Message-Passing Computing

Basics of Message-Passing Programming

Programming Options

Programming a message-passing multicomputer can be achieved by

1. Designing a special parallel programming language

2. Extending the syntax/reserved words of an existing sequential high-level language to handle message passing

3. Using an existing sequential high-level language and providing a library of external procedures for message passing

Here, we will concentrate upon the third option.

Necessary to say explicitly what processes are to be executed, when to pass messages between concurrent processes, and what to pass in the messages.

Two primary methods are needed in this form of a message-passing system:

1. A method of creating separate processes for execution on different computers

2. A method of sending and receiving messages

Single Program Multiple Data (SPMD) model

Different processes are merged into one program.

Within the program are control statements that will customize the code; i.e. select different parts for each process.

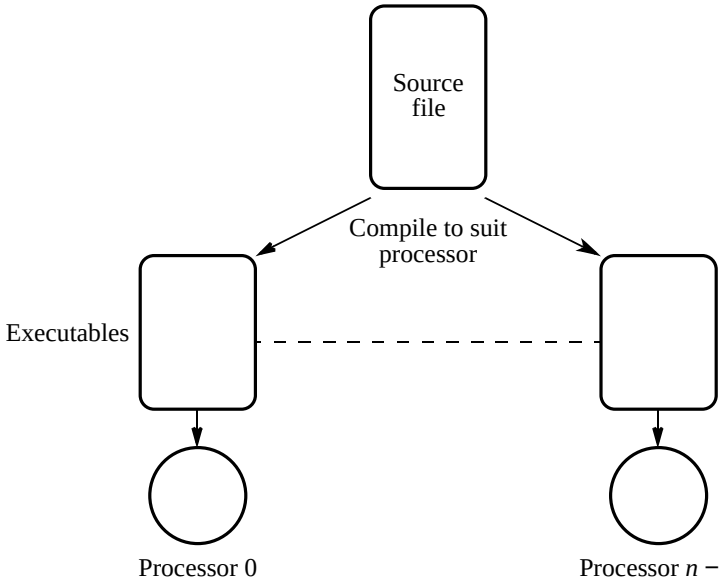


Figure 2.1 Single program, multiple data operation.

Multiple Program Multiple Data (MPMD) Model

Completely separate and different program is written for different processors.

The master-slave approach is usually taken whereby a single processor executes a master program (the master process) and other processes are started from within the master process.

Starting these processes is relatively expensive in computational effort.

An example of a library call for dynamic process creation might be of the form

```
spawn(name_of_process);
```

which immediately starts another process, and both the calling process and the called process proceed together:

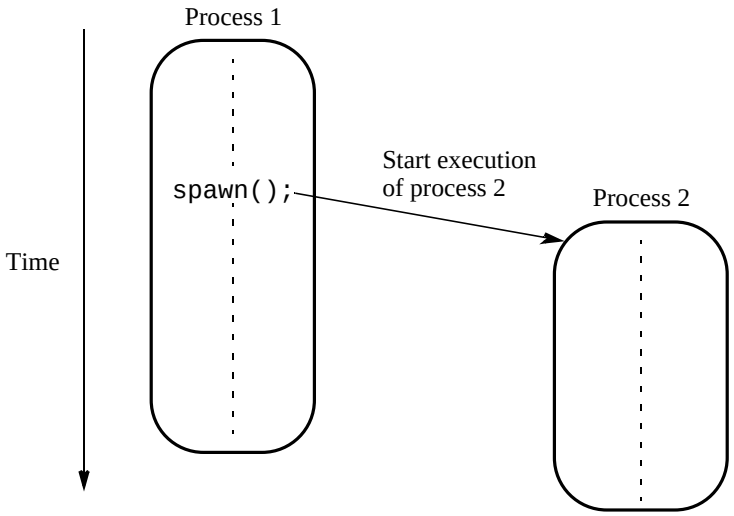


Figure 2.2 Spawning a process.

Message-Passing Routines

Basic Send and Receive Routines

Often have the form

```
send(&x, destination_id);
```

in the source process and the call

```
recv(&y, source_id);
```

in the destination process, to send the data `x` in the source process to `y` in the destination process:

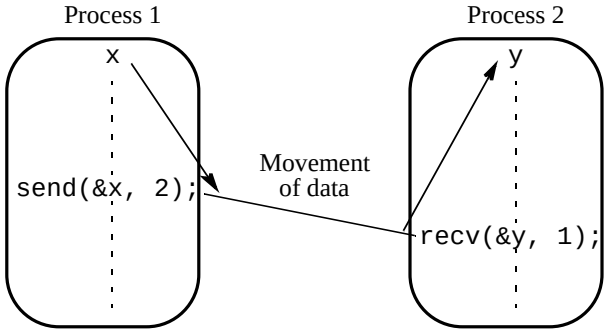


Figure 2.3 Passing a message between processes using `send()` and `recv()` library calls.

