

# Heuristic Algorithm for Coordinating Smart Houses in MicroGrid.

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**Abstract**—This work presents a framework for efficiently managing the energy needs of a set of houses connected in a micro-grid configuration. The micro-grid consists of houses and local renewable plants, each seen as independent agents with their specific goals. In particular houses have the option to buy energy from the national grid or from the local renewable plants. We discuss a practical heuristic that leads to energy allocation schedules that are cost-effective for the individual houses and profitable for the local plants. We present experiments describing the benefits of our proposal. The results illustrate that houses and micro plants can make considerable saving when they work in micro grid compared with working alone.

## I. INTRODUCTION

The world's energy needs are ever increasing [1], [2] and the investment in new power plants is not going to cover the future demand [3]. The power sector can be improved in many ways, using more renewable resources, or resorting to more efficient and environmentally friendly power plants. Also, Demand Side Management and Demand Response could encourage consumers to modify their energy usage behaviour.

The concept of Smart Grid is relatively new. The Smart Grid is an enhanced electrical grid in which information and communication technology is used to improve the power system and increase the profit of consumers, distributors and generation companies. The key features of such infrastructure are reliability, flexibility, efficiency, sustainability, peak curtailment, and demand response. The Smart Grid is also market enabling, it provides a platform for advanced services, and increases the manageability of the available resources. To exploit the Smart Grid in residential buildings, we need new technologies such as integrated communications, sensing and measurements, smart meters, advanced control, advanced components, power generation, and smart appliances [4]. Smart micro-grids [5] can be defined as a set of houses containing loads (appliances) and co-located resources (such as small PV arrays, or wind plants) working as a single controllable system [6]. Smart micro-grids also offer the possibility to export the surplus of locally generated power to the national grid.

There are plenty of studies that investigate methods for optimizing the cost of electricity in stand-alone residential buildings, based on electricity price, availability of renewable power, or user preferences. These studies use different algorithms to achieve their goals. For example, studies [7]–[10] use algorithms that find the optimal cost of electricity, whereas [11]–[13] use heuristic methods which only guarantee suboptimal cost. Sharing

local renewable power in small communities is an active research area. Study [14], uses Mixed Integer Linear Programming (MILP) to compare the cost of 20 houses working individually and the same houses working in a micro-grid setup. Although this study adds some knowledge to the field, it does not tackle the important issue of computation time. In studies of this type the time complexity of the particular algorithm increases with the system's granularity or the number of available appliances. The authors are only able to present examples that allocates resources over relatively large time slots. Furthermore, in this study each house uses only two appliances. Study [1] investigates the sharing of local renewable energy in a micro-grid. A greedy energy search algorithm is used to match the predicted renewable power with the predicted house consumption. The proposed approach also minimizes the power loss incurred while transferring electricity power along power lines by choosing the nearest house to share renewable power with. Unfortunately the proposed algorithm does not scale well with the length of the time slots.

In this work, we investigate the effectiveness of a MILP-based strategy that can be used to solve a particular energy allocation problem within a given micro-grid. Our contribution is summarized in the following points:

- Design of a general micro-grid management system. Including fairly general notion of appliance and house to power source interaction.
- Design of a MILP heuristic used to solve a multi-objective energy allocation problem within such micro-grids.
- Preliminary empirical evaluation of our strategy.

The rest of this paper is organized as follows: Section 2 details the system definition and modeling of system entities, MILP formulation is presented in section 3, and the fourth section illustrates the results which are followed by discussions in Section 5. Finally, the paper ends with conclusions.

## II. ALLOCATION PROBLEM

In this section we present the formalization of the computational problem discussed in this paper.

