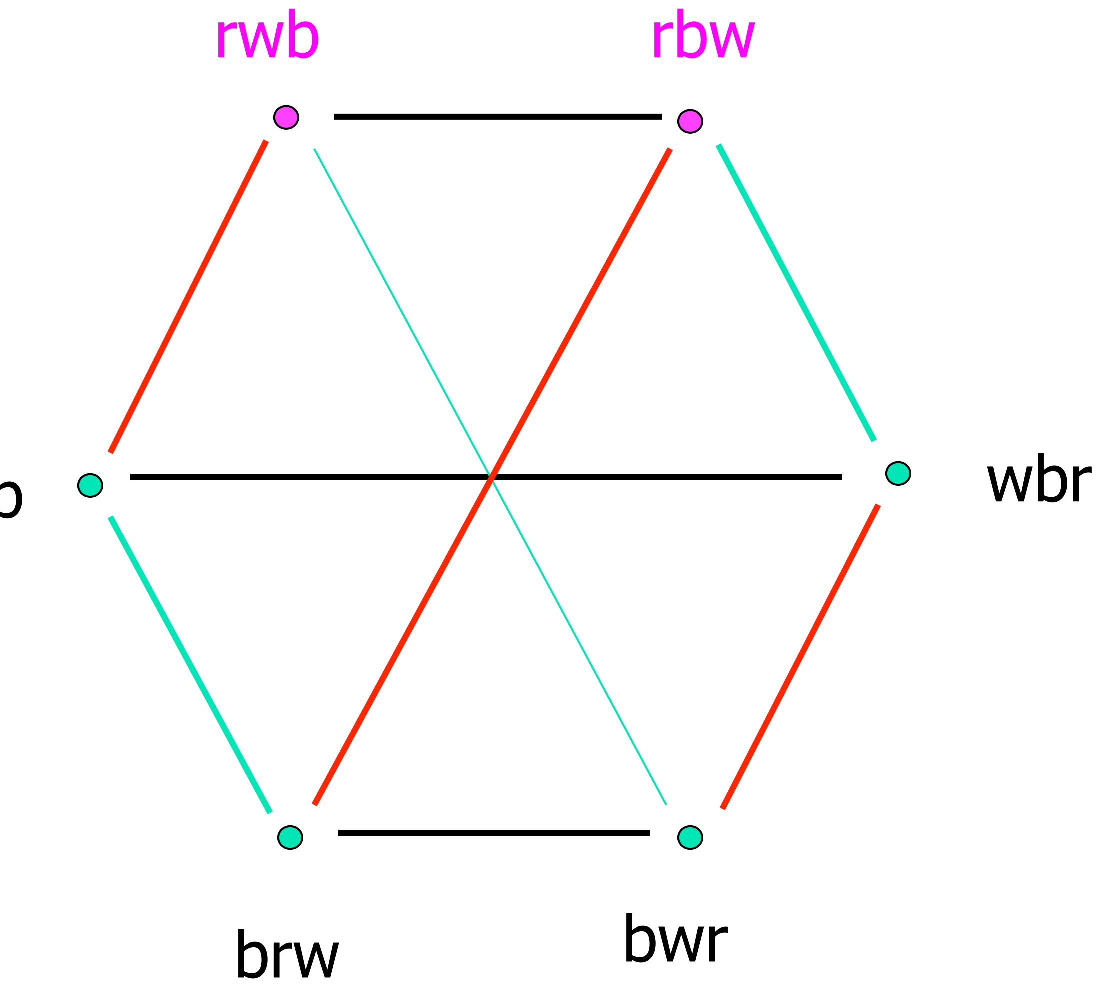
- $K_1(K_2\neg r_2) \wedge K_1(\neg K_2 r_1 \wedge \neg K_2 r_3)$
- $M, rwb \models K_1 K_2 \neg r_2$, since
  - $M, rwb \models K_2 \neg r_2$, since
    - $M, rwb \models \neg r_2$ and
    - $M, bwr \models \neg r_2$
  - $M, rbw \models K_2 \neg r_2$, since
    - $M, rwb \models \neg r_2$ and
    - $M, bwr \models \neg r_2$

Properties of HEXA



- $K_1(K_2\neg r_2) \wedge K_1(\neg K_2 r_1 \wedge \neg K_2 r_3)$
- $M, rwb \models K_1(\neg K_2 r_1 \wedge \neg K_2 r_3)$
- since
  - $M, rwb \models \neg K_2 r_1 \wedge \neg K_2 r_3$ and
  - $M, rbw \models \neg K_2 r_1 \wedge \neg K_2 r_3$

Properties of HEXA

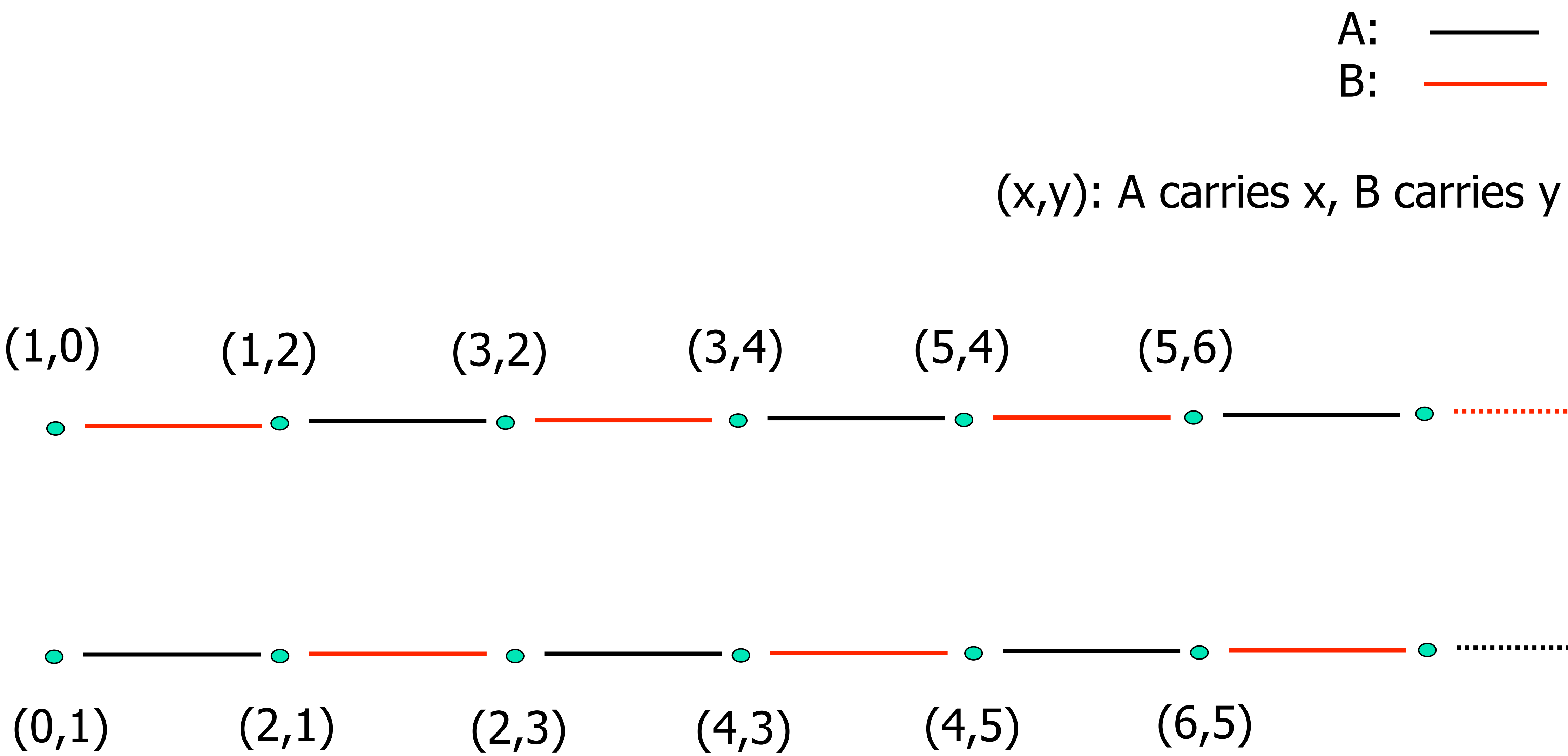


- $K_1(K_2\neg r_2) \wedge K_1(\neg K_2 r_1 \wedge \neg K_2 r_3)$
- $M, rwb \models K_1(\neg K_2 r_1 \wedge \neg K_2 r_3)$
- since
  - $M, rwb \models \neg K_2 r_1 \wedge \neg K_2 r_3$ since
    - $M, bwr \models \neg r_1$ and
    - $M, rwb \models \neg r_3$
  - $M, rbw \models \neg K_2 r_1 \wedge \neg K_2 r_3$ since
    - $M, wbr \models \neg r_1$ and
    - $M, rbw \models \neg r_3$

Exercise 3

- Ann and Bob each have a number on their head: Ann can only see Bob's, who can only see Ann's. However, it is known by everybody that the numbers are successors of each other, i.e. they are n and n + 1 for some n.
- Draw the appropriate model.

