COMP521: Knowledge Representation
Model Solution 2nd Test (December 2011)

1. Consider the following ABox: 25marks

ABox	$\text{john} : \text{employee} \sqcap \forall \text{hasCar.fast}$
	$\text{bmw}_1 : \neg\text{fast}$
	$(\text{john}, \text{bmw}_1) : \text{hasCar}$

Give a tableau derivation which determines whether this ABox is satisfiable.

$\Delta_1 = \{\text{john} : \text{employee} \sqcap \forall \text{hasCar.fast}, \text{bmw}_1 : \neg\text{fast}, (\text{john}, \text{bmw}_1) : \text{hasCar}\}$
$\Rightarrow_\sqcap \Delta_2 = \{\text{john} : \text{employee}, \text{john} : \forall \text{hasCar.fast}\} \cup \Delta_1$
$\Rightarrow_\forall \Delta_3 = \{\text{bmw}_1 : \text{fast}\} \cup \Delta_2$
$\Rightarrow_\bot \Delta_4 = \{\text{bmw}_1 : \bot\} \cup \Delta_3$

Since Δ_4 contains $\text{bmw}_1 : \bot$ and no other constraint system is derivable, the ABox above is unsatisfiable.

2. (Interpreations) 25 marks
A terminological interpretation $\mathcal{I}$ over a signature $\Sigma = (\mathsf{O}, \mathsf{C}, \mathsf{R})$ is an ordered pair $(\Delta, \cdot^{\mathcal{I}})$ consisting of

- a domain Δ (an arbitrary set)
- an interpretation function $\cdot^{\mathcal{I}}$

Explain how this interpretation is used to give a meaning to the concept $\exists R.C$ and to the concept $\bot$.

$$(\exists R.C)^{\overline{\mathcal{I}}} = \{x \mid \text{exists } y,\ (x,y) \in R^{\mathcal{I}} \text{ and } y \in C^{\overline{\mathcal{I}}}\}$$

Also, $\bot^I = \emptyset$.

3. Let the knowledge base $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ be given by 25marks

TBox	$\text{cool} \dot{\sqsubseteq} \text{fast} \sqcup \text{cabrio}$	$\text{nice} \doteq \exists \text{hasCar.cool}$
	$\text{popular} \doteq \text{student} \sqcap \forall \text{hasFriend.nice}$	
ABox	$\text{andy} : \neg\text{popular}$	$\text{andy} : \text{student}$

Eliminate all primitive concept definitions, expand the knowledge base, and transform all concepts into negation normal form.

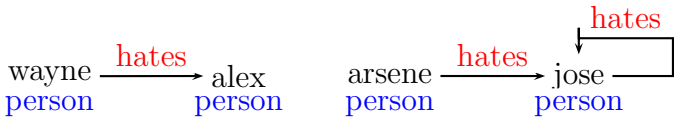
TBox	$\text{cool} \doteq (\text{fast} \sqcup \text{cabrio}) \sqcap \text{cool}^*$
	$\text{nice} \doteq \exists \text{hasCar.}((\text{fast} \sqcup \text{cabrio}) \sqcap \text{cool}^*)$
	$\text{popular} \doteq \text{student} \sqcap \forall \text{hasFriend.}(\exists \text{hasCar.}((\text{fast} \sqcup \text{cabrio}) \sqcap \text{cool}^*))$
ABox	$\text{andy} : \neg\text{student} \sqcup \exists \text{hasFriend.} \forall \text{hasCar.}((\neg\text{fast} \sqcap \neg\text{cabrio}) \sqcup \neg\text{cool}^*)$
	$\text{andy} : \text{student}$

4. (Terminological Interpretations) 25 marks

Consider the TBox

$$\text{arrogant} \doteq \text{person} \sqcap \forall \text{hates.arrogant}$$

together with the ABox depicted by the following graph

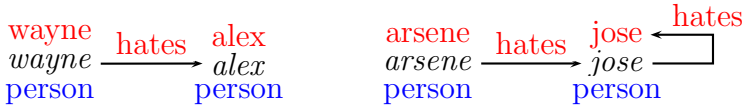


Demonstrate whether you can construct terminological interpretations for this knowledge base, such that the interpretation of arrogant is identical to one of the following sets:

(a) $\emptyset$
(b) $\{alex\}$
(c) $\{alex, wayne\}$
(d) $\{wayne, alex, arsene\}$
(e) $\{wayne, alex, jose\}$
(f) $\{wayne, alex, arsene, jose\}$

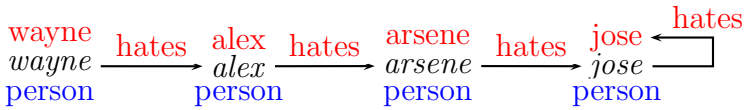
Note that we are free to construct the interpretations as long as they are compatible with the graph. The only case we are not able to do so, is in case (d).