### A. The micro-grid

A micro-grid consists of a set of houses  $\mathcal{H}$ , and a set of micro-generation power plants (or generators)  $\mathcal{R}$  (see the example in Fig.(1)). Some houses (like H2, H3, or H4 in Fig.(1)) may be directly connected to a generator, and therefore they are able

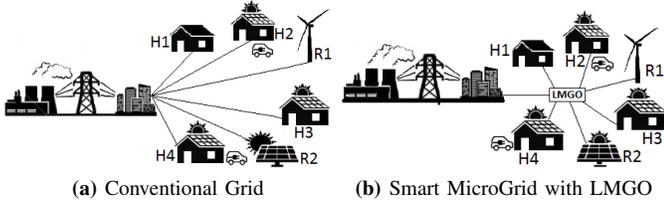


Fig. 1: Conventional grid vs smart MicroGrid

to receive energy from it in a particularly efficient way, but in general the houses in the system may receive their power from any of the generators in the micro-grid or the National Electricity Grid (NEG). The energy exchange within a micro-Grid is controlled by a Local micro-Grid Optimizer (LMGO). The power plants generate energy which can be either used by the houses in the micro-grid or exported to the NEG. Fig.(2) describes the possible energy exchanges between a house, a generator and the NEG. Houses (and their appliances) can only use electricity. The electricity comes in the house either from a generator (internal to the house or external) or from the grid. The labels on the arcs represent the unit cost that the entity at the end of the arrow will have to pay to the entity at the other end to get electricity from it. We assume that the energy produced by a generator  $r$  can be sent to a house  $h$  at a unit cost  $\gamma_r^h$  or exported to the NEG at a cost  $\zeta_r$ . Alternatively, a house can buy energy from the NEG at a cost  $\lambda_h$ . All costs might change over time (hence the dependence on a time parameter  $t$ ).

### B. Appliances

Each house  $h \in \mathcal{H}$  is equipped with a set of appliances  $\mathcal{A}_h = \{A_1, A_2, \dots, A_{m_h}\}$ , Appliances in a micro-grid are the main energy outlets. We assume that the appliances in the system can be easily switched on or off without disrupting their functionalities. Washing machines, cookers, air-conditioning (AC) units, battery chargers are examples of suitable appliances whereas TV sets or Computers do not fit into such framework. Moreover we assume that the appliances in a micro-grid can either be interruptible or uninterruptible, uniphase or multiphase. Interruptible appliances are designed to be switched ON/OFF at any time. Appliances of this type include heaters, cookers, or air-conditioning (AC) units. Uninterruptible appliances are not designed to be switched OFF once they have been switched ON until they finish a particular task. Washing machines are good examples of uninterruptible appliances. Heaters are also examples of uniphase appliance. Any such appliance can either be OFF or ON and when it is ON it uses approximately a constant amount of power (nominal power). Note that the restriction to the use of uniphase appliances is that only have a single ON state, without loss of generality, appliances that can run at one of several power levels can be

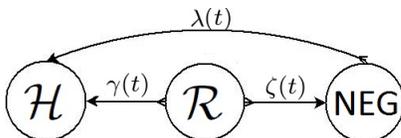


Fig. 2: Diagram shows exchange local renewable power among micro-grid components.

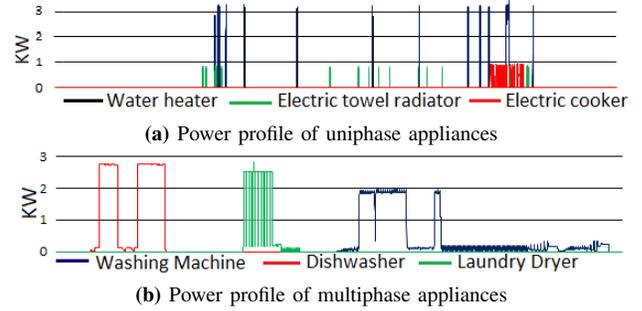


Fig. 3: Uniphase vs Multiphase appliances

simulated by a combination of several uniphase appliances as defined in this paper. Multiphase appliances work in different phases, each using a certain amount of power. A washing machine typically uses a lot of energy at the beginning of a wash cycle, to heat up the water, then uses little energy for some time, and a bit more again during the final spins. Fig.(3.(b)) shows the typical power profile of three different multiphase appliances. Within a given phase, multiphase appliances cannot be switched off. We also assume that some appliances may have constraints on how often they are run while others might be controlled by environmental factors such as the level of charge of a battery, or particular desired values of room temperatures.