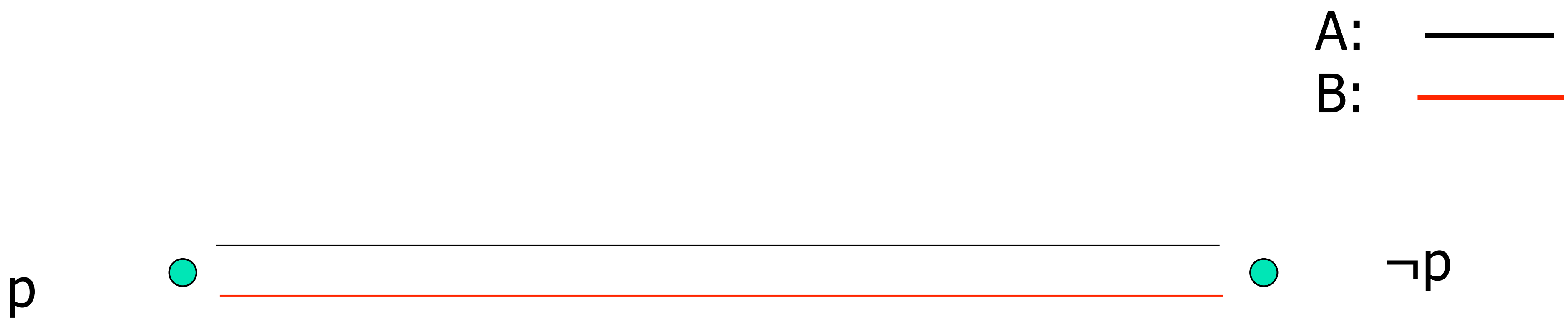
Exercise 3



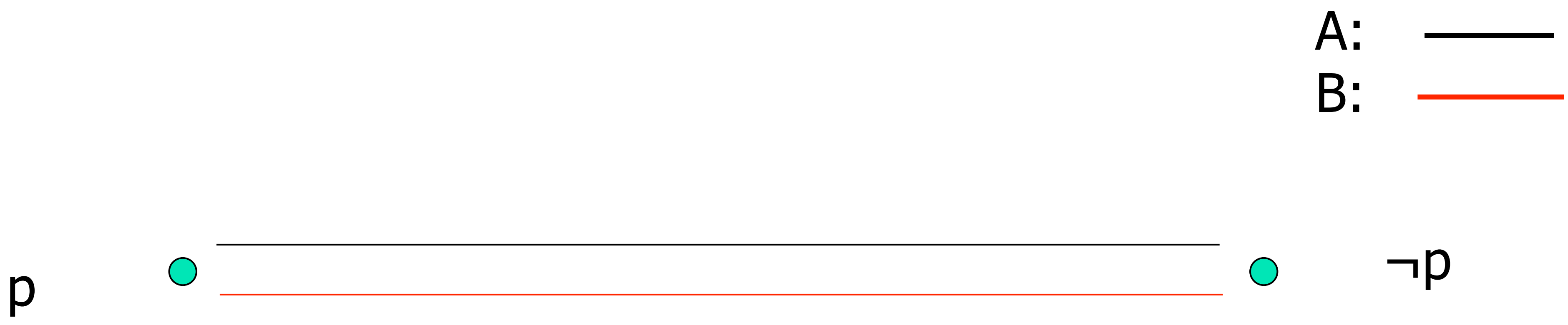
Exercise 4

- Ann and Bob are in a bar, in front of them is an envelop for Ann, with either an invitation to go out in Amsterdam (p) or one to give a lecture (¬p)
  - Draw the corresponding Kripke model whe nobody has looked in the envelop yet
  - Suppose A reads the letter in the envelop, and B sees this. Draw the new model.

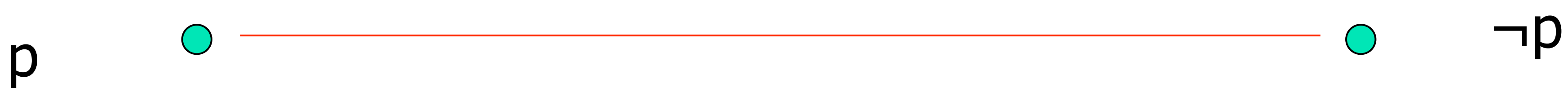
Exercise 4



Exercise 4



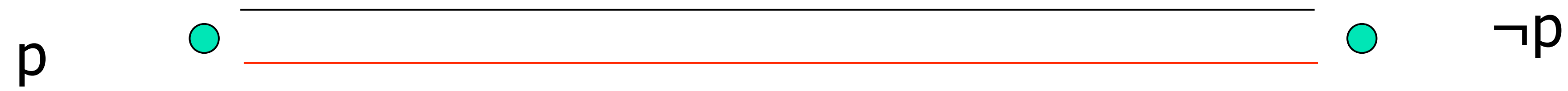
after A reads, and B sees that:



Exercise 4

A:

B:



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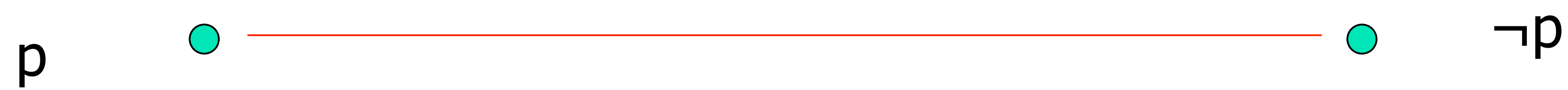
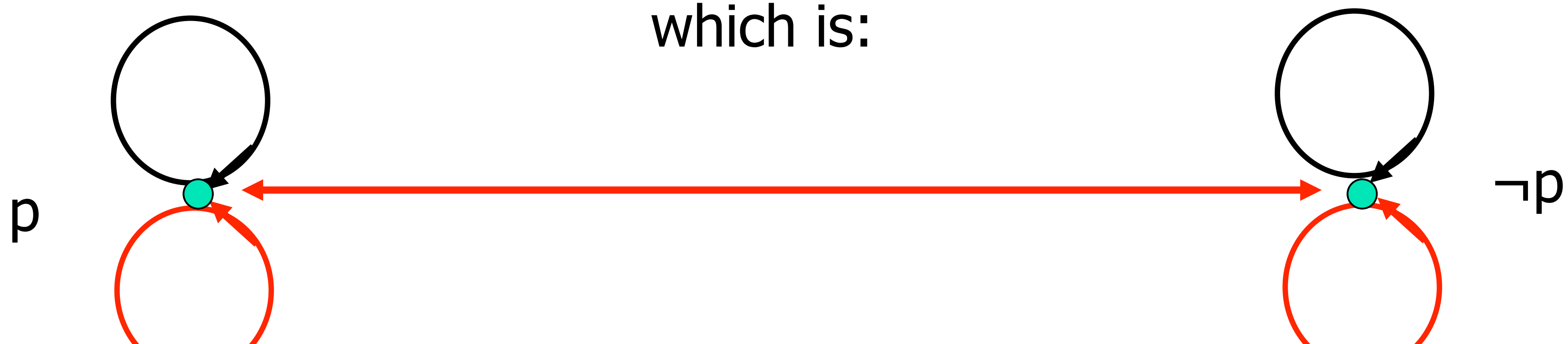


which is:



Exercise 5

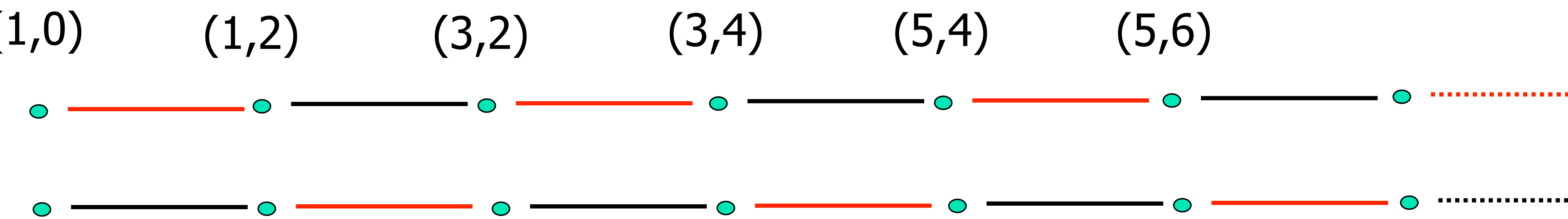
let a_n be: “An has number n”,similarly for b_n “Bob has number n

5a Show: M, (3,4) $\models \neg K_B b_4 \wedge K_A \neg K_B b_4$

let $K_A a$ denote: A knows what her number is, similarly for $K_B b$

5b Show: M,(1,2) $\models M_B K_A a$, or equivalently: M,(3,4) $\models \neg K_B \neg K_A a$

5c Show: M,(3,4) $\models M_B M_A M_B K_A a$, or equivalently: M,(3,4) $\models \neg K_B K_A K_B \neg K_A a$



Exercise 6

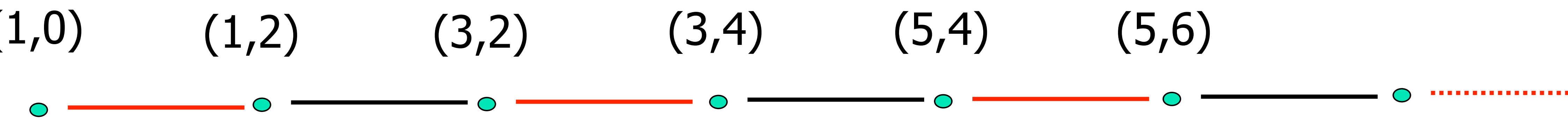
et a_n be: "An has number n",similarly for b_n "Bob has number n. Suppose the real world is (3,4). Call the model M (see below).

6a. Show M, (3,4) $|= \neg M_B M_A K_B b \wedge M_B M_A M_B K_A a$

6b. Suppose Ann says: I don't know my number
Draw the new model M_1. Show M_1, (3,4) $|= \neg M_B K_A a \wedge M_B M_A K_B b$

6c. Suppose now Bob says: I don't know my number
Draw the new model M_2. Show M_2, (3,4) $|= \neg K_B b \wedge M_B K_A a$

6d. Suppose now Ann says: I don't know my number
Draw the new model M_3. Show M_3, (3,4) $|= K_B b$!!



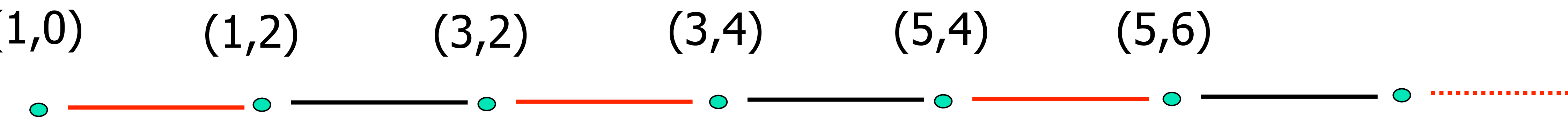
Exercise 6

Let a_n be: “An has number n”,similarly for b_n “Bob has number n. Suppose the real world is (3,4). Call the model M (see below).

6a. Show M, (3,4) $|\!=\neg\, M_B M_A K_B b \wedge M_B M_A M_B K_A a$

First of all, note that $K_B b$ is true nowhere: Bob is uncertain everywhere
This explains M, (3,4) $|\!=\neg\, M_B M_A K_B b$ (in fact M,x $|\!=\neg\, M_B M_A K_B b$ for all x)

Moreover, $K_A a$ is true only in (1,0). From (3,4) there is a B-A-B path to (1,0).



