

Optimised functional translation and resolution

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Prover: We facilitate modal theorem proving in a first-order resolution calculus implemented in SPASS Version 0.77 [4]. SPASS uses ordered resolution and ordered factoring, it supports splitting and branch condensing (splitting amounts to case analysis while branch condensing resembles branch pruning in the Logics Workbench), it has an extensive set of reduction rules including tautology deletion, subsumption and condensing, and it supports dynamic sort theories by additional inference and reduction rules.

The translation we use is the *optimised functional translation* [2]. It maps normal propositional modal logics into a class of *path logics*. Path logics are clausal logics over the language of the monadic fragment of sorted first-order logic with a special binary function symbol for defining accessibility. Clauses of path logics are restricted in that only Skolem terms which are constants may occur and the prefix stability property holds. Ordinary resolution without any refinement strategies is a decision procedure for the path logics associated with $K(m)$ and $KT(m)$ [3]. Our decision procedure for $S4$ uses an a priori term depth bound.

Availability: SPASS and a routine for the translation of modal formulae are available from <http://www.mpi-sb.mpg.de/~hustadt/mdp>

Advantages of the prover: SPASS is a fast and sophisticated state-of-the-art first-order theorem prover. Ordered inference rules and splitting are of particular importance when treating satisfiable formulae, while unit propagation and branch condensing are important for benchmarks based on randomly generated modal formulae [1].

Advantages of translation approaches: In its most general form the translation approach can deal with any complete, finitely axiomatizable, normal modal logic. Moreover, any first-order theorem prover can be used, that is, we may substitute SPASS with another theorem prover (not necessarily a resolution theorem prover). The relational and optimised functional translation approach are refinements of the general translation approach towards efficient modal theorem proving. The optimised functional translation is applicable to many propositional modal logics, including K , KT , and $S4$ and their multi-modal versions, but notably also to some second-order modal logics, like KM [2]. A general result shows that any first-order resolution theorem prover (with condensing) provides a decision procedure for a variety of modal logics [3].

Hardware: Sun Ultra 1 Model 170E (167 MHz UltraSPARC processor, 512 KB second level cache), 192 MB main memory.

Results: On classes of provable formulae (in the first column), the combination of the optimised functional translation approach and SPASS has little difficulty. Notable exceptions are the classes k_ph_p , kt_ph_p , $s4_ph_p$, k_branch_p and $s4_ipc_p$. Observe that SPASS solves more formulae in $s4_branch_p$ than in either kt_branch_p or k_branch_p . While for the basic modal logic, the classes of non-provable formulae (in the second column) are not harder than the classes of provable formulae, we see a noticeable difference between kt_dum_n , kt_poly_n , and kt_t4p_n and the corresponding classes of provable KT-formulae. However, the results are still acceptable. In contrast, for the classes $s4_45_n$, $s4_grz_n$, $s4_s5_n$, and $s4_t4p_n$ of non-provable S4-formulae the performance is unsatisfactory. We attribute this to using superposition and not E -unification, and to enforcing termination by an explicit term depth bound instead of a loop check. For the classes $s4_45_n$ and $s4_s5_n$ there are trivial satisfiability checks on the clause level which are not implemented in SPASS.

k_branch_p	9	k_branch_n	9
k_d4_p	> 20	k_d4_n	18
k_dum_p	> 20	k_dum_n	> 20
k_grz_p	> 20	k_grz_n	> 20
k_lin_p	> 20	k_lin_n	> 20
k_path_p	20	k_path_n	20
k_ph_p	6	k_ph_n	9
k_poly_p	16	k_poly_n	17
k_t4p_p	> 20	k_t4p_n	19
kt_45_p	17	kt_45_n	6
kt_branch_p	13	kt_branch_n	9
kt_dum_p	17	kt_dum_n	9
kt_grz_p	> 20	kt_grz_n	> 20
kt_md_p	16	kt_md_n	20
kt_path_p	> 20	kt_path_n	16
kt_ph_p	5	kt_ph_n	12
kt_poly_p	16	kt_poly_n	3
kt_t4p_p	> 20	kt_t4p_n	7
$s4_45_p$	9	$s4_45_n$	0
$s4_branch_p$	> 20	$s4_branch_n$	4
$s4_grz_p$	14	$s4_grz_n$	0
$s4_ipc_p$	6	$s4_ipc_n$	> 20
$s4_md_p$	9	$s4_md_n$	10
$s4_path_p$	15	$s4_path_n$	> 20
$s4_ph_p$	5	$s4_ph_n$	5
$s4_s5_p$	> 20	$s4_s5_n$	1
$s4_t4p_p$	11	$s4_t4p_n$	0

Table 1: Performance indices

References

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