For the purpose of this study we assume that each appliance  $A$  operates in  $\Delta_A > 0$  (nominal) phases and for each appliance, it is possible to define a power profile vector  $(\alpha_1, \dots, \alpha_{\Delta_A})$  describing its energy needs. We assume that  $\tau_{\min}$  is length of the shortest phase. Each  $\alpha_j$  is a non-negative real number, corresponding to the average amount of power used by the appliance during its  $j$ th phase. When switched on appliance  $A$  progresses through each of its phases, starting from phase 1 up until phase  $\Delta_A$  at which point the appliance is switched OFF. We also assume that for each appliance we know whether it is interruptible or not, and the number of times it must be used,  $n_A$ . Note that such model fits the different types of appliances described before.

### C. Optimization Problem

From the discussion so far it is evident that a micro-grid consists of distinct agents each with their own goals and priorities: houses need energy to run their set of appliances according to pre-defined plans, generators produce energy that can be sold to the houses in the micro-grid or the NEG; houses want to purchase cheap energy whereas generators want to maximize their profit. In this setting we can associate a cost function  $\Psi_h$  to each house  $h \in \mathcal{H}$ :

$$\Psi_h = \int \lambda_h L_g^h dt + \sum_r \int \gamma_r^h G_r^h dt, \quad (1)$$

where  $L_g^h$  describes the amount of energy from the NEG used by house  $h$  over time, and  $G_r^h$  the amount of energy generated from plant  $r$  used by  $h$ . Similarly, we can associate a profit function  $\Xi_r$  to each  $r \in \mathcal{R}$ :

$$\Xi_r = \int \zeta_r E_g^r dt + \sum_h \int \gamma_r^h G_r^h dt. \quad (2)$$

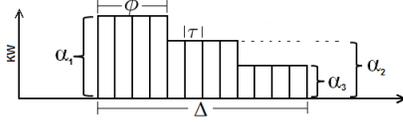


Fig. 4: Multiphase appliance modeling

In such formula,  $E_g^r$  describes the amount of energy produced by  $r$  that is sold to the NEG. The problem of allocating energy to houses in a micro-grid in a way that is cost effective for the houses and profitable for the grid's power plants can then be cast as a multi-objective optimization problem [15].

$$\min(\Psi_h : h \in \mathcal{H}; -\Xi_r : r \in \mathcal{R}) \quad (3)$$

In this paper we present a practical solution to this problem and provide some evidence of its effectiveness.

### III. MILP FORMULATION

In this Section we present our heuristic algorithm for finding a feasible solution for the multi-objective optimization problem defined in Section II-C. We start by providing a (multi-objective) mathematical programming formulation of the problem.

#### A. Appliances modeling and linear constraints

In what follows we assume that each instance of the problem is solved over a fixed time horizon (say 24 hours) and that time within such horizon is divided into a finite set of time slots,  $\mathcal{T} = \{t_1, t_2, \dots, t_T\}$ , all of length  $\tau$  with  $0 < \tau < \tau_{\min}$ . We assume that  $\tau$  divides the length of each phase within the system. We identify the  $m_h$  appliances in house  $h$  with the numbers  $1, 2, \dots, m_h$ . Without loss of generality, we also assume that each appliance  $i$  runs through  $\Delta_i^h$  (real) phases, of length  $\tau$ . Note that real phases may be much shorter than the nominal phases mentioned in Section II-B. Thus we assume that real phases are grouped into clutches corresponding to the nominal phases and the appliances are uninterruptible within each clutch (Fig.(4) shows an appliance with three clutches). We use a dedicated binary variable  $x_{i,j}^h(t)$  for appliance  $i$  in (real) phase  $j$ . The variable holds the appliance ON/OFF state at time  $t$ .