Synchronous Message Passing

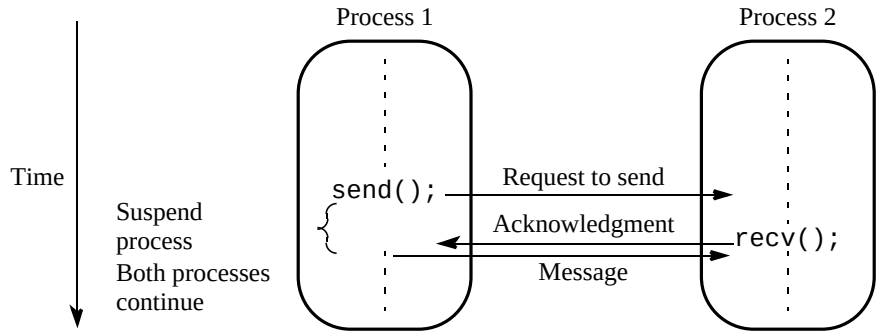
Routines that actually return when the message transfer has been completed.

Do not need message buffer storage. A synchronous send routine could wait until the complete message can be accepted by the receiving process before sending the message.

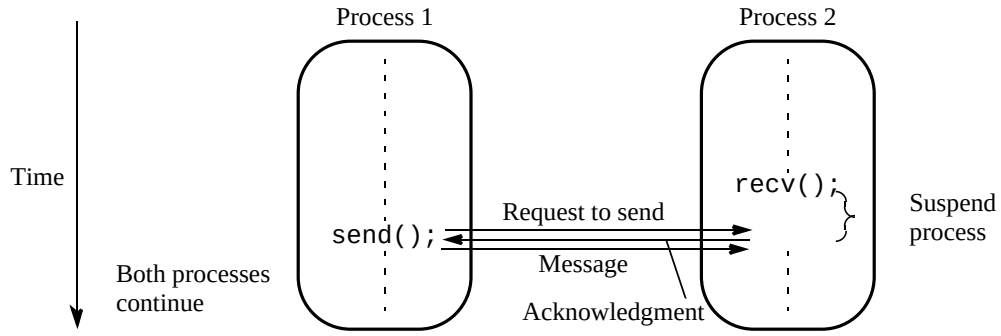
A synchronous receive routine will wait until the message it is expecting arrives.

Synchronous routines intrinsically perform two actions: They transfer data and they synchronize processes.

Suggest some form of signaling, such as a three-way protocol:



(a) When `send()` occurs before `recv()`



(b) When `recv()` occurs before `send()`

Figure 2.4 Synchronous `send()` and `recv()` library calls using a three-way protocol.

Blocking and Nonblocking Message Passing

Blocking - has been used to describe routines that do not return until the transfer is completed.

The routines are “blocked” from continuing.

In that sense, the terms *synchronous* and *blocking* were synonymous.

Non-blocking - has been used to describe routines that return whether or not the message had been received.

The terms *blocking* and *nonblocking* redefined in systems such as MPI:

MPI Definitions of Blocking and Non-Blocking

Blocking - return after their local actions complete, though the message transfer may not have been completed.

Non-blocking - return immediately. Assumed that the data storage being used for the transfer is not modified by the subsequent statements prior to the data storage being used for the transfer, and it is left to the programmer to ensure this.

How message-passing routines can return before the message transfer has been completed

Generally, a *message buffer* is needed between the source and destination to hold message:

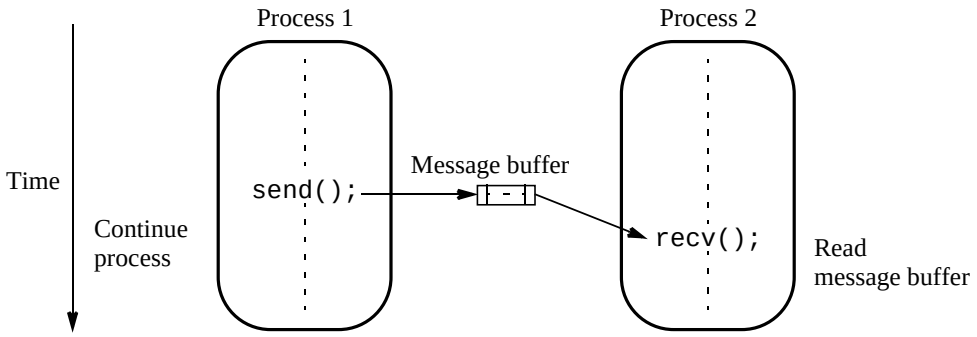


Figure 2.5 Using a message buffer.

Message buffer is used to hold messages being sent prior to being accepted by `recv()`.

For a receive routine, the message has to have been received if we want the message.

If `recv()` is reached before `send()`, the message buffer will be empty and `recv()` waits for the message.

For a send routine, once the local actions have been completed and the message is safely on its way, the process can continue with subsequent work.

In this way, using such send routines can decrease the overall execution time.

