Greedy algorithms

1. Consider the set of items $S = \{a, b, c, d, e, f, g, h\}$ where the items have the following (benefit, weight) values:

a	$(14, 3)$
b	$(5, 1)$
c	$(10, 6)$
d	$(12, 4)$
e	$(8, 2)$
f	$(10, 4)$
g	$(16, 8)$
h	$(9, 9)$

   Solve the Fractional Knapsack Problem for this set of items, where the maximum total allowed weight is $W_{\max} = 15$.

2. Let $T = \{(1, 3), (2, 4), (3, 5), (2, 7), (4, 6), (5, 6), (3, 7), (5, 8), (6, 10), (7, 9), (8, 10)\}$ denote the (start, finish) times for a collection of 11 tasks.

   (a) Solve the Interval Scheduling Problem for this collection of tasks (i.e. find the maximum number of tasks that can be scheduled on a single machine, and give a set of compatible tasks that achieves this maximum).

   (b) Solve the Task Scheduling Problem for this collection of tasks (i.e. find the minimum number of machines required to complete all tasks, and give a schedule for doing so).

3. Describe an efficient greedy algorithm for making change for a specified value using a minimum number of coins, assuming that there are four denominations of currency of coins with values $25, 10, 5, 1$ (American quarters, dimes, nickels, and pennies, respectively).

   Argue why your algorithm is correct.

4. Give an example set of coins (i.e. specify their values) so that a greedy change making algorithm will not always use the minimum number of coins. Illustrate this by showing a value for which a greedy algorithm like that in the above question doesn't give the minimum number of coins.

   (Assume that the smallest denomination of coins in your set has value 1, so that a solution always exists.)

5. In the **art gallery guarding problem** we are given a line L that represents a long (straight) hallway in an art gallery. We are also given a set $X = \{x_1, x_2, \ldots, x_n\}$ of real numbers that represent locations

where paintings are hung in the hallway. Suppose that a single guard can protect all the paintings within distance at most 1 of his or her position (on both sides).

Design an algorithm for finding a placement of the guards that uses the minimum number of guards to guard all the paintings with positions in X.