$$P_{i,j}^h(t) = \alpha_{i,j}^h \cdot x_{i,j}^h(t) \in \{0, \dots, \alpha_{\Delta_i^h}^h\}. \quad (4)$$

We also assume that appliance  $i$  in  $h$  can only be run between time slot  $t_s^{h,i}$  and  $t_f^{h,i}$  (with  $t_s^{h,i} \leq t_f^{h,i}$ ), in a so called comfort interval specified by the user, if needed. We model this using the following constraints

$$\sum_{t=0}^{t_s^{h,i}-1} x_{i,j}^h(t) + \sum_{t=t_f^{h,i}+1}^{t_T} x_{i,j}^h(t) = 0, \quad (5)$$

where either sums may be empty if  $t_s^{h,i} = t_1$  or  $t_f^{h,i} = t_T$ . If both equalities hold (say if the user does not specify a comfort interval) the constraints vanish. To enforce that appliance  $i$  in

$h$  runs  $n_i^h$  times in  $\{t_s^{h,i}, \dots, t_f^{h,i}\}$ , we need the following constraints

$$\sum_{t \in \{t_s^{h,i}, \dots, t_f^{h,i}\}} x_{i,j}^h(t) = n_i^h. \quad (6)$$

Phases can be kept in order imposing the following constraint,

$$\sum_{t \in \mathcal{T}} [t \cdot x_{i,j+1}^h(t) - t \cdot x_{i,j}^h(t)] \geq 1. \quad (7)$$

and to prevent interruption between any two consecutive phases, we use constraint (7) with “=” replacing “ $\geq$ ”.

As mentioned before, the operation of some appliances depends on external conditions rather than initial user demands. For instance charging a battery depends on the battery charging state  $\Theta_i^h(t)$  and its charging rate, whereas the operations of an Air Conditioning (AC) unit depends on the room temperature,  $T_{in}^{h,i}(t)$ , the outside temperature and the device heating or cooling power [8]. Appropriate constraints in such cases replace those in (6). In the case of batteries, we need to use Eqs.(8) and (9).

$$\Theta_i^h(t) = \Theta_i^h(t-1) + \frac{1}{4} \cdot \pi \cdot P_{i,1}^h(t) \quad \forall t : t \in \{t_s^{h,i}, \dots, t_f^{h,i}\} \quad (8)$$

$$\Theta_i^h(t_s^{h,i}) = \underline{\beta}_i^h, \quad \Theta_i^h(t_f^{h,i}) = \overline{\beta}_i^h \quad (9)$$

where  $\underline{\beta}_i^h$  is the initial state of charge of the battery,  $\overline{\beta}_i^h$  is the desired final state of charge of the battery (usually full), and  $\pi$  is the battery charging efficiency. In the case of heating/cooling units, the main task of the given unit is to keep the room temperature within the comfort level  $[T_{min}^{h,i}, T_{max}^{h,i}]$  during  $b_i^h$  specified time intervals  $I_1^h, \dots, I_{b_i^h}^h$ . The relationship between room temperature and the power allocated to the appliance is shown in Eq. (10).

$$T_{in}^{h,i}(t) = \rho \cdot T_{in}^{h,i}(t-1) + (1-\rho) \left[ T_{out}(t) - \frac{\eta}{\kappa} P_{i,1}^h(t) \right] \quad (10)$$

$$T_{min}^{h,i} \leq T_{in}^{h,i}(t) \leq T_{max}^{h,i} \quad \forall t : t \in I_1^h \cup \dots \cup I_{b_i^h}^h$$

where  $\rho$  is the appliance inertia,  $\eta$  is efficiency of the system (with  $\eta > 0$  for a heating appliance and  $\eta < 0$  in the case of cooling),  $\kappa$  is the thermal conductivity,  $T_{out}(t)$  is outside temperature at time  $t$ .

