

# Sincere-Strategy Preference-Based Approval Voting Fully Resists Constructive Control and Broadly Resists Destructive Control

Gábor Erdélyi Markus Nowak Jörg Rothe

Institut für Informatik, Heinrich-Heine-Universität Düsseldorf, Germany

COMSOC-08, Liverpool, September 2008

# Outline

- 1 Introduction
- 2 Approval Voting and its Versions
  - Approval Voting (AV)
  - Sincere-Strategy Preference-Based Approval Voting (SP-AV)
- 3 Results

Parts of this paper were presented at MFCS 2008.

# Introduction

- Artificial systems:
  - Hybridization - Fully resists control (Hemaspaandra, Hemaspaandra, Rothe - IJCAI 2007)

# Introduction

- Artificial systems:
  - Hybridization - Fully resists control (Hemaspaandra, Hemaspaandra, Rothe - IJCAI 2007)
- Natural systems:

Condorcet	$3 \times R$	$4 \times I$	$7 \times V$
Approval	$4 \times R$	$9 \times I$	$9 \times V$
Llull	$14 \times R$	$0 \times I$	$8 \times V$
Copeland	$15 \times R$	$0 \times I$	$7 \times V$
Plurality	$16 \times R$	$0 \times I$	$6 \times V$

# Introduction

- Artificial systems:
  - Hybridization - Fully resists control (Hemaspaandra, Hemaspaandra, Rothe - IJCAI 2007)
- Natural systems:

Condorcet     $3 \times R$      $4 \times I$      $7 \times V$

Approval     $4 \times R$      $9 \times I$      $9 \times V$

Llull         $14 \times R$      $0 \times I$      $8 \times V$

Copeland     $15 \times R$      $0 \times I$      $7 \times V$

Plurality     $16 \times R$      $0 \times I$      $6 \times V$

- Can we do better?

# Voting Systems

- Set of candidates and voters:
  - $C = \{c_1, \dots, c_m\}$
  - $V = \{v_1, \dots, v_n\}$
- Voter preferences over  $C$  can be represented as
  - preference lists (rankings)
  - approval/disapproval vectors
- Voting rule aggregates the preferences and outputs the set of winners
  - unique winner
  - nonunique winner

# Approval Voting

- Introduced by Brams and Fishburn
- Each voter specifies his or her 0 – 1 approval vector:
  - 1 represents approval
  - 0 represents disapproval
- Ignores preference rankings
- The winners are the candidates with the highest score

# Example for Approval Voting

## Example

- Set of voters:
  - $V = \{v_1, \dots, v_{10}\}$
- Set of candidates:
  - $c_1 = \text{chicken}$
  - $c_2 = \text{fish}$
  - $c_3 = \text{pork}$
  - $c_4 = \text{rump steak}$
  - $c_5 = \text{tofu}$

# Example for Approval Voting

## Example

- Set of voters:
  - $V = \{v_1, \dots, v_{10}\}$
- Set of candidates:
  - $c_1 = \text{chicken}$
  - $c_2 = \text{fish}$
  - $c_3 = \text{pork}$
  - $c_4 = \text{rump steak}$
  - $c_5 = \text{tofu}$
- The votes:
  - $v_1 = v_2 = 00010$
  - $v_3 = 00110$
  - $v_4 = 11110$
  - $v_5 = v_6 = v_7 = v_8 = 11011$
  - $v_9 = 01001$
  - $v_{10} = 00001$

# Example for Approval Voting

## Example

The result of the voting:

- $score(c_1) = 5$
- $score(c_2) = 6$
- $score(c_3) = 2$
- $score(c_4) = 8$
- $score(c_5) = 6$

● The votes:

- $v_1 = v_2 = 00010$
- $v_3 = 00110$
- $v_4 = 11110$
- $v_5 = v_6 = v_7 = v_8 = 11011$
- $v_9 = 01001$
- $v_{10} = 00001$