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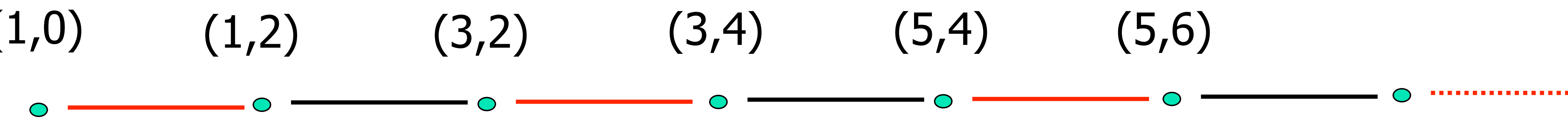
6a. Show M, (3,4) $|= \neg\, M_B M_A K_B b \wedge M_B M_A M_B K_A a$

Moreover, $K_A a$ is true only in (1,0). From (3,4) there is a B-A-B path to (1,0). More precisely:

M, (1,0) $|= K_A a$
M, (1,2) $|= M_B K_A a$
M, (3,2) $|= M_A M_B K_A a$
M, (3,4) $|= M_B M_A M_B K_A a$



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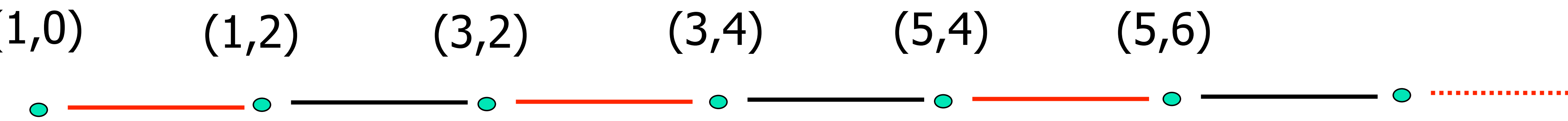
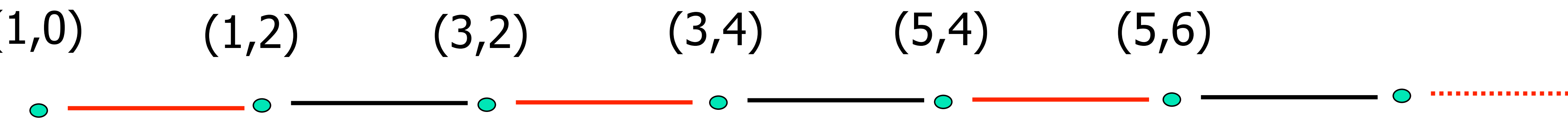
Moreover, $K_A a$ is true only in (1,0). From (3,4) there is a B-A-B path to (1,0). So, M, (3,4) $|= M_B M_A M_B K_A a$



Exercise 6

et a_n be: “An has number n”,similarly for b_n “Bob has number n. Suppose the
real world is (3,4). Call the model M (see below).

6b. Suppose Ann says: I don’t know my number
Draw the new model M_1. Show M_1, (3,4) $\models \neg M_B K_A a \wedge M_B M_A K_B b$



Exercise 6

Let a_n be: “An has number n”,similarly for b_n “Bob has number n. Suppose the real world is (3,4). Call the model M (see below).

6b. Suppose Ann says: I don’t know my number
Draw the new model M_1. Show M_1, (3,4) $\models \neg M_B K_A a \wedge M_B M_A K_B b$

There is no point at which $K_A a$ holds, so M_1, (3,4) $\models \neg M_B K_A a$

$K_B b$ is true at (1,2), and there is a B-A path to (1,2), hence
M_1, (3,4) $\models M_B M_A K_B b$

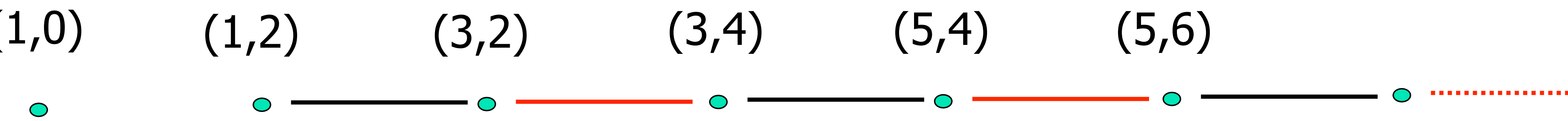
M_1, (1,2) $\models K_B b$, thus M_1, (3,2) $\models M_A K_B b$ thus M_1, (3,4) $\models M_B M_A K_B b$

(1,0) (1,2) (3,2) (3,4) (5,4) (5,6)

Exercise 6

et a_n be: "An has number n",similarly for b_n "Bob has number n. Suppose the real world is (3,4). Call the model M_1 (see below).

5c. Suppose now Bob says: I don't know my number
Draw the new model M_2. Show M_2, (3,4) $\models \neg K_B b \wedge M_B K_A a$

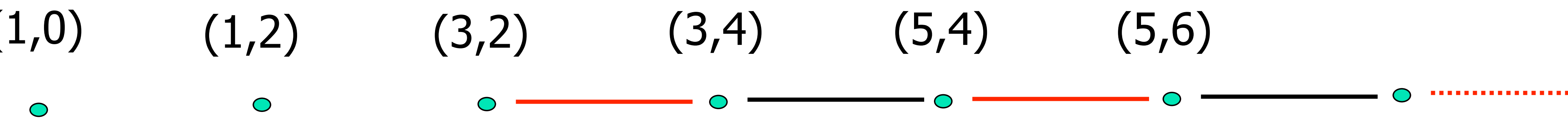


Exercise 6

et a_n be: "An has number n",similarly for b_n "Bob has number n. Suppose the real world is (3,4). Call the model M_2 (see below).

6c. Suppose now Bob says: I don't know my number

Draw the new model M_2. Show M_2, (3,4) $\models \neg K_B b \wedge M_B K_A a$



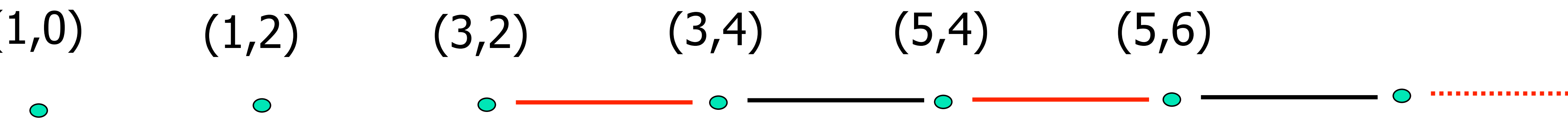
Exercise 6

et a_n be: “An has number n”,similarly for b_n “Bob has number n. Suppose the real world is (3,4). Call the model M_2 (see below).

5c. Suppose now Bob says: I don’t know my number
Draw the new model M_2. Show M_2, (3,4) $\models \neg K_B b \wedge M_B K_A a$

There is no point where $K_B b$ is true, hence (3,4) $\models \neg K_B b$

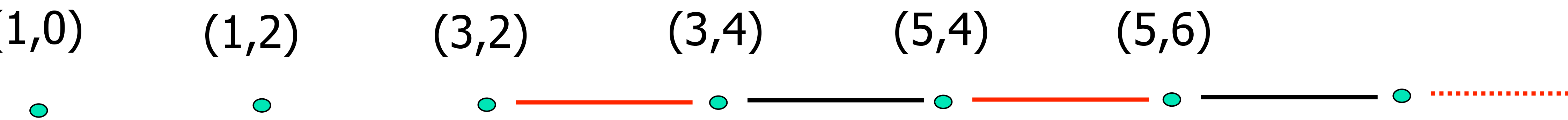
$K_A a$ is true at (3,2), hence (3,4) $\models M_B K_A a$



Exercise 6

et a_n be: "An has number n",similarly for b_n "Bob has number n. Suppose the real world is (3,4). Call the model M_2 (see below).

6d. Suppose now Ann says: I don't know my number
Draw the new model M_3. Show M_3, (3,4) $\models K_B b$!!

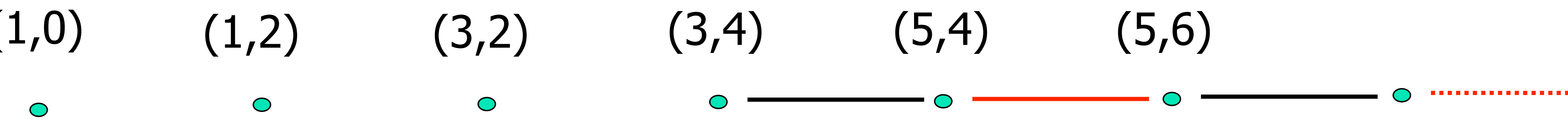


Exercise 6

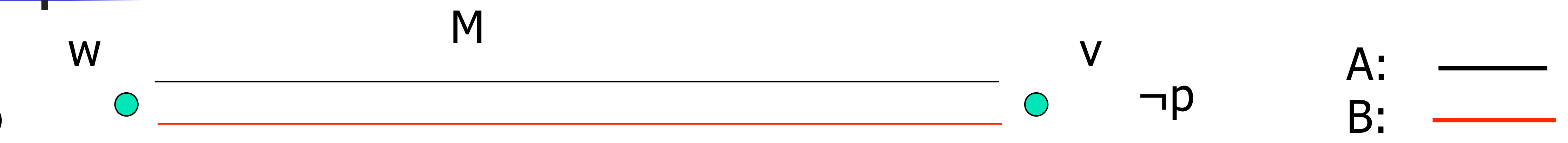
et a_n be: “An has number n”,similarly for b_n “Bob has number n. Suppose the real world is (3,4). Call the model M_3 (see below).

6d. Suppose now Ann says: I don’t know my number
Draw the new model M_3. Show M_3, $(3,4) \models K_B b$!!

This is clear: in (3,4), Bob has no doubt!