In practice, buffers can only be of finite length and a point could be reached when the send routine is held up because all the available buffer space has been exhausted.

It may be necessary to know at some point if the message has actually been received, which will require additional message passing.

Message Selection

So far, we have described messages being sent to a specified destination process from a specified source process.

Wide Card - A special symbol or number to allow the destination to accept messages from any source.

Message Tag

Used to differentiate between different types of messages being sent.

Example

To send a message, `x`, with message tag 5 from a source process, 1, to a destination process, 2, and assign to `y`, we might have

```
send(&x, 2, 5);
```

in the source process and

```
recv(&y, 1, 5);
```

in the destination process. The message tag is carried within the message.

If special type matching is not required, a *wild card* message tag is used, so that the `recv()` will match with any `send()`.

More powerful message selection mechanism is needed to differentiate between messages being sent between library routines and those being passed between user processes. This mechanism will be developed later.

Broadcast

Sending the same message to all the processes concerned with the problem.

M*ulticast* - sending the same message to a defined group of processes.

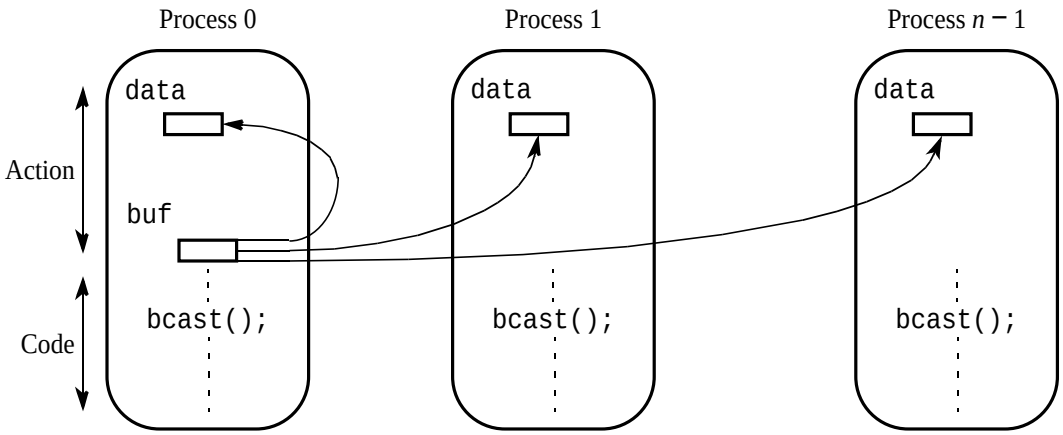


Figure 2.6 Broadcast operation.

Process 0 is identified as the *root process* within the broadcast parameters. The root process holds the data to be broadcast in `buf`.

Figure 2.6 shows each process executing the same `bcast()` routine, which is convenient for the SPMD model in which each process has the same program. It also shows the root receiving the data, but this depends upon the message-passing system.

Alternative arrangement - for the source to execute a broadcast routine and destination processes to execute regular message-passing receive routines.

Broadcast action does not occur until all the processes have executed their broadcast routine, and the broadcast operation will have the effect of synchronizing the processes.

Scatter

Sending each element of an array of data in the root to a separate process.

The contents of the *i*th location of the array is sent to the *i*th process.

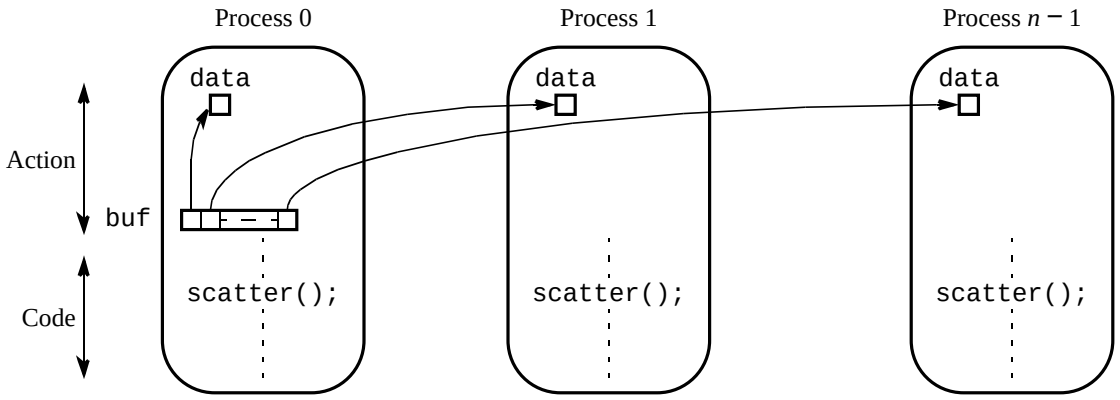


Figure 2.7 Scatter operation.

Common startup requirement.

Gather

Having one process collect individual values from a set of processes.

Gather is essentially the opposite of scatter.

The data from the *i*th process is received by the root process and placed in the *i*th location of array set aside to receive the data.