# Rules

- Proposed by Brams and Sanver
- Each voter has a preference ranking, a tie free linear ordering of all candidates:

$$c_4 > c_1 > c_3 > c_5 > c_2 > c_6$$

- Line between acceptable and unacceptable candidates:

$$c_4 \ c_1 \ | \ c_3 \ c_5 \ c_2 \ c_6$$

- The winners are the candidates with the highest score

# Notations

- The set of candidates  $S_v$  that voter  $v$  approves of is an AV strategy
- The list of all strategies is an AV strategy profile

# Conventions

- Admissibility:
  - $v$ 's most preferred candidate  $\in S_v$
  - $v$ 's least preferred candidate  $\notin S_v$
- Sincerity: no gaps in the approval strategies
- Sincere strategy is always admissible for at least 2 candidates if  $\emptyset \neq S_v \neq C$

# Example for SP-AV

## Example

- $V_1 = V_2 = C_4 \mid C_1 C_3 C_5 C_2$
- $V_3 = C_3 C_4 \mid C_2 C_5 C_1$
- $V_4 = C_3 C_1 C_2 C_4 \mid C_5$
- $V_5 = V_6 = C_5 C_4 C_1 C_2 \mid C_3$
- $V_7 = V_8 = C_1 C_5 C_2 C_4 \mid C_3$
- $V_9 = C_2 C_5 \mid C_1 C_3 C_4$
- $V_{10} = C_5 \mid C_1 C_4 C_3 C_2$

# Example for SP-AV

## Example

- $V_1 = V_2 = c_4 \mid c_1 c_3 c_5 c_2$
- $V_3 = c_3 c_4 \mid c_2 c_5 c_1$
- $V_4 = c_3 c_1 c_2 c_4 \mid c_5$
- $V_5 = V_6 = c_5 c_4 c_1 c_2 \mid c_3$
- $V_7 = V_8 = c_1 c_5 c_2 c_4 \mid c_3$
- $V_9 = c_2 c_5 \mid c_1 c_3 c_4$
- $V_{10} = c_5 \mid c_1 c_4 c_3 c_2$

### The result of the voting:

- $score(c_1) = 5$
- $score(c_2) = 6$
- $score(c_3) = 2$
- $score(c_4) = 8$
- $score(c_5) = 6$

# Violations Against the Conventions

- Violations against admissible AV strategies in control via:
  - Deleting Candidates,
  - Partition of Candidates,
  - Partition of Voters.

# Example for Re-Enforcing the Conventions

## Example

### Nonvegetarian food:

●  $V_1 = V_2 = c_4 \mid c_1 c_3 c_2$

●  $V_3 = c_3 c_4 \mid c_2 c_1$

●  $V_4 = c_3 c_1 c_2 \mid c_4$

←

$V_4 = c_3 c_1 c_2 c_4 \mid$

●  $V_5 = V_6 = c_4 c_1 c_2 \mid c_3$

●  $V_7 = V_8 = c_1 c_2 c_4 \mid c_3$

●  $V_9 = c_2 \mid c_1 c_3 c_4$

●  $V_{10} = c_1 \mid c_4 c_3 c_2$

←

$V_{10} = \mid c_1 c_4 c_3 c_2$

# Example for Re-Enforcing the Conventions

## Example

### Nonvegetarian food:

- $V_1 = V_2 = C_4 \mid C_1 C_3 C_2$
- $V_3 = C_3 C_4 \mid C_2 C_1$
- $V_4 = C_3 C_1 C_2 \mid C_4$
- $V_5 = V_6 = C_4 C_1 C_2 \mid C_3$
- $V_7 = V_8 = C_1 C_2 C_4 \mid C_3$
- $V_9 = C_2 \mid C_1 C_3 C_4$
- $V_{10} = C_1 \mid C_4 C_3 C_2$

