Simulated Annealing for the Permanent

The *permanent* of an $n \times n$ (non-negative) matrix A is defined as

$$per(A) = \sum_{\sigma} \prod_{i=1}^{n} a_{i,\sigma(i)},$$

where the sum is over all permutations σ of the set $\{1, \ldots, n\}$.

Despite its similarity to the definition of the determinant of a matrix, the permanent is a classical #P-complete problem [7]. Thus, while the determinant can be efficient calculated, there is no general efficient algorithm for computing the permanent of an arbitrary matrix.

However, like many hard problems, there is still high interest in good methods for *estimating* the permanent because of its occurrence in many different problems. For example, if A is a 0/1 matrix, then it represents the adjacency matrix of a bipartite graph, and the permanent counts the number of *perfect matchings* of that graph. The permanent also arises in other areas, e.g. statistical physics [4], computer vision [6] and statistics [5].

Current algorithms for estimating the permanent are based on the so-called *Markov chain Monte Carlo* method, a randomized approach based on sampling perfect and near-perfect matchings in bipartite graphs. Jerrum, Sinclair, and Vigoda [3] presented a simulated annealing method for estimating the permanent of a non-negative matrix with running time $O(n^{10} \log^3 n)$ for 0/1 matrices.

A recent paper by Bezáková, et. al. [1] proposes a new "cooling scheme" for the simulated annealing algorithm that achieves a $O(n^7 \log^4 n)$ randomized algorithm for the permanent of a 0/1 matrix.

This project will involve the implementation, coding, and testing of this approximation algorithm in a programming language such as C/C++ or Java. It will essentially focus on the case of 0/1 matrices as this is the most common and widely known case. The project will involve two main parts:

1. Implementation of the Markov chain algorithm for sampling (weighted) perfect and near-perfect matchings in a bipartite graph.

2. Combining the first part with the "cooling schedule" for simulated annealing to obtain the approximation algorithm for the permanent of a desired 0/1 matrix.

It is also possible that we may consider the use of appropriate classes or add-ons for handling large integers or extremely small numbers as this could be useful as n, the size of the matrices, gets large. This aspect will be further considered in the course of the project.

Familiarity with probability and/or combinatorics (especially graph theory) is a bonus, but not strictly required for this project. Relevant background necessary for the implementation of the Markov chain sampling algorithm will be provided though a combination of reference materials (some listed in the bibliography) and instruction by the supervisor. However, excellent programming skills are certainly required. Some programming assistance can be provided by the supervisor if the language used is C/C++, but that's not his field of expertise.

References

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