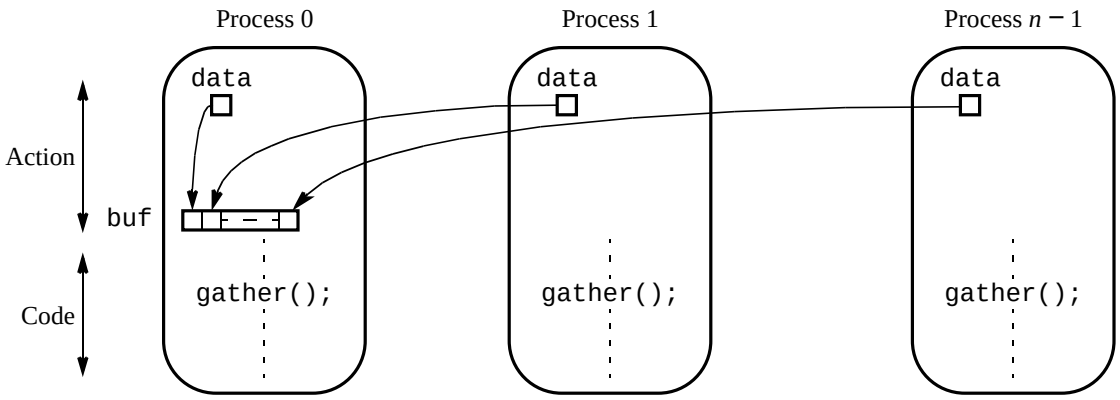


Figure 2.8 Gather operation.

Normally used after some computation has been done by these processes.

Reduce

Gather operation combined with a specified arithmetic or logical operation.

Example, the values could be gathered and then added together by the root:

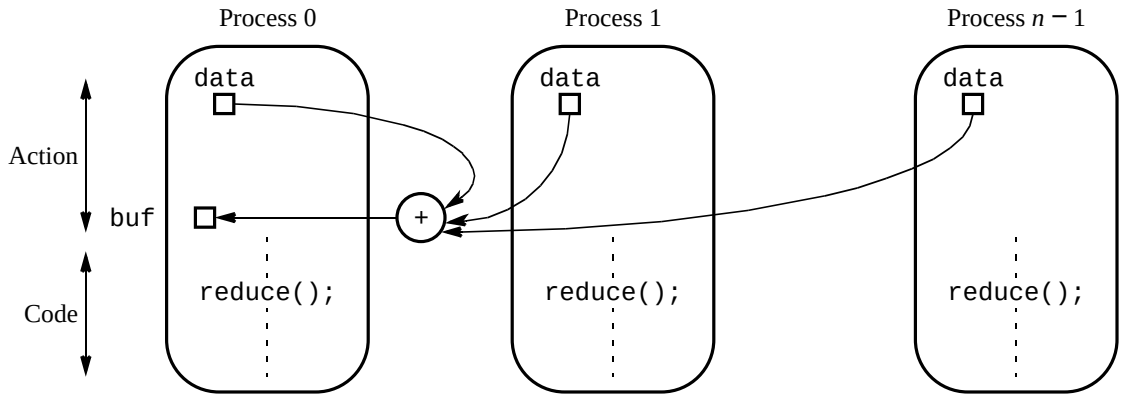


Figure 2.9 Reduce operation (addition).

Using Workstation Clusters

Software Tools

PVM (Parallel Virtual Machine) - Perhaps the first widely adopted attempt at using a workstation cluster as a multicomputer platform developed by Oak Ridge National Laboratories.

Provides for a software environment for message passing between homogeneous or heterogeneous computers and has a collection of library routines that the user can employ with C or FORTRAN programs.

Available at no charge.

MPI (Message Passing Interface) - standard developed by group of academics and industrial partnersto foster more widespread use and portability.

Several free implementations exist

PVM

The programmer decomposes the problem into separate programs. Each program is written in C (or Fortran) and compiled to run on specific types of computers in the network.

The set of computers used on a problem first must be defined prior to running the programs.

The most convenient way of doing this is by creating a list of the names of the computers available in a *hostfile*. The hostfile is then read by PVM.

The routing of messages between computers is done by PVM daemon processes installed by PVM on the computers that form the virtual machine:

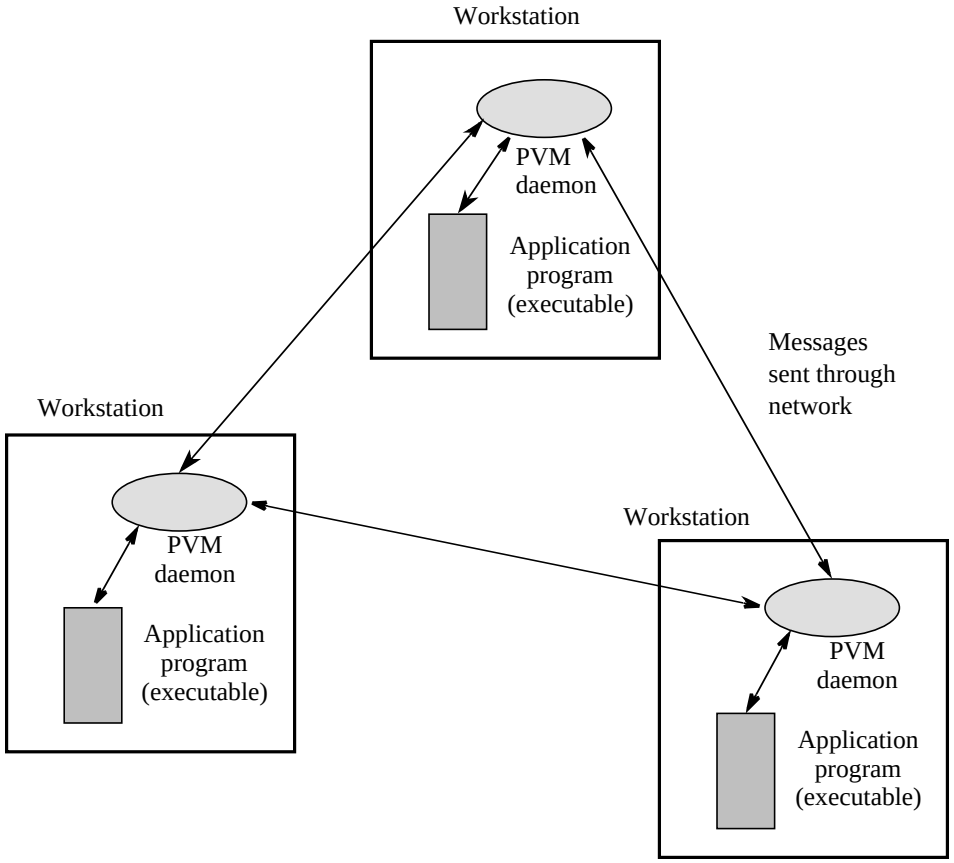


Figure 2.10 Message passing between workstations using PVM.

Number of Processes Greater Than Number of Processors

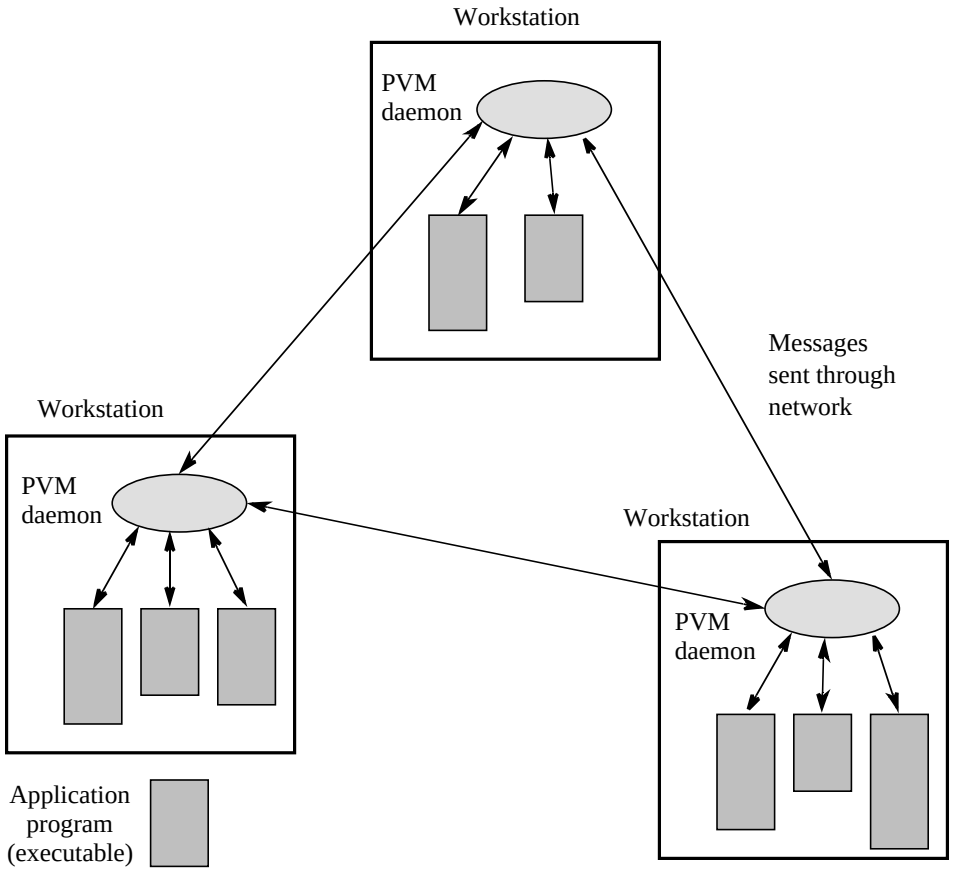


Figure 2.11 Multiple processes allocated to each processor (workstation).