### Result of $(C_1, V)$ :

- $score(c_1) = 7$
- $score(c_2) = 6$
- $score(c_3) = 2$
- $score(c_4) = 7$

# Example for Re-Enforcing the Conventions

## Example

●  $V_1 = V_2 = C_4 \mid C_1 C_5$

●  $V_3 = C_4 \mid C_5 C_1$

●  $V_4 = C_1 C_4 \mid C_5$

●  $V_5 = V_6 = C_5 C_4 \mid C_1$

●  $V_7 = V_8 = C_1 C_5 \mid C_4$

●  $V_9 = C_5 \mid C_1 C_4$

●  $V_{10} = C_5 \mid C_1 C_4$



$V_5 = V_6 = C_5 C_4 C_1 \mid$



$V_7 = V_8 = C_1 C_5 C_4 \mid$

# Example for Re-Enforcing the Conventions

## Example

- $V_1 = V_2 = C_4 \mid C_1 C_5$
- $V_3 = C_4 \mid C_5 C_1$
- $V_4 = C_1 C_4 \mid C_5$
- $V_5 = V_6 = C_5 C_4 \mid C_1$
- $V_7 = V_8 = C_1 C_5 \mid C_4$
- $V_9 = C_5 \mid C_1 C_4$
- $V_{10} = C_5 \mid C_1 C_4$

The result of the second stage:

- $score(c_1) = 3$
- $score(c_4) = 6$
- $score(c_5) = 6$

# Plurality and Approval

## Theorem

Control by	Plurality		AV	
	Constr.	Destr.	Constr.	Destr.
<i>Adding an Unlimited Number of Candidates</i>	R	R	I	V
<i>Adding a Limited Number of Candidates</i>	R	R	I	V
<i>Deleting Candidates</i>	R	R	V	I
<i>Partition of Candidates</i>	TE: R	TE: R	TE: V	TE: I
	TP: R	TP: R	TP: I	TP: I
<i>Run-off Partition of Candidates</i>	TE: R	TE: R	TE: V	TE: I
	TP: R	TP: R	TP: I	TP: I
<i>Adding Voters</i>	V	V	R	V
<i>Deleting Voters</i>	V	V	R	V
<i>Partition of Voters</i>	TE: R	TE: R	TE: R	TE: V
	TP: V	TP: V	TP: R	TP: V

# Results

## Theorem

Control by	SP-AV		AV		Plurality	
	Constr.	Destr.	Constr.	Destr.	Constr.	Destr.
<i>Adding an Unlimited Number of Candidates</i>	R	R	I	V	R	R
<i>Adding a Limited Number of Candidates</i>	R	R	I	V	R	R
<i>Deleting Candidates</i>	R	R	V	I	R	R
<i>Partition of Candidates</i>	TE: R. TP: R	TE: R TP: R	TE: V TP: I	TE: I TP: I	TE: R TP: R	TE: R TP: R
<i>Run-off Partition of Candidates</i>	TE: R TP: R	TE: R TP: R	TE: V TP: I	TE: I TP: I	TE: R TP: R	TE: R TP: R
<i>Adding Voters</i>	R	V	R	V	V	V
<i>Deleting Voters</i>	R	V	R	V	V	V
<i>Partition of Voters</i>	TE: R TP: R	TE: V TP: R	TE: R TP: R	TE: V TP: V	TE: R TP: V	TE: R TP: V

# Proof Technique

- Resistancy results follow via reduction from Hitting Set and Exact Cover by 3-Sets
- Vulnerability results follow via polynomial time Algorithm
- Some results are straightforward modifications of results and constructions from Hemaspaandra, Hemaspaandra, Rothe - Anyone but him
- But some results require new constructions

# Contrast

## Table

*Number of resistances, immunities, and vulnerabilities to our 22 control types.*

<i>Number of</i>	<i>Condorcet</i>	<i>Approval</i>	<i>Llull</i>	<i>Copeland</i>	<i>Plurality</i>	<i>SP-AV</i>
<i>resistances</i>	3	4	14	15	16	19
<i>immunities</i>	4	9	0	0	0	0
<i>vulnerabilities</i>	7	9	8	7	6	3

# Summary

- SP-AV offers:
  - Full resistance to constructive control
  - Full resistance to candidate control
  - More resistances than is currently known for any other natural voting system with a polynomial-time winner problem
  - Fewer vulnerabilities than is currently known for any other natural voting system with a polynomial-time winner problem

# Thank you very much!