(a) $\text{arrogant}^{\mathcal{I}} = \emptyset$
$\text{person}^{\mathcal{I}} = \{wayne, alex, arsene, jose\}$
$\text{hates}^{\mathcal{I}} = \{(wayne, alex), (arsene, jose), (jose, jose)\}$



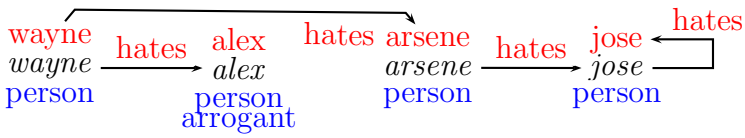
Does not satisfy the knowledge base,
since $alex$ has to be an element of $\text{arrogant}^{\mathcal{I}}$ by the definition of arrogant (because $alex \in \text{person}^{\mathcal{I}}$ and there is nobody related to $alex$ via $\text{hates}^{\mathcal{I}}$, so $alex \in (\forall \text{hates.arrogant})^{\overline{\mathcal{I}}}$) and, consequently, $wayne$ is an element of $\text{arrogant}^{\mathcal{I}}$ as well.
This is not the interpretation we need.

(a) $\text{arrogant}^{\mathcal{I}} = \emptyset$
$\text{person}^{\mathcal{I}} = \{wayne, alex, arsene, jose\}$
$\text{hates}^{\mathcal{I}} = \{(wayne, alex), (alex, arsene), (arsene, jose), (jose, jose)\}$



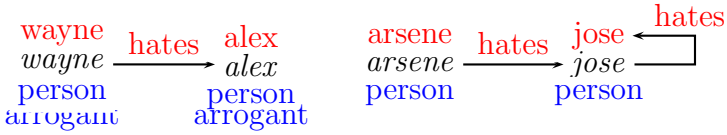
Does satisfy the knowledge base,
since every element a of the domain is related to some element b via $\text{hates}^{\mathcal{I}}$ such that $b \in \neg\text{arrogant}^{\overline{\mathcal{I}}}$.
This is one interpretation which meets our requirement.

(b) $\text{arrogant}^{\mathcal{I}} = \{alex\}$
$\text{person}^{\mathcal{I}} = \{wayne, alex, arsene, jose\}$
$\text{hates}^{\mathcal{I}} = \{(wayne, alex), (wayne, arsene), (arsene, jose), (jose, jose)\}$



Does satisfy the knowledge base,
since *alex* is not related via hates$^{\mathcal{I}}$ to any element a of the domain such that $a \notin$ arrogant$^{\mathcal{I}}$, while all other elements are related to an element that is not in arrogant$^{\mathcal{I}}$.

(c) $\text{arrogant}^{\mathcal{I}} = \{alex, wayne\}$
$\text{person}^{\mathcal{I}} = \{wayne, alex, arsene, jose\}$
$\text{hates}^{\mathcal{I}} = \{(wayne, alex), (arsene, jose), (jose, jose)\}$

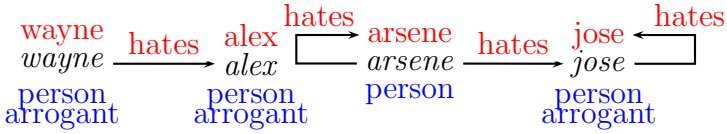


Does satisfy the knowledge base,
since in contrast to (b) *wayne* is now only related to *alex* via hates$^{\mathcal{I}}$ and *alex* is in arrogant$^{\mathcal{I}}$, so *wayne* has to be in arrogant$^{\mathcal{I}}$ as well. *arsene* and *jose* are both still related to an element of domain not in arrogant$^{\mathcal{I}}$.

(d) $\text{arrogant}^{\mathcal{I}} = \{alex, wayne, arsene\}$
$\text{person}^{\mathcal{I}} = \{wayne, alex, arsene, jose\}$
$\text{hates}^{\mathcal{I}} \supset \{(wayne, alex), (arsene, jose), (jose, jose)\}$

There is no interpretation with $arrogant^{\mathcal{I}} = \{alex, wayne, arsene\}$ that satisfies the knowledge base, since $arsene \in$ arrogant$^{\mathcal{I}}$ implies any element related to *arsene* via hates$^{\mathcal{I}}$ has to be in arrogant$^{\mathcal{I}}$. Obviously, *jose* violates this constraint.
(Note $\text{arsene}^{\mathcal{I}} = arsene \neq jose = \text{jose}^{\mathcal{I}}$ by the unique name assumption)

(e) $\text{arrogant}^{\mathcal{I}} = \{alex, wayne, jose\}$
$\text{person}^{\mathcal{I}} = \{wayne, alex, arsene, jose\}$
$\text{hates}^{\mathcal{I}} = \{(wayne, alex), (arsene, arsene), (arsene, jose), (jose, jose)\}$



Does satisfy the knowledge base,
since except for *arsene* all other elements of the domain are only related via hates$^{\mathcal{I}}$ to elements in arrogant$^{\mathcal{I}}$; *arsene*, on the other hand, is related via hates$^{\mathcal{I}}$ to an element which is not in arrogant$^{\mathcal{I}}$, namely, himself.

(f) $\text{arrogant}^{\mathcal{I}} = \{alex, wayne, arsene, jose\}$
$\text{person}^{\mathcal{I}} = \{wayne, alex, arsene, jose\}$
$\text{hates}^{\mathcal{I}} = \{(wayne, alex), (arsene, jose), (jose, jose)\}$

Does satisfy the knowledge base,
since every element a of the domain is only related via hates$^{\mathcal{I}}$ to elements of the domain that are in arrogant$^{\mathcal{I}}$.