#### B. Objective Function and Additional Constraints

For the purpose of our experiments we simplify the general model presented in Section II-C. The cost function in Eq.(1) is replaced by the linear function

$$\Psi_h = \sum_{t \in \mathcal{T}} \left\{ \lambda_h(t) L_g^h(t) + \sum_{r \in \mathcal{R}} [\gamma_r^h(t) G_r^h(t)] \right\} \quad \forall h : h \in \mathcal{H}, \quad (11)$$

and similarly, the profit function in Eq.(2) is replaced by

$$\Xi_r = \sum_{t \in \mathcal{T}} \left\{ \zeta_r(t) E_g^r(t) + \sum_{h \in \mathcal{H}} [\gamma_r^h(t) G_r^h(t)] \right\} \quad \forall r : r \in \mathcal{R}, \quad (12)$$

with the proviso that if  $r$  belongs to  $h$  then  $\gamma_r^h(t) = 0 \quad \forall t$ , and  $\Psi_h$  is the right-hand side of (11) minus  $\Xi_r$ .

Few constraints need to be added to the system. There are the renewable power constraints

$$E_g^r(t) + \sum_{h \in \mathcal{H}} G_r^h(t) = P_r(t) \quad \forall t : t \in \mathcal{T}, \forall r : r \in \mathcal{R}, \quad (13)$$

where  $P_r(t)$  is the renewable power generated by  $r$ , and power balance equations, enforcing that the allocated power at any time slot,  $t$ , must equal power demand at that time

$$L_g^h(t) + \sum_{r \in \mathcal{R}} G_r^h(t) = \sum_{i \in \mathcal{A}_h} \sum_{j=0}^{\Delta_i^h} P_{i,j}^h(t), \quad \forall t : t \in \mathcal{T}, \quad (14)$$

The key idea is to reduce the multi-objective problem to a single objective one using a heuristic inspired by the so called  $\epsilon$ -constraints method [16]. An off-the-shelf MILP solver can then be used to find a feasible allocation. To this purpose we consider the MILP obtained by using the constraints listed in the previous sections along with the following objective function,

$$\text{Min} \left\{ \sum_{i=1}^{|\mathcal{H}|} \Psi_i - \sum_{i=1}^{|\mathcal{R}|} \Xi_i \right\} \quad (15)$$

Extra constraints

$$\Psi_h \leq \tilde{\Psi}_h \quad \forall h : h \in \mathcal{H} \quad (16)$$

and

$$\Xi_r \geq \tilde{\Xi}_r \quad \forall r : r \in \mathcal{R} \quad (17)$$

are also added, where  $\tilde{\Psi}_h$ , and  $\tilde{\Xi}_r$  are the optimal costs of the energy allocation problem for house  $h$  and renewable plant  $r$  considered as isolated units connected solely to the NEG.

### C. MILP-based Heuristic

Let MINCOST denote the version of our problem restricted to a single house, with  $m$  uniphase appliances, to be allocated in one of two possible time slots. Also assume that the available renewable power is always  $\frac{1}{2} \sum_{i=1}^m \alpha_i$ , and the NEG electricity price is  $\lambda > 0$ . A straightforward reduction from the PARTITION problem [17] shows that MINCOST is NP-hard. Therefore there is little hope that the MILP defined in the previous section might be solved quickly if the number of appliances is large. In our experiments we resort to an MILP-based heuristic algorithm to get a feasible solution in acceptable time. The basic idea is to use an off-the-shelf LP-solver to generate a feasible solution but without running the optimization process to completion. The LP-solver uses dual relaxation to find a lower bound on the optimum and stops as soon as the difference between the cost of the best feasible solution so far and the lower bound on the optimum becomes smaller than a predefined threshold. Also, we can put time limit or deadline to stop the algorithm.