Basic Message-Passing Routines

All PVM send routines are nonblocking (or asynchronous in PVM terminology) while PVM receive routines can be either blocking (synchronous) or nonblocking.

The key operations of sending and receiving data are done through message buffers.

PVM uses a *message tag* (`msgtag`), attached to a message to differentiate between types of messages being sent. Both message tag and source wild cards are available.

`pvm_psend()` and `pvm_precv()`

If the data being sent is a list of items of the same data type, the PVM routines `pvm_psend()` and `pvm_precv()` can be used.

A parameter in `pvm_psend()` points to an array of data in the source process to be sent, and a parameter in `pvm_precv()` points to where to store the received data:

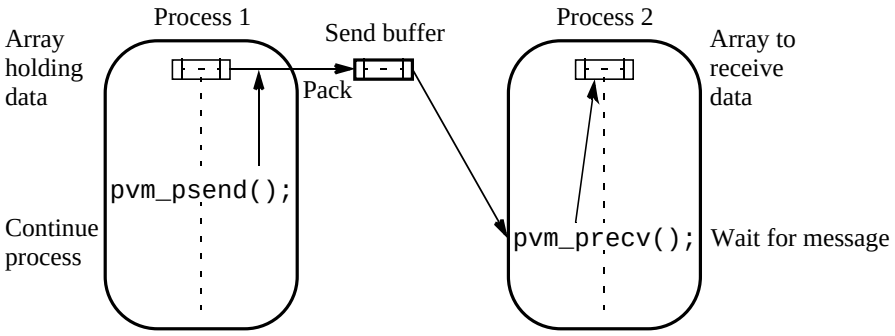


Figure 2.12 `pvm_psend()` and `pvm_precv()` system calls.

Full list of parameters for `pvm_psend()` and `pvm_precv()`:

```
pvm_psend(int dest_tid, int msgtag, char *buf, int len, int datatype)

pvm_precv(int source_tid, int msgtag, char *buf, int len, int datatype)
```

Sending Data Composed of Various Types

The data has to be packed into a PVM send buffer prior to sending the data. The receiving process must unpack its receive message buffer according to the format in which it was packed.

Specific packing and unpacking routines for each datatype

The basic message-passing routines for packed messages are `pvm_send()` (nonblocking), `pvm_recv()` (blocking), and `pvm_nrecv()` (nonblocking).

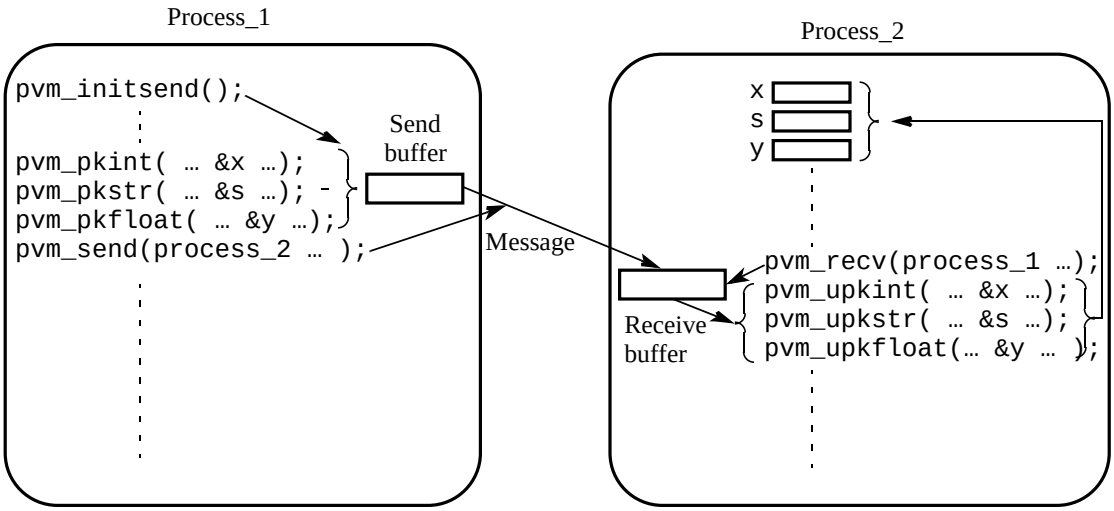


Figure 2.13 PVM packing messages, sending, and unpacking.

Broadcast, Multicast, Scatter, Gather, and Reduce

In PVM, broadcast, scatter, gather, and reduce operations (`pvm_bcast()`, `pvm_scatter()`, `pvm_gather()`, and `pvm_reduce()`, respectively) are used with a group of processes after the group is formed.

A process joins the named group by calling `pvm_joingroup()`.

The `pvm_bcast()`, when called, would send a message to each member of the named group.

Similarly, `pvm_gather()` would collect values from each member of the named group.

The PVM multicast operation, `pvm_mcast()`, is not a group operation. It is generally used to send the contents of the send buffer to each of a set of processes that are defined in a `task_ID` array (but not to itself even if it is named in the array).

