

A Qualitative Vickrey Auction

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Vickrey versus Qualitative Vickrey

Vickrey's sealed-bid
second-price single item
auction

- bids are prices
- outcome: winner has highest bid, price of second-highest bid
- bidding private value is a dominant strategy

Qualitative Vickrey auction

- bids are alternatives
- outcome: winner has highest ranked bid, alternative at least as high as second-highest
- bidding highest acceptable alternative is a dominant strategy

Motivating Example: Buy a Super-computer

Limited budget (e.g. from a project) to buy a super-computer

- 1 Announce ranking of alternatives (including budget) to suppliers
- 2 Request one (sealed) proposal from each supplier
- 3 Select winner: supplier with most preferred proposal
- 4 Select deal (by supplier): higher preferred than second-ranked proposal



Outline

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 - Summary
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Definitions and Assumptions

Notation and Definitions

- An *outcome* is an alternative and a winner: $(a, i) \in A \times N$.
- Center's order over $A \times N$ is given by a linear order \geq .
- Bidder i 's preferences over $A \times N$ are given by a weak order \succsim_i .

Assumptions

- Bidder i can only bid from $A \times \{i\}$.
- Bidder i is indifferent between outcomes where winner is not i .
- Assume each bidder has at least one *acceptable* outcome, where an outcome (a, i) is acceptable to i if $(a, i) \succsim_i (x, j)$ for $j \neq i$.

The Qualitative Vickrey Auction

The *qualitative Vickrey auction* follows the following protocol:

- 1 The order \succeq of the center is publicly announced.
- 2 Each bidder i submits a sealed bid $(a, i) \in A \times \{i\}$.
- 3 The bidder i^* who submitted the bid ranked highest in \succeq is the winner.
- 4 The winner i^* may choose from $A \times \{i^*\}$ any outcome ranked at least as high as *second-highest* bid in \succeq .

Example of a Qualitative Vickrey Auction

$$(a,1) > (a,2) > (a,3) > (b,1) > (b,2) > \dots > (c,1) > \dots > (d,3)$$

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Adequate Strategies

A strategy for i is *adequate* if

- 1 i bids acceptable outcome ranked highest in \succeq , and
- 2 if i wins the auction, i selects outcome she prefers most (in \succsim_i) from those ranked higher in \succeq than the second-highest bid.

Example of Using an Adequate Strategy

$$(a,1) > (a,2) > (a,3) > (b,1) > (b,2) > \dots > (c,1) > \dots > (d,3)$$

\succsim_1	\succsim_2	\succsim_3
$(c,1)$	$(d,2)$	$(d,3)$
$(d,1)$	$(b,2)$	$(x,i) \notin A \times \{3\}$
$(x,i) \notin A \times \{1\}$	$(a,2)$	$(a,3)$
$(b,1)$	$(x,i) \notin A \times \{2\}$	$(c,3)$
$(a,1)$	$(c,2)$	$(b,3)$

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Example of Using an Adequate Strategy

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$(x,i) \notin A \times \{1\}$	$(a,2)$	$(a,3)$
$(b,1)$	$(x,i) \notin A \times \{2\}$	$(c,3)$
$(a,1)$	$(c,2)$	$(b,3)$

Adequate Strategies are Dominant

Theorem

Adequate strategies are dominant.

Proof.

(sketch)

- Let (a, i) be acceptable outcome (to i) ranked highest in \succeq .
- Let (a', j) be highest-ranked bid by $j \neq i$.
- Two cases:
 - ① $(a', j) > (a, i)$: i should bid below (a', j) in \succeq , because if i wins, she can only select unacceptable outcomes, and
 - ② $(a, i) > (a', j)$: i should bid above (a', j) in \succeq , because then outcome can be highest in \succsim_i which is above (a', j) .
- In both cases, optimal strategy for i is to bid (a, i) .



DSE is Not Strongly Pareto Efficient

$$(a,1) > (a,2) > (a,3) > (b,1) > (b,2) > \dots > (c,1) > \dots > (d,3)$$

\succsim_1	\succsim_2	\succsim_3
$(b,1)$	$(b,2)$	$(d,3)$
$(x,i) \notin A \times \{1\}$	$(x,i) \notin A \times \{2\}$	$(a,3)$
\vdots	\vdots	$(x,i) \notin A \times \{3\}$
		\vdots

Bidder 3 will win with outcome $(a,3)$, while

- ❶ $(d,3)$ is strictly higher preferred by bidder 3, and
- ❷ all other bidders are indifferent.

Other Properties

The dominant strategy equilibrium is

- Weakly Pareto efficient: no outcome is *strictly* preferred by *all* bidders.
- Strongly Pareto efficient when center is also considered: other outcome is either worse for center, or for winner.
- Weakly monotonic: if a bidder moves the equilibrium outcome (a^*, i^*) up in its order, the outcome of the mechanism stays the same.

Summary

- A class of auctions without money, similar to Vickrey's second-price auction
- A dominant strategy equilibrium that is
 - weakly Pareto efficient (but not strongly),
 - strongly Pareto efficient when center is also considered, and
 - weakly monotonic.
- In paper:
 - Escape Gibbard-Satterthwaite by restricting bidders' preferences (distinct acceptable outcomes and indifferent among non-winning)
 - Drop assumption that each bidder has an acceptable outcome

Future Work

- Prove that the Vickrey auction with money is a special case (where \geq is the standard order over prices)
- Show relation to multi-attribute auctions
- Study other qualitative auctions (e.g. English, multi-unit, online)
- Characterise instances of these mechanisms (parameterised by \geq)
- Find more interesting applications without money transfers (e.g. grids)