## IV. EMPIRICAL EVALUATION

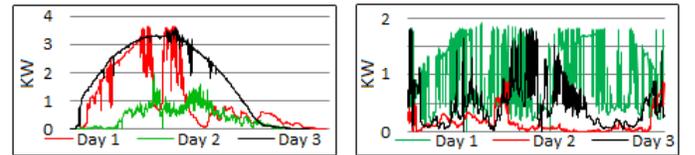
All the experiments in this work have been done on a PC with an Intel(R) core(TM) i7-2600 CPU @ 3.4 GHZ, RAM is 16 GB, 64-bit Operating System (windows 7). In addition, Gurobi [18] has been used to solve LP and MILP problems, whereas the Java was the main tools to build our model. Three case studies will



Fig. 5: Electricity Price, one fixed pricing and two dynamic schemes.

be demonstrated to illustrate the advantages and disadvantages of our approach.

In all case studies  $\tau = 5$  minutes. The electricity prices  $\lambda_h(t)$  used in the experiments (assumed to be fixed for every house  $h$ ) are shown in Fig.(5). We assume that  $\zeta_r(t) = 4.5$  P/KWH,  $\gamma_r^h(t) = 8.5$  P/KWH for all  $r, h$  and  $t$ .  $\pi = 0.8$ ,  $\rho = 0.96$ ,  $\eta = 30$  KW/°C,  $\kappa = 0.98$ ,  $T_{min} = 18.0$ °C, and  $T_{max} = 22.0$ °C. Fig.(6) shows solar and wind power generated in Liverpool, UK (53°24'N 2°59'W), using 3.5KW/H PV array and 2KW/H wind turbine. These data will be approximated and scaled up/down to model different sets of PV arrays and wind turbines. Fig.(7) illustrates the outside temperature.



(a) Solar power for three days in (b) Wind power for three days in January, April, May, and June 2012 March, and June

Fig. 6: Renewable Power, for different three days in April, May and June 2012 in Liverpool.

### A. First case study

The main goals of this case study is to show the effect of renewable power demand on saving or profit.

1) *Input setting* : 20 houses with variable renewable power generation capacities, see Table (I), and three independent renewable plants (PV array with maximum generation capacity = 5KW/H, two wind turbines, with 1KW/H, 10KW/H generation capacity, respectively) will be used to investigate the performance of our algorithm. Dynamic pricing 1 in Fig.(5) will be used. We will use three scenarios (low demand, medium demand, and high demand) in this case study to examine the effect of electricity demand on saving. Due to the page limit, we can not fit all the input data for 20 houses. Therefore, we had to put details appliances in each house and time comfortable interval for each appliances in a technical report [19]. The power profiles of uninterruptible appliances are shown in Table.(II), whereas the interruptible appliances are given in Table (III).

2) *Findings*: Fig.(8a) displays the average profit of three scenarios, low demand, medium demand, and high demand. The

Table I: PV array generation capacity of houses

House No	5,10,15	1,6,11,16,19	2,7,12,17,20	3,8,13,18	4,9,14
Capacity	0.0 KW	1.0 KW	1.5 KW	2.0 KW	2.5 KW

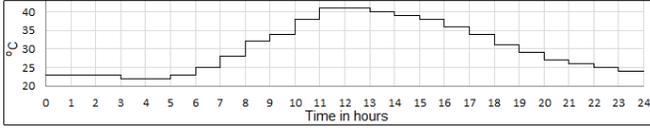


Fig. 7: Predicted outside temperature

houses with high demand, in general, can make more profit because the relationship between saving and renewable power consumption is positive. In contrast, Fig(8b) shows the relative MILP Gap (duality gap) of the three scenarios. MILP Gap of high demand scenario is still above 100% after 30 minutes of running time that means the solution found could be far from optimality, it could be so close to optimality though. In addition, the first and second scenarios are so close to optimality because MILP gap is less than 1 %.