Exercise 7



or any item, start from M again.

a. Suppose Ann reads the letter aloud: it says p! Draw the model M_1

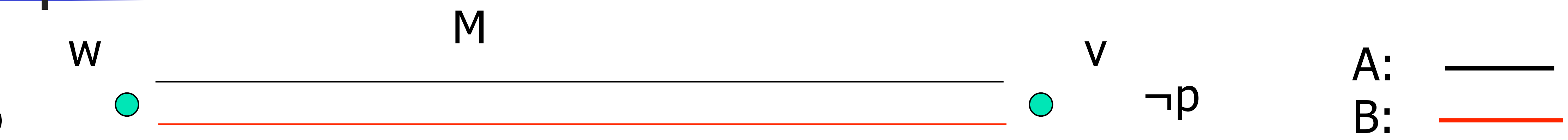
b. Bob holds it for possible that A read the letter. Draw M_2

c. An outsider tells A and B that one of them has read the letter. Draw M_3

d. An outsider tells A and B that one of them may have read the letter. Draw M_4

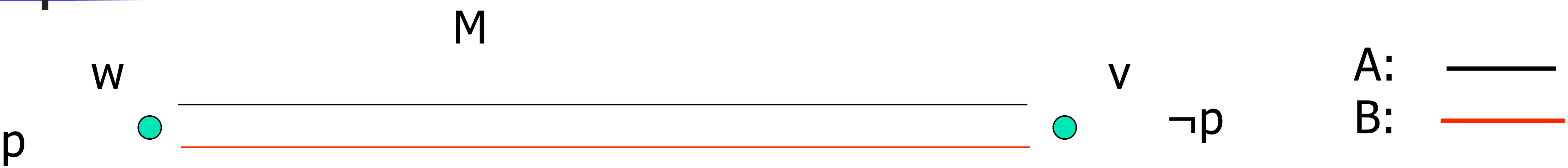
e. An outsider tells A and B that some (0, 1 or 2) of them may have read the let
Draw M_5

Exercise 7



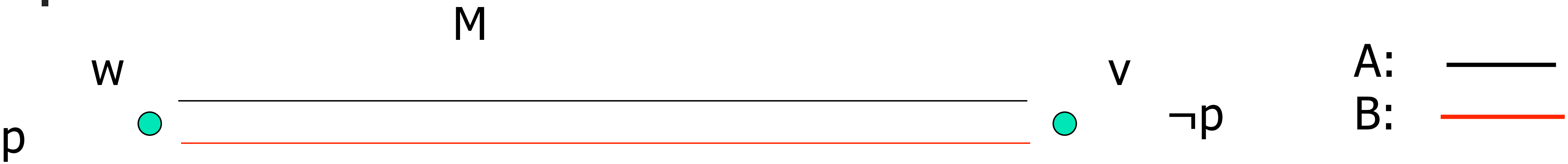
a. Suppose Ann reads the letter aloud: it says p! Draw the model M_1

Exercise 7

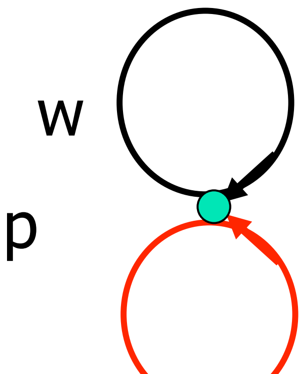


a. Suppose Ann reads the letter aloud: it says p! Draw the model M_1

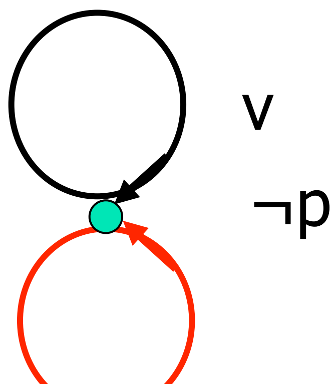
Exercise 7



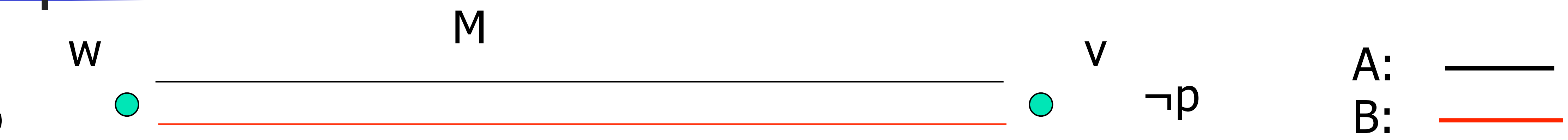
a. Suppose Ann reads the letter aloud: it says p! Draw the model M_1



which is actually:

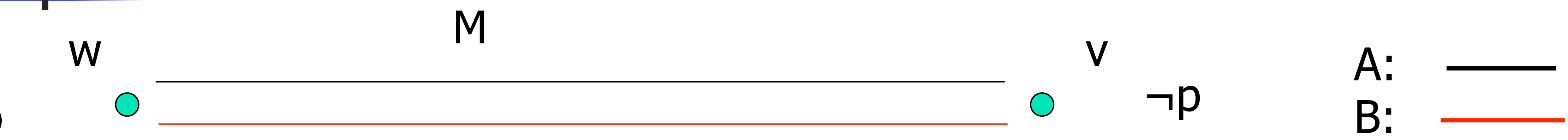


Exercise 7



b. Bob holds it for possible that A read the letter. Draw M_2

Exercise 7



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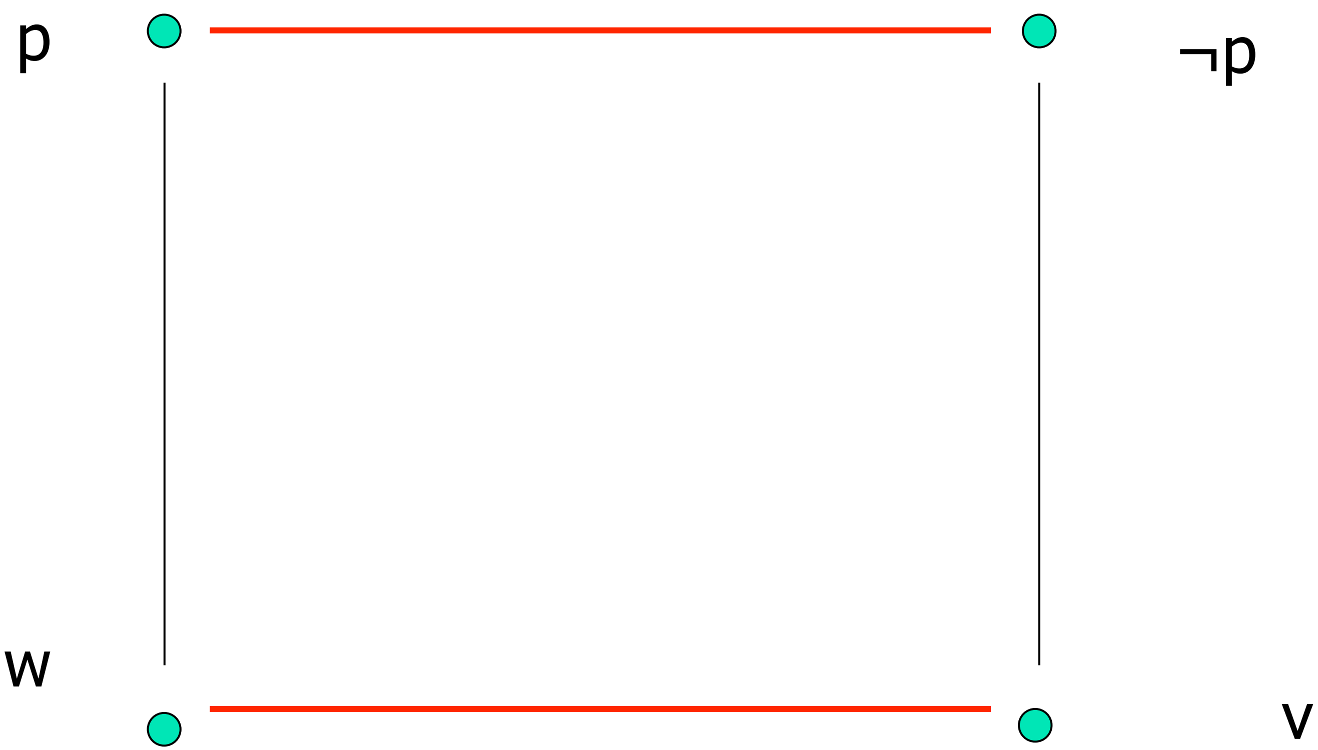
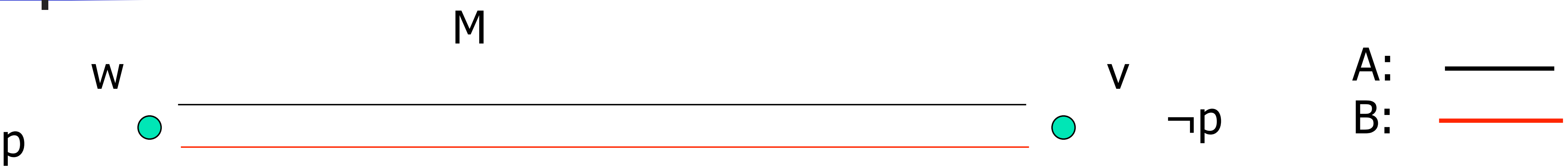
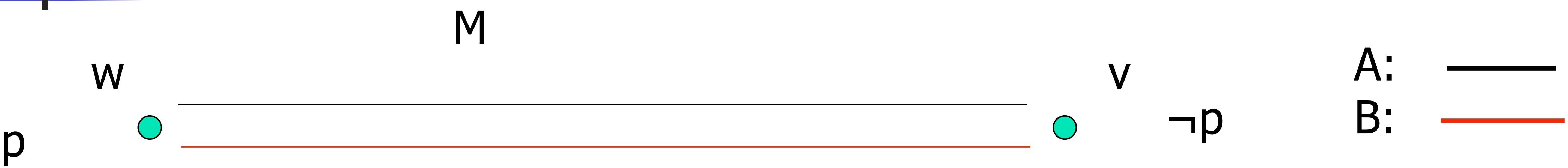