Master

```
#include <stdio.h>
#include <stdlib.h>
#include <pvm3.h>
#define SLAVE “spsum”
#define PROC 10
#define NELEM 1000
main() {
  int mytid,tids[PROC];
  int n = NELEM, nproc = PROC;
  int no, i, who, msgtype;
  int data[NELEM],result[PROC],tot=0;
  char fn[255];
  FILE *fp;
  mytid=pvm_mytid();/*Enroll in PVM */

/* Start Slave Tasks */
  no=
   pvm_spawn(SLAVE,(char**)0,0,“”,nproc,tids);
  if (no < nproc) {
     printf(“Trouble spawning slaves \n”);
     for (i=0; i<no; i++) pvm_kill(tids[i]);
     pvm_exit(); exit(1);
  }

/* Open Input File and Initialize Data */
  strcpy(fn,getenv(“HOME”));
  strcat(fn,”/pvm3/src/rand_data.txt”);
  if ((fp = fopen(fn,”r”)) == NULL) {
     printf(“Can’t open input file %s\n”,fn);
     exit(1);
  }
  for(i=0;i<n;i++)fscanf(fp,”%d”,&data[i]);

/* Broadcast data To slaves*/
  pvm_initsend(PvmDataDefault);
  msgtype = 0;
  pvm_pkint(&nproc, 1, 1);
  pvm_pkint(tids, nproc, 1);
  pvm_pkint(&n, 1, 1);
  pvm_pkint(data, n, 1);
  pvm_mcast(tids, nproc, msgtag);


/* Get results from Slaves*/
  msgtype = 5;
  for (i=0; i<nproc; i++){
     pvm_recv(-1, msgtype);
     pvm_upkint(&who, 1, 1);
     pvm_upkint(&result[who], 1, 1);
     printf(“%d from %d\n”,result[who],who);
  }

/* Compute global sum */
  for (i=0; i<nproc; i++) tot += result[i];
  printf (“The total is %d.\n\n”, tot);

 pvm_exit(); /* Program finished. Exit PVM */
 return(0);
```

Broadcast data

Receive results

Slave

```
#include <stdio.h>
#include “pvm3.h”
#define PROC 10
#define NELEM 1000

main() {
   int mytid;
   int tids[PROC];
   int n, me, i, msgtype;
   int x, nproc, master;
   int data[NELEM], sum;

   mytid = pvm_mytid();

/* Receive data from master */
   msgtype = 0;
   pvm_recv(-1, msgtype);
   pvm_upkint(&nproc, 1, 1);
   pvm_upkint(tids, nproc, 1);
   pvm_upkint(&n, 1, 1);
   pvm_upkint(data, n, 1);

/* Determine my tid */
   for (i=0; i<nproc; i++)
      if(mytid==tids[i])
         {me = i;break;}

/* Add my portion Of data */
   x = n/nproc;
   low = me * x;
   high = low + x;
   for(i = low; i < high; i++)
      sum += data[i];

/* Send result to master */
   pvm_initsend(PvmDataDefault);
   pvm_pkint(&me, 1, 1);
   pvm_pkint(&sum, 1, 1);
   msgtype = 5;
   master = pvm_parent();
   pvm_send(master, msgtype);

/* Exit PVM */
   pvm_exit();
   return(0);
}
```

Figure 2.14 Sample PVM program.

MPI

MPI is a “standard” that has implementations. MPI has a large number of routines (over 120 and growing)

Process Creation and Execution

Creating and starting MPI processes is purposely not defined in the MPI standard and will depend upon the implementation.

A significant difference from PVM is that only static process creation is supported in MPI version 1. This means that all the processes must be defined prior to execution and started together. Use the SPMD model of computation.

Communicators

Defines the *scope* of a communication operation.

Processes have ranks associated with the communicator.

Initially, all processes are enrolled in a “universe” called `MPI_COMM_WORLD`, and each process is given a unique rank, a number from 0 to $n - 1$, where there are n processes.

Other communicators can be established for groups of processes.

Using the SPMD Computational Model

To facilitate this within a single program, statements need to be inserted to select which portions of the code will be executed by each processor.

Hence, the SPMD model does not preclude a master-slave approach, just that both the master code and the slave code must be in the same program:

```
main (int argc, char *argv[])
{
   MPI_Init(&argc, &argv);
   .
   .
   MPI_Comm_rank(MPI_COMM_WORLD, &myrank);    /* find process rank */
   if (myrank == 0)
      master();
   else
      slave();
   .
   .
   MPI_Finalize();
}
```

where `master()` and `slave()` are procedures to be executed by the master process and slave process, respectively.

Global and Local Variables

Any global declarations of variables will be duplicated in each process.