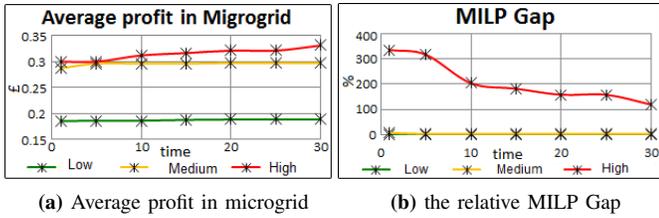


Fig. 8: The result of low demand, medium demand, and high demand.

Fig. (9) depicts profit stability in low and high demand scenarios. The relationship between run time and average profit of all entities is positive, but it does not always hold for each entity. For instance, House No.17 in Fig.(9b) made more profit after 1, 5, and 10 minutes of calculation time than after 15 minutes but in general the average profit increases with time until it reach optimality. Fig.(10) illustrates the profit made by each component in micro-grid in three scenarios. Note that the first five house in medium demand scenarios made more profit than the first five houses in high demand scenario which conflict with the result in Fig.(8a) that is because in medium demand scenario, the first five houses have 7 to 8 appliances, see [19], and MILP gap of medium, Fig(8b), shows that the solution of medium demand is so close to optimality whereas in high demand it is not, which explain we the first five house in medium

Table II: Multiphase uninterruptible appliances

Appliance	$\alpha$ in KW	3.2	0.28	0	3.2	0.28
Laundry Dryer	$\phi$ in minutes	15	10	5	20	10
Dishwasher	$\alpha$ in KW	0.2	2.7	0.2	2.7	0.2
	$\phi$ in minutes	5	15	15	20	5
Washing Machine	$\alpha$ in KW	2.2	0.28	2.2	0.28	-
	$\phi$ in minutes	10	20	10	20	-

Table III: Interruptible appliances

Interruptible appliances	$\alpha$	Depend on
Water heater	3.1 KW/t	-
Electric Towel Radiator	1.5 KW/t	-
Electric cooker	2.5 KW/t	-
Plug-in Hybrid Electric Vehicle	0.35 KW /t	$\Theta(t_s)=2.0, \Theta(t_f)=16.0$
Air conditioner	2.3 KW/t	$T_{min}=18, T_{max}=22$

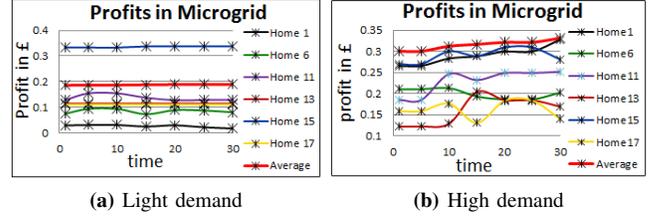


Fig. 9: Profit stability in micro grid of first and third scenarios.

demand made more profit then their counterpart in high demand. Also, there is couple of house made more profit in low demand scenarios than in others that is because this is a sup-optimal solution.

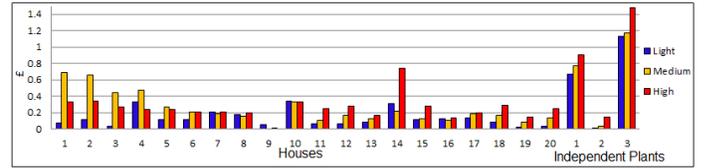


Fig. 10: The profit of entity in micro-grid in three scenarios.

### B. Second case study

The main goal of this case study is to investigate the effect of electricity pricing on the profit.

1) *Input settings:* We will use almost the same input data that have been used in third scenario (high demand) in first case study but we will repeat the experiment with three pricing schemes in Fig(5).

2) *Findings:* Fig.(11) shows the profit percentage made by each entity in micro grid with three pricing scheme. We used MIP gap = 25% to stop searching. The result illustrates that micro grid entities can make more profit in dynamic pricing schemes than in fixed pricing scheme.