Exercise 7

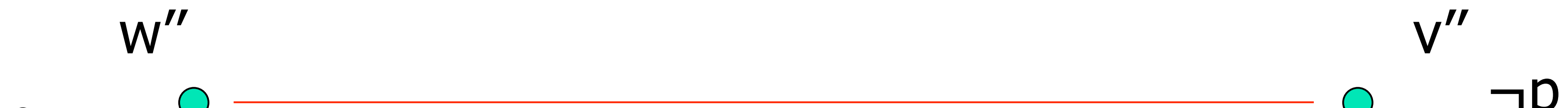


c. An outsider tells A and B that one of them has read the letter. Draw M_3

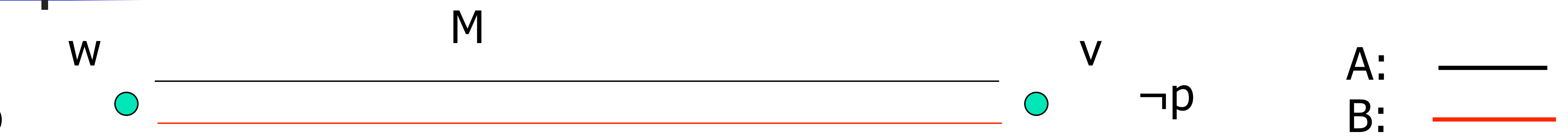
Exercise 7



c. An outsider tells A and B that one of them has read the letter. Draw M_3

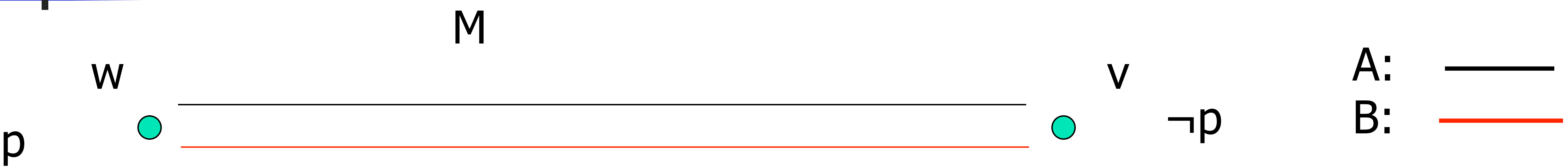


Exercise 7

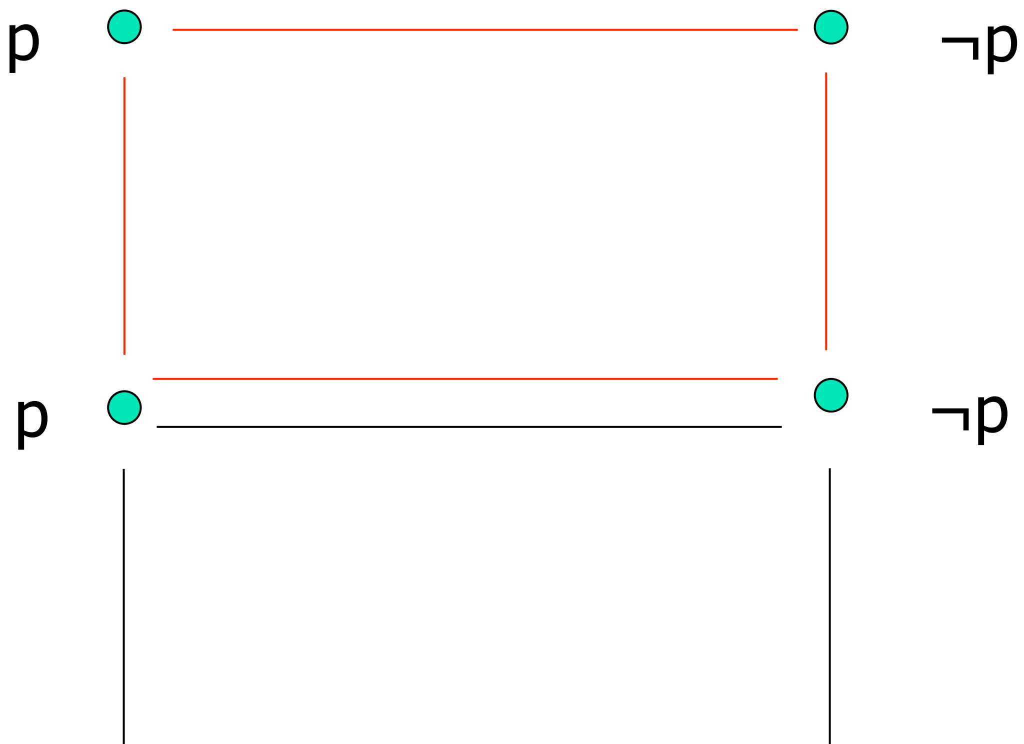


d. An outsider tells A and B that one of them may have read the letter. Draw M_4

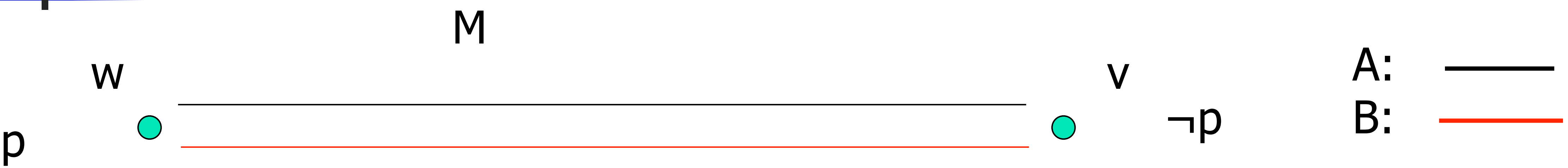
Exercise 7



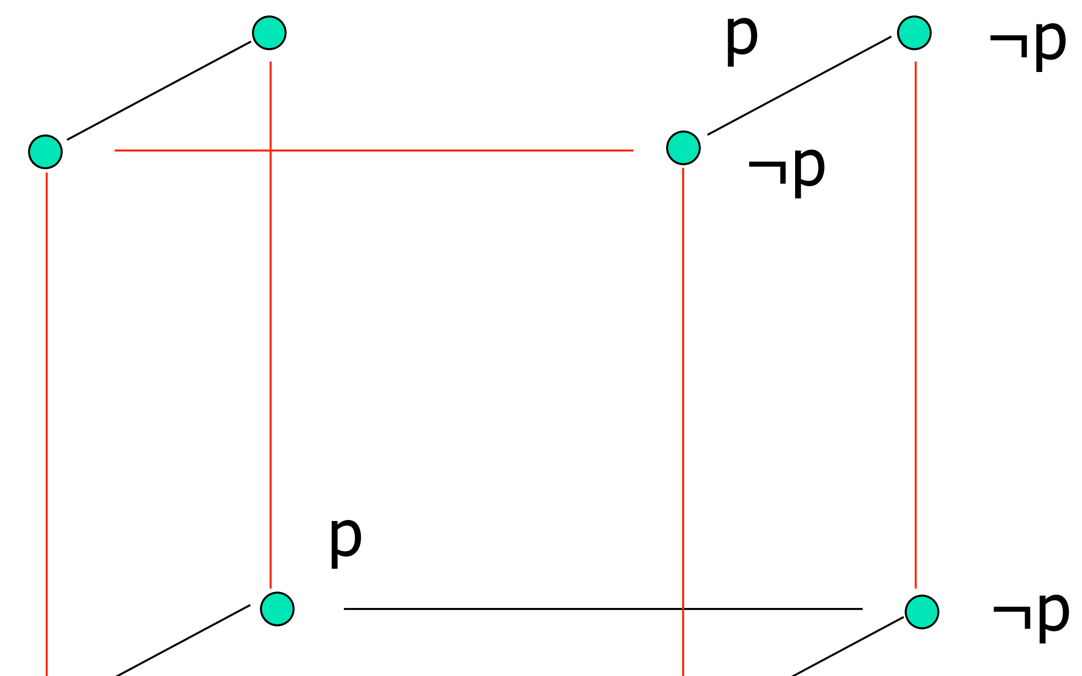
d. An outsider tells A and B that one of them may have read the letter. Draw M_4



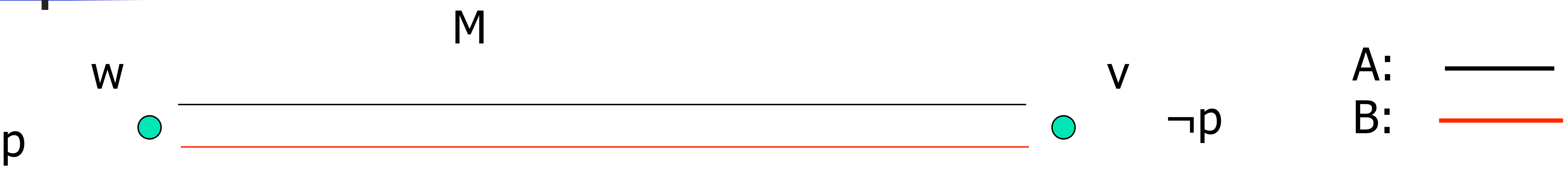
Exercise 7



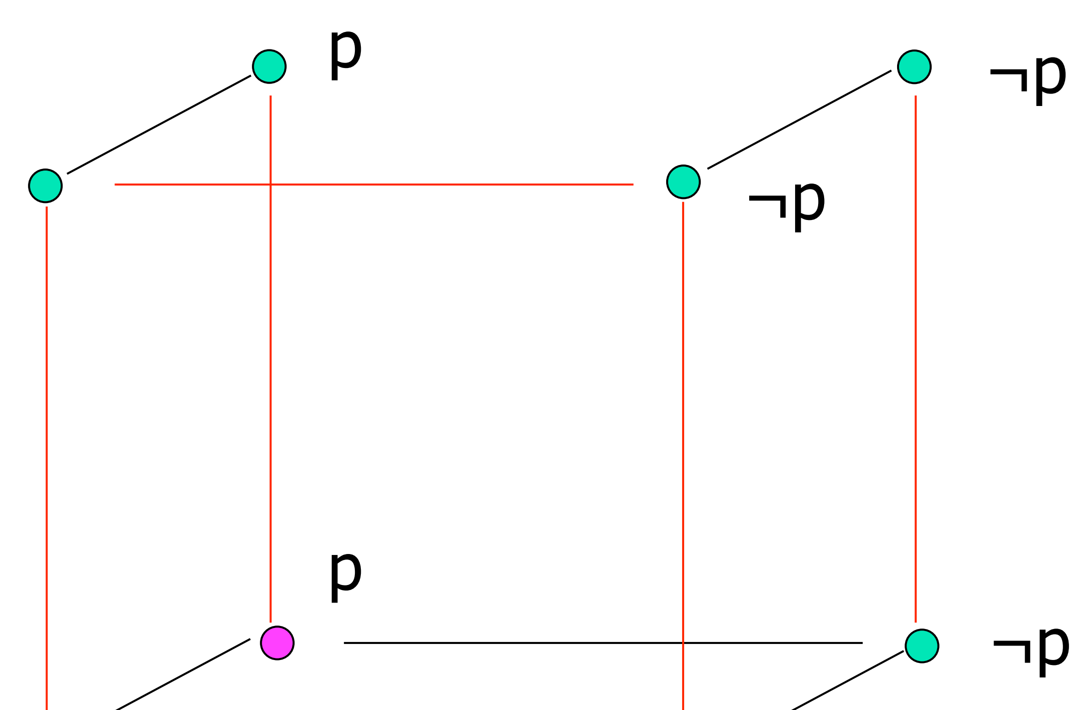
e. An outsider tells A and B that some (0, 1 or 2) of them may have read the let