### C. Third case study

The main goal of this case study is to examine the effect of number of houses in micro grid on the performance of our algorithm. Two scenarios will be demonstrated here.

1) *Input settings:* In the first scenario, we will use up to 20 identical houses (have the same power demand), each house has 8 different appliances, nominal power of appliances and comfortable time of each house are exactly the same in first case study. Each house equipped with PV array (2.5KW). There is no independent micro plant in this scenario.

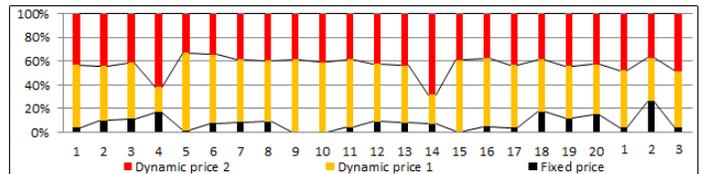


Fig. 11: The profit percentage of micro grid using three pricing schemes.

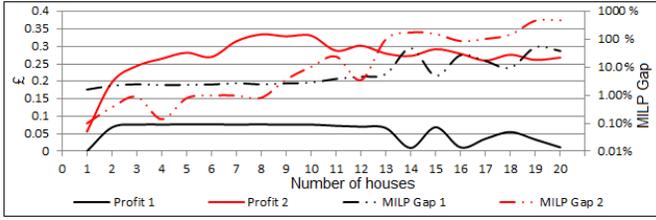


Fig. 12: Average cost and MILP Gap of houses in two scenarios

In the second scenario, we will use up to 20 house and three independent micro plant in micro grid. We used the same data in third scenario in first case study. The only difference here is that we vary the number of houses from 1 to 20.

2) *Findings*: Fig.(12) shows the average profit of houses, and the MILP gap in two scenarios. The profit increases up to a point in both scenarios then it start decreasing with fluctuation that is because the calculation time is fixed to 5 minutes, deadline=300 sec, for all experiments, whereas the number of houses is varied. Moreover, micro grid with two houses will reach optimality or near-optimality faster than micro grid with 20 houses. In addition, it shows that the relationship between MILP Gap and number of houses is positive, and the relationship between MILP Gap and profit is negative. This explains why the curve of profit starts decreasing when MILP Gap curve start increasing. Black curves present the results of first scenario whereas the red curves present the results of second scenario.

## V. DISCUSSION AND CONCLUSION

### A. Fairness issues

By converting the problem from multi objectives to single objective one and using  $\epsilon$ -constraint method, fairness issue could raise. Therefore, we modified the objective function of  $\epsilon$ -constraint method, Eq.(15), to tackle this fairness issue . Fig.(10) shows the individual profit of each entity. As we can see, house no. 5, 10, and 15, which are the houses that does not have PV array, made profit higher than house no. 9 (equipped with PV array). The main reason for this fairness issue is that this solution is suboptimal. Further work is needed to cope with this issue.

### B. Profit stability

The stability of the profit depends on size of the problem (number of integer variables) and MILP Gap, if problem is small, the algorithm will reach optimality/near-optimality relatively fast and the profit will be almost stable, and vice versa. See Fig.(8) and Fig.(9). Fig.(8b) shows, after 30 minutes of run time, the MILP gap still around 100%, which means that the optimal solution may be still far on optimality, it could be so close, though.

### C. Scalability

Increasing the number of entities in micro-grid does not always increase the profits of these entities. Fig.(12) shows that the relationship between the number of houses and profit is not

always positive. As we can see in both scenarios, the relationship between the number of houses and the average profit is positive up to a point, then it becomes negative, that is because we increase the number of houses whereas run time is fixed.

To conclude, this work illustrates how an appropriate using MILP Heuristic can be for solving huge optimization problem, the results shows that the sub-optimal cost of each house in micro grid is cheaper than the optimal cost of each house working alone.

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