Exercise 7



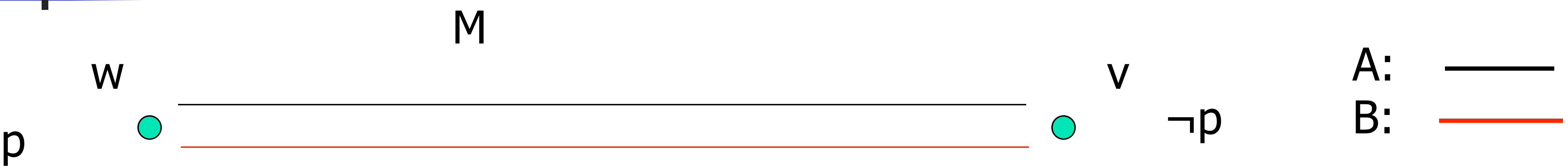
e. An outsider tells A and B that some (0, 1 or 2) of them may have read the let



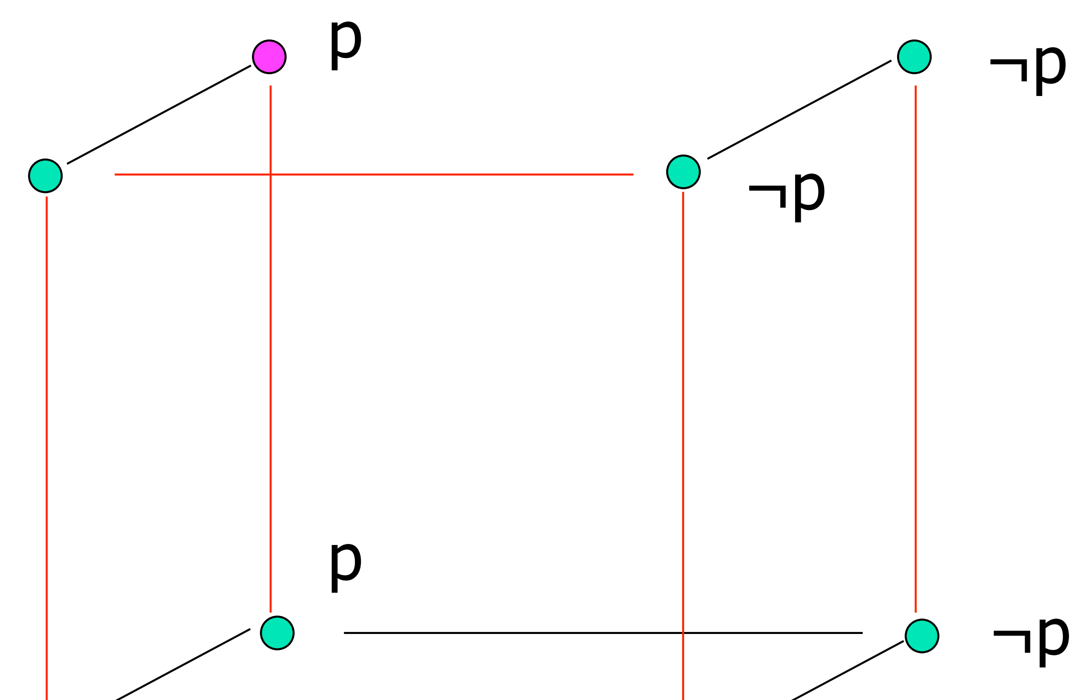
Suppose only B read the letter: p

$K_B p \wedge M_B K_A p \wedge M_B \neg K_A p$

Exercise 7



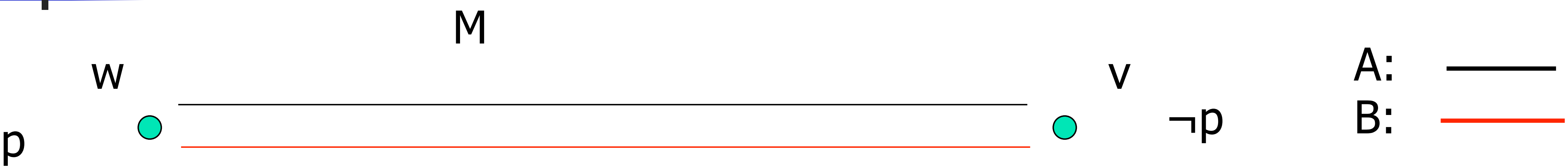
e. An outsider tells A and B that some (0, 1 or 2) of them may have read the let



Suppose A and B read the letter: p

$K_B p \wedge K_A p \wedge M_B \neg K_A p \wedge M_A \neg K_B p$

Exercise 7



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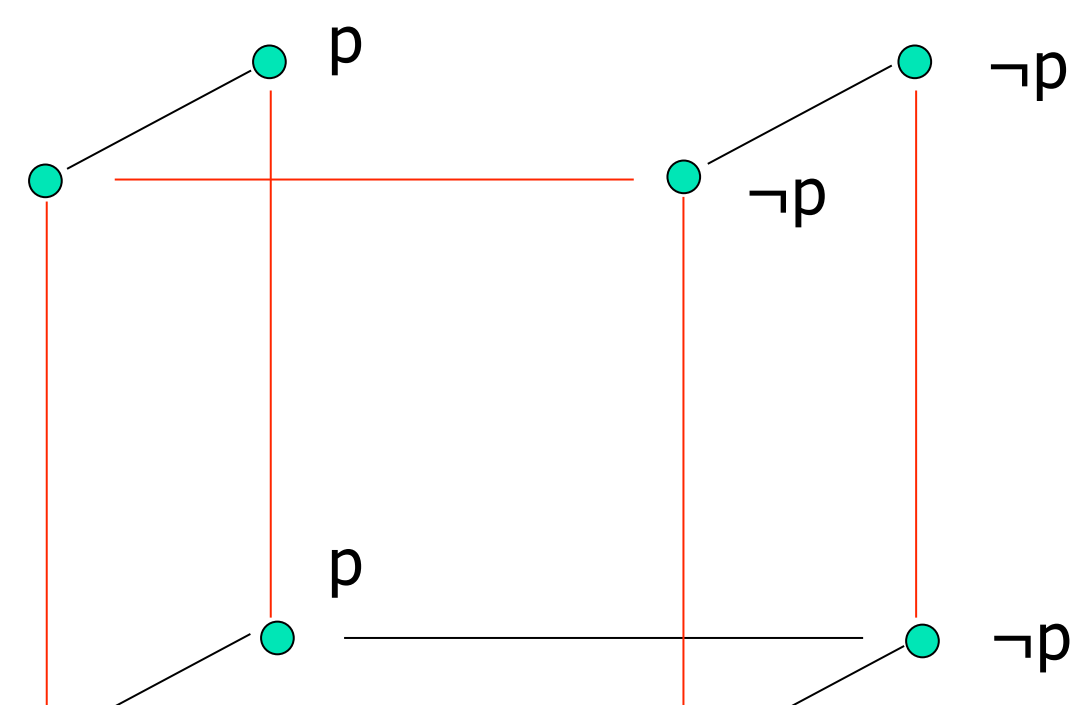


Suppose nobody read the letter: p

$\neg K_B p \wedge \neg K_A p \wedge M_B K_A p \wedge M_A K_B p$

Different Modal Systems

- Propositional Tautologies
- $K(\phi \rightarrow \psi) \rightarrow (K\phi \rightarrow K\psi)$
- $K\phi \rightarrow \phi$
- $K\phi \rightarrow KK\phi$
- $\neg K\phi \rightarrow K\neg K\phi$
- $\vdash \phi \Rightarrow \vdash K\phi$
- $\vdash \phi, \vdash \phi \rightarrow \psi \Rightarrow \vdash \psi$

Different Modal Systems

- Propositional Tautologies — A1
- $K_i(\phi \rightarrow \psi) \rightarrow (K_i\phi \rightarrow K_i\psi)$ — A2
- $K_i\phi \rightarrow \phi$ — A3
- $K_i\phi \rightarrow K_iK\phi$ — A4
- $\neg K\phi \rightarrow K\neg K\phi$ — A5
- $\vdash \phi \Rightarrow \vdash K_i\phi$ — Nec

Different Modal Systems

K	$K_i(\phi \rightarrow \psi) \rightarrow (K_i\phi \rightarrow K_i\psi)$	A2
T	$K_i\phi \rightarrow \phi$	A3
4	$K_i\phi \rightarrow K_iK_i\phi$	A4
5	$\neg K_i\phi \rightarrow K_i\neg K_i\phi$	A5

Different Modal Systems

- Propositional tautologies, Nec
- $K_i(\phi \rightarrow \psi) \rightarrow (K_i\phi \rightarrow K_i\psi)$

$K_{(m)}$

- $K_i\phi \rightarrow \phi$
- $K_i\phi \rightarrow K_iK_i\phi$
- $\neg K_i\phi \rightarrow K_i\neg K_i\phi$

Different Modal Systems

- Propositional tautologies, Nec
- $K_i(\phi \rightarrow \psi) \rightarrow (K_i\phi \rightarrow K_i\psi)$
- $K_i\phi \rightarrow \phi$
- $K_i\phi \rightarrow K_iK_i\phi$
- $\neg K_i\phi \rightarrow K_i\neg K_i\phi$

$\mathbf{K_{(m)}}$

$\mathbf{T_{(m)}}$

Different Modal Systems

- Propositional tautologies, Nec
- $K_i(\phi \rightarrow \psi) \rightarrow (K_i\phi \rightarrow K_i\psi)$
- $K_i\phi \rightarrow \phi$
- $K_i\phi \rightarrow K_iK_i\phi$
- $\neg K_i\phi \rightarrow K_i\neg K_i\phi$

$K_{(m)}$

$T_{(m)}$

$S4_{(m)}$

Different Modal Systems

- Propositional tautologies, Nec
- $K_i(\phi \to \psi) \to (K_i\phi \to K_i\psi)$
- $K_i\phi \to \phi$
- $K_i\phi \to K_iK_i\phi$
- $\neg K_i\phi \to K_i\neg K_i\phi$

$K_{(m)}$

$T_{(m)}$

$S4_{(m)}$

S

Different Modal Systems

- Propositional tautologies, Nec
- $K_i(\phi \rightarrow \psi) \rightarrow (K_i\phi \rightarrow K_i\psi)$
- $\neg K_i \bot$
- $K_i\phi \rightarrow K_iK_i\phi$
- $\neg K_i\phi \rightarrow K_i\neg K_i\phi$

$K_{(m)}$

$KD_{(m)}$

$KD4_{(m)}$

KD

Different Semantic Systems

- Kripke Models
- Accessibility Relations R_i
- R_i is reflexive
- R_i is transitive
- R_i is Euclidean

$\mathcal{K}_{(m)}$

$\mathcal{T}_{(m)}$

$\mathcal{S}4_{(m)}$

$\mathcal{S}$

Different Semantic Systems

- Kripke Models
- Accessibility Relations R_i
- R_i is serial
- R_i is transitive
- R_i is Euclidean

$\mathcal{K}_{(m)}$

$\mathcal{KD}_{(m)}$

$\mathcal{KD}4_{(m)}$

$\mathcal{KD}5$

Relating the Systems

$$
\begin{array}{ccccccccc}
) \models \varphi & \Rightarrow & \mathcal{KD}_{(m)} \models \varphi & \Rightarrow & \mathcal{T}_{(m)} \models \varphi & \Rightarrow & \mathcal{S}4_{(m)} \models \varphi & \Rightarrow & \mathcal{S}5_{(m)} \mid \\
\Updownarrow & & \Updownarrow & & \Updownarrow & & \Updownarrow & & \Updownarrow \\
) \vdash \varphi & \Rightarrow & \mathbf{KD_{(m)}} \vdash \varphi & \Rightarrow & \mathbf{T_{(m)}} \vdash \varphi & \Rightarrow & \mathbf{S4_{(m)}} \vdash \varphi & \Rightarrow & \mathbf{S5_{(m)}}
\end{array}
$$

Example Derivation

$\mathbf{S5_{(m)}} \vdash \neg K_i\phi \leftrightarrow K_i\neg K_i\phi$

- $\neg K_i\phi \rightarrow K_i\neg K_i\phi$ A5

Example Derivation

$\mathbf{S5_{(m)}} \vdash \neg K_i\phi \leftrightarrow K_i\neg K_i\phi$

- $\neg K_i\phi \rightarrow K_i\neg K_i\phi$ A5
- $K_i\neg K_i\phi \rightarrow \neg K_i\phi$ A3

Example Derivation

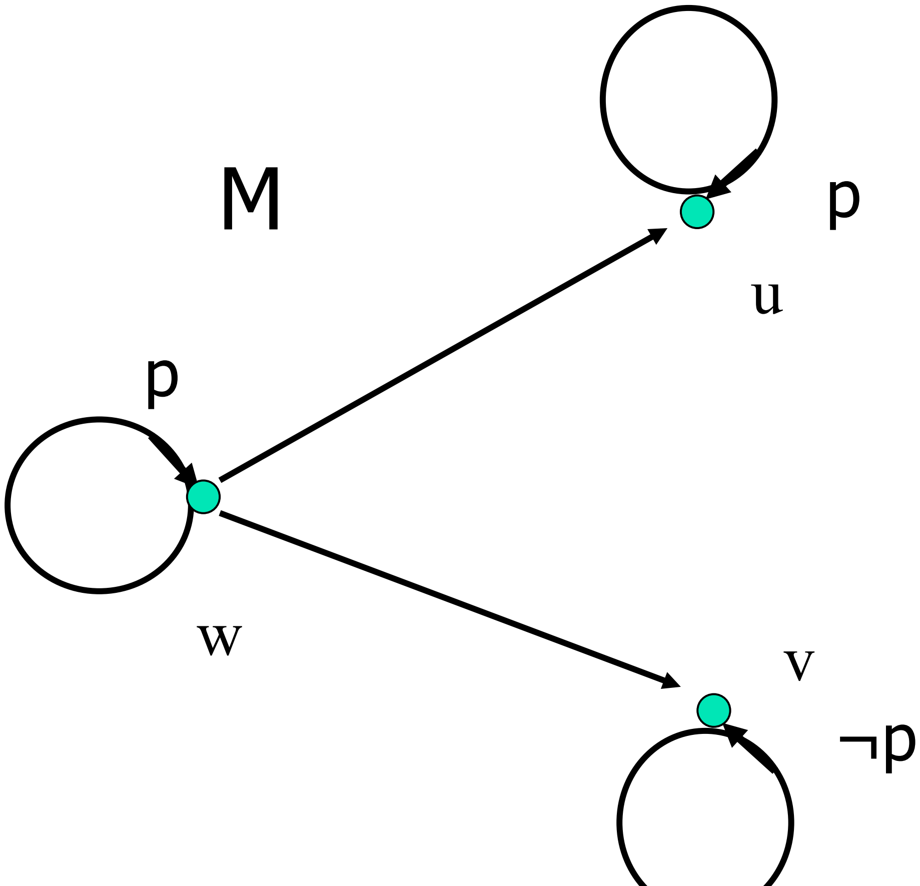
$\mathbf{S5_{(m)}} \vdash \neg K_i\phi \leftrightarrow K_i\neg K_i\phi$

- $\neg K_i\phi \rightarrow K_i\neg K_i\phi$ A5
- $K_i\neg K_i\phi \rightarrow \neg K_i\phi$ A3
- $\neg K_i\phi \leftrightarrow K_i\neg K_i\phi$ 1,2,

Counterexamples

$\mathbf{S4_{(m)}} \vdash \neg K_i\phi \leftrightarrow K_i\neg K_i\phi$?

Answer is NO! Because $S4_{(m)} \not\models \neg K_i\phi \leftrightarrow K_i\neg K_i\phi$!!!!!!!!!



$M, \not\models \neg K_i\phi \leftrightarrow K_i\neg K_i\phi$

Because

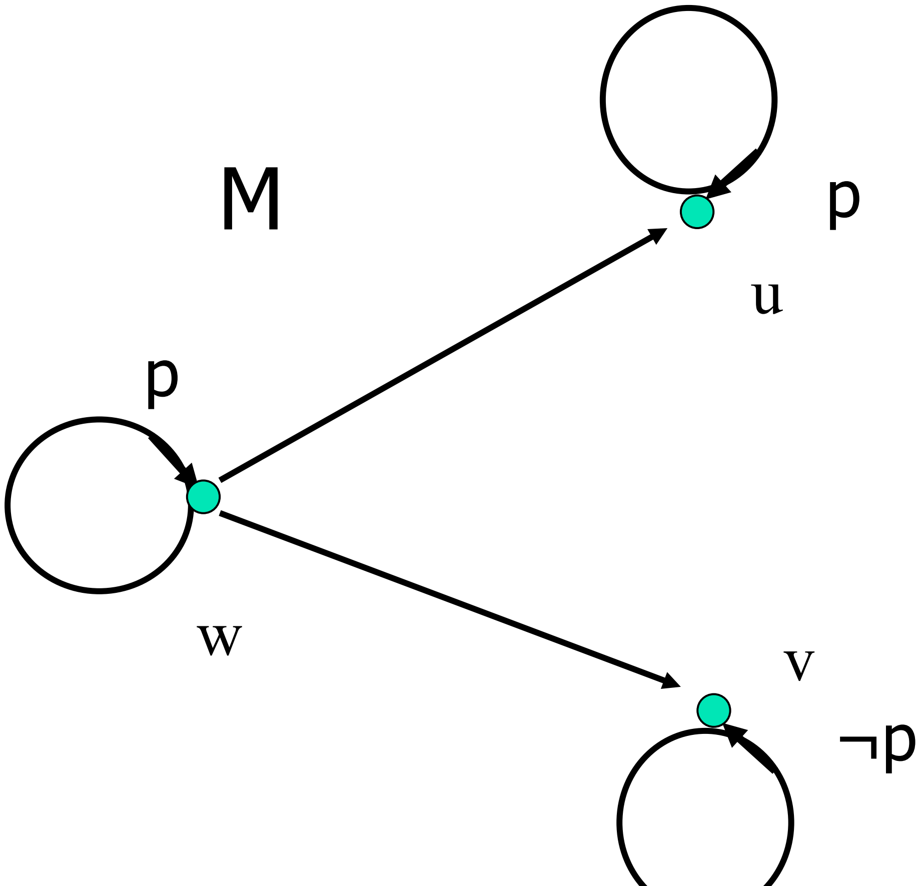
$M,w \models \neg K_i p$

Because $M,v \models \neg p$

Counterexamples

$\mathbf{S4}_{(\mathbf{m})} \vdash \neg K_i\phi \leftrightarrow K_i\neg K_i\phi$?

Answer is NO! Because $S4_{(m)} \not\models \neg K_i\phi \leftrightarrow K_i\neg K_i\phi$!!!!!!!!!



$M \not\models \neg K_i\phi \leftrightarrow K_i\neg K_i\phi$

Because

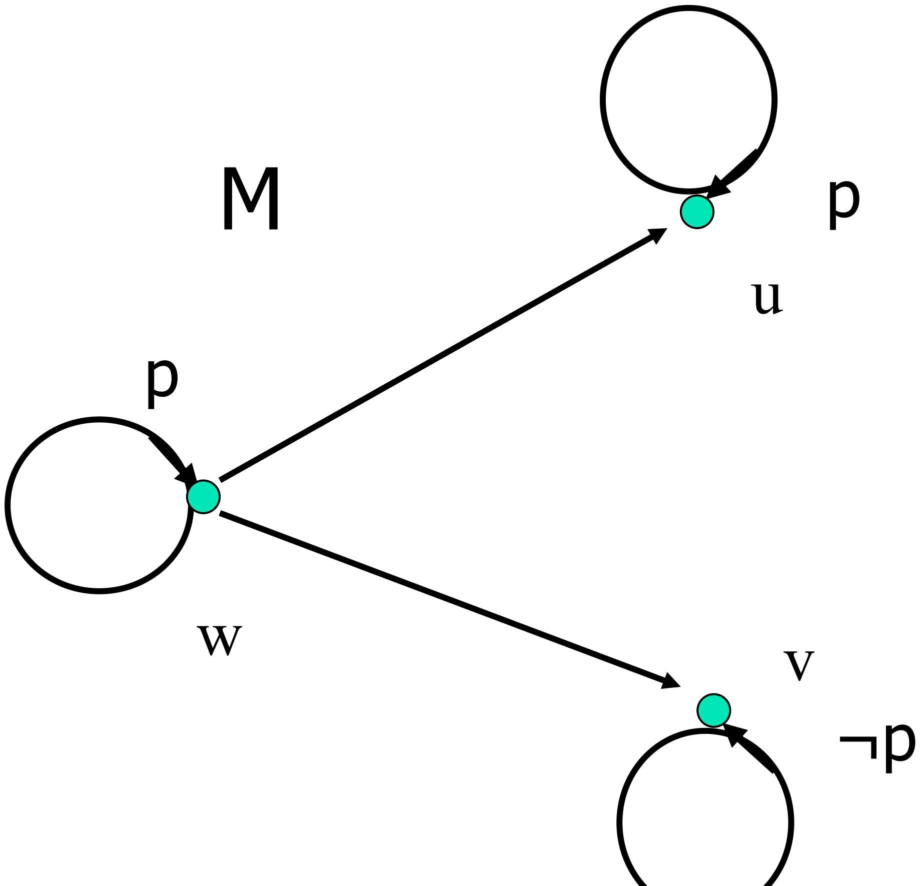
$M,w \models \neg K_i p$, and
$M,w \models \neg K_i\neg K_i p$

because this is $M,w \models M_i K_i p$
and $M,u \models K_i p$

Counterexamples

$\mathbf{S4_{(m)}} \vdash \neg K_i\phi \leftrightarrow K_i\neg K_i\phi$?

Answer is NO! Because $S4_{(m)} \not\models \neg K_i\phi \leftrightarrow K_i\neg K_i\phi$!!!!!!!!!



$M \not\models \neg K_i\phi \leftrightarrow K_i\neg K_i\phi$

Because

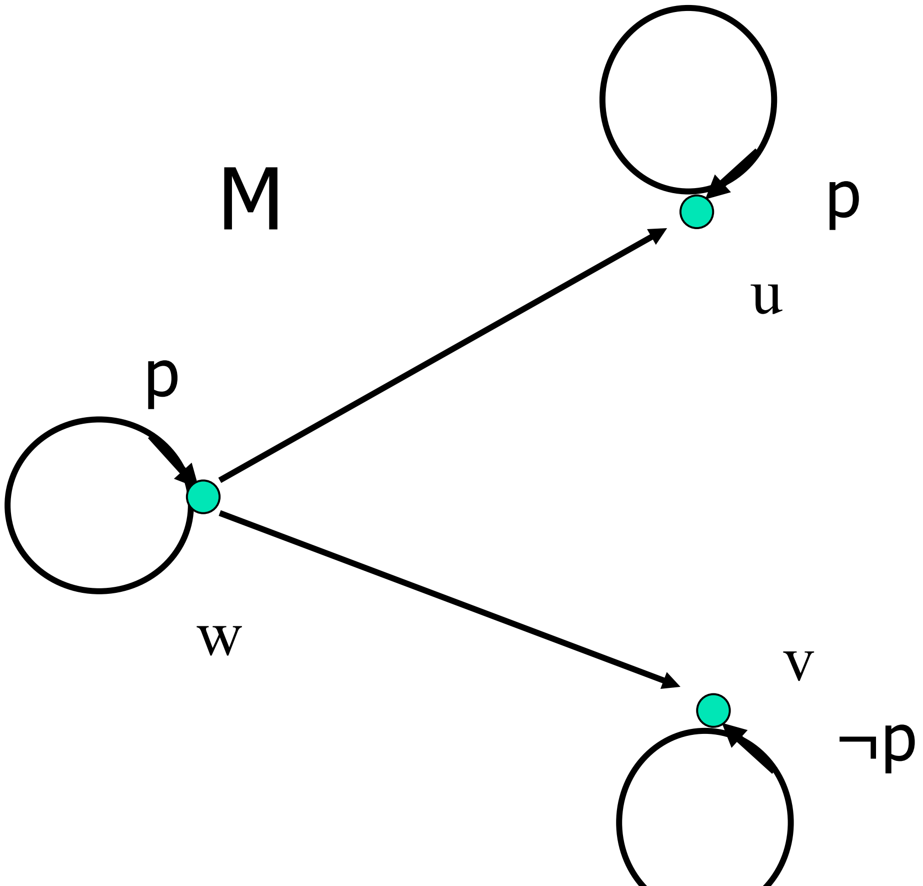
$M,w \models \neg K_i p$, and
$M,w \models \neg K_i \neg K_i p$

So $M,w \models \neg(\neg K_i p \leftrightarrow K_i\neg K_i p$

Counterexamples

$\mathbf{S4_{(m)}} \vdash \neg K_i\phi \leftrightarrow K_i\neg K_i\phi$?

Answer is NO! Because $S4_{(m)} \not\models \neg K_i\phi \leftrightarrow K_i\neg K_i\phi$!!!!!!!!



$M \not\models \neg K_i\phi \leftrightarrow K_i\neg K_i\phi$

and

$M \in S4_{(m)}$

hence $S4_{(m)} \not\models \neg K_i\phi \leftrightarrow K_i\neg$

Validities

We already derived: $\mathbf{S5_{(m)}} \vdash \neg K_i\phi \leftrightarrow K_i\neg K_i\phi$
We could also have proved: $\mathcal{S}5_{(m)} \models \neg K_i\phi \leftrightarrow K_i\neg K_i\phi$

t of all, $\neg K_i\phi \leftrightarrow K_i\neg K_i\phi$

quivalent to $\neg\neg K_i\phi \leftrightarrow \neg K_i\neg K_i\phi$

quivalent to $K_i\phi \leftrightarrow \neg K_i\neg K_i\phi$

quivalent to $K_i\phi \leftrightarrow M_iK_i\phi$

Validities

We already derived: $\mathbf{S5_{(m)}} \vdash \neg K_i\phi \leftrightarrow K_i\neg K_i\phi$
We could also have proved: $\mathcal{S5}_{(m)} \models \neg K_i\phi \leftrightarrow K_i\neg K_i\phi$

First of all, $\neg K_i\phi \leftrightarrow K_i\neg K_i\phi$ is equivalent to $K_i\phi \leftrightarrow M_iK_i\phi$

Let $M \in \mathcal{S5}_{(m)}$ and w a world in M. To prove: $M,w \models K_i\phi \leftrightarrow M_iK$

Suppose $M,w \models K_i\phi$

Since R_i is reflexive, we have R_iww and hence $M,w \models M_iK_i\phi$

This proves $M,w \models K_i\phi \to M_iK_i\phi$

Validities

We already derived: **S5**$_{\mathbf{(m)}}$ $\vdash \neg K_i\phi \leftrightarrow K_i\neg K_i\phi$

We could also have proved: $\mathcal{S5}_{(m)} \models \neg K_i\phi \leftrightarrow K_i\neg K_i\phi$

First of all, $\neg K_i\phi \leftrightarrow K_i\neg K_i\phi$ is equivalent to $K_i\phi \leftrightarrow M_iK_i\phi$

et M $\in \mathcal{S5}_{(m)}$ and w a world in M. To prove: M,w $\models K_i\phi \leftrightarrow M_iK$

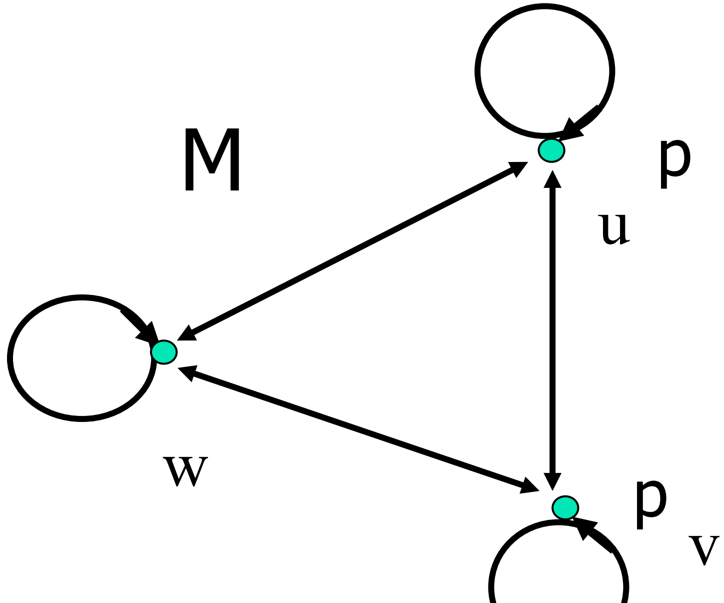
uppose M,w $\models M_iK_i\phi$

here is u, $R_i$wu and M,u $\models K_i\phi$

ake v, $R_i$wv. To prove: M,v $\models \phi$

ince R_i is Euclidean, we have $R_i$uv. So M,v $\models \phi$

This proves M,w $\models M_iK_i\phi \rightarrow K_i\phi$



Tactics

Let $\mathbf{X_{(m)}}$ be any logic, for which $\mathbf{X_{(m)}} \vdash \varphi \Leftrightarrow \mathcal{X}_{(m)} \models \varphi$

To prove $\mathbf{X_{(m)}} \vdash \psi$

either use the axioms and rules of $\mathbf{X_{(m)}}$

or use completeness and show $\mathcal{X}_{(m)} \models \varphi$

To prove $\mathbf{X_{(m)}} \nvdash \psi$

use soundness and show $\mathcal{X}_{(m)} \not\models \varphi$

Tactics

Let $\mathbf{X}_{(\mathbf{m})}$ be any logic, for which $\mathbf{X}_{(\mathbf{m})} \vdash \varphi \Leftrightarrow \mathcal{X}_{(m)} \models \varphi$

A formula ψ is $\mathcal{X}_{(m)}$-satisfiable if there is a $\mathsf{M} \in \mathcal{X}_{(m)}$ so that $\mathsf{M},\mathsf{w} \models \psi$

A formula ψ is $\mathbf{X}_{(\mathbf{m})}$ –consistent if

$\mathbf{X}_{(\mathbf{m})} \not\vdash \neg\psi$ if

$\mathcal{X}_{(m)} \not\models \neg\psi$ if

for some $\mathsf{M} \in \mathcal{X}_{(m)}$ we have $\mathsf{M},\mathsf{w} \models \psi$

Tactics

Let $\mathbf{X}_{\mathbf{(m)}}$ be any logic, for which $\mathbf{X}_{\mathbf{(m)}} \vdash \varphi \Leftrightarrow \mathcal{X}_{(m)} \models \varphi$

A formula ψ is $\mathcal{X}_{(m)}$-valid if
for all $\mathsf{M} \in \mathcal{X}_{(m)}$: $\mathsf{M,w} \models \psi$

A formula ψ is $\mathbf{X}_{\mathbf{(m)}}$ –derivable if
$\mathbf{X}_{\mathbf{(m)}} \vdash \psi$ if

Exercise 8

Are the following formulas valid with respect to $\mathbf{K}_{\mathbf{(m)}}$? $\mathbf{T}_{\mathbf{(m)}}$? $\mathbf{S4}_{\mathbf{(m)}}$? $\mathbf{S5}_{\mathbf{(m)}}$?

(a) $\phi \rightarrow K_i\phi$

(b) $\phi \rightarrow K_i\neg K_i \neg \phi$

(c) $K_i\phi \rightarrow \neg K_i \neg \phi$

(d) $\neg K_i\phi \rightarrow K_i \neg \phi$