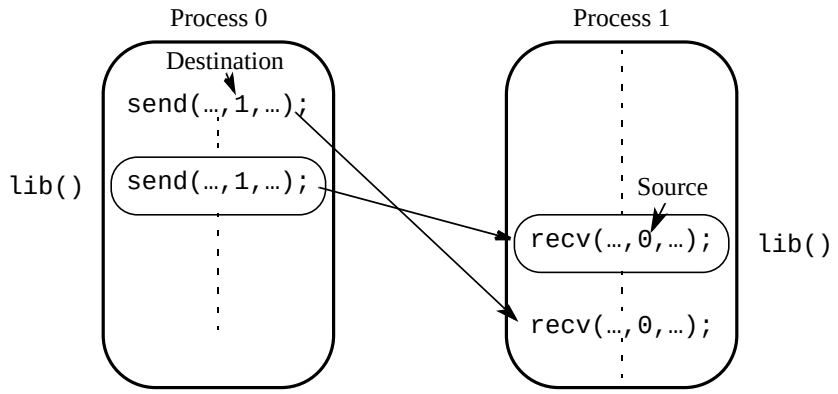
Variables that are not to be duplicated will need to be declared within code only executed by that process.

For example,

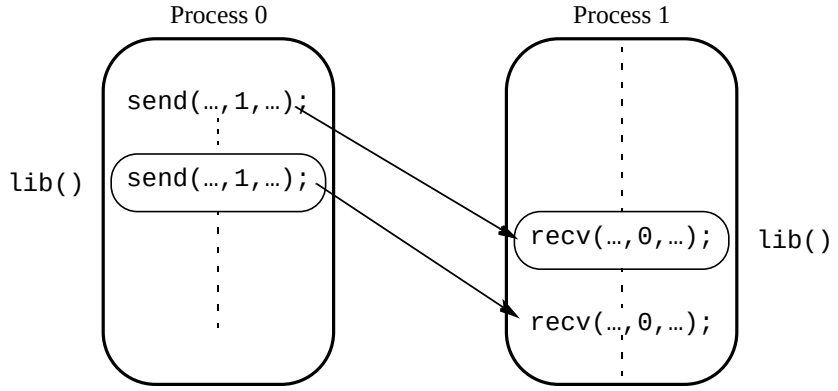
```
MPI_Comm_rank(MPI_COMM_WORLD, &myrank);/* find process rank */
if (myrank == 0) {            /* process 0 actions/local variables */
   int x, y;
      .
      .
} else if (myrank == 1) {     /* process 1 actions/local variables */
   int x, y;
      .
      .
}
```

Here, `x` and `y` in process 0 are different local variables from `x` and `y` in process 1.

Unsafe Communication Environment



(a) Intended behavior



(b) Possible behavior

Figure 2.15 Unsafe message passing with libraries.

In this figure, process 0 wishes to send a message to process 1, but there is also message passing between library routines as shown.

Even though each `send`/`recv` pair has matching source and destination, incorrect message passing occurs.

The use of wild cards makes incorrect operation or deadlock even more likely.

Suppose that in one process a nonblocking receive has wild cards in both the tag and source fields. A pair of other processes call library routines that require message passing.

The first send in this library routine may match with the non-blocking receive that is using wild cards, causing erroneous actions.

Solution

Communicators - used in MPI for all point-to-point and collective MPI message-passing communications.

A communicator is a *communication domain* that defines a set of processes that are allowed to communicate between themselves.

In this way, the communication domain of the library can be separated from that of a user program.

Each process has a rank within the communicator, an integer from 0 to $n - 1$, where there are n processes.

Communicator Types

Intracommunicator - for communicating within a group

Intercommunicator - for communication between groups.

A *group* is used to define a collection of processes for these purposes. A process has a unique *rank* in a group (an integer from 0 to $m - 1$, where there are m processes in the group).

A process could be a member of more than one group.

Default intracommunicator - `MPI_COMM_WORLD`, exists as the first communicator for all the processes existing in the application.

New communicators are created based upon existing communicators. A set of MPI routines exists for forming communicators from existing communicators (and groups from existing groups).

Point-to-Point Communication

Message tags are present, and wild cards can be used in place of the tag (`MPI_ANY_TAG`) and in place of the source in receive routines (`MPI_ANY_SOURCE`).

PVM style packing and unpacking data is generally avoided by the use of an MPI datatype being defined in the send/receive parameters together with the source or destination of the message.

Blocking Routines

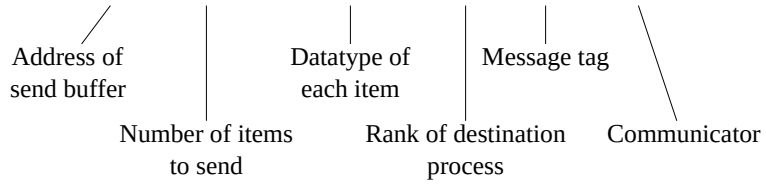
Return when they are locally complete - when the location used to hold the message can be used again or altered without affecting the message being sent.

A blocking send will send the message and return. This does not mean that the message has been received, just that the process is free to move on without adversely affecting the message.

A blocking receive routine will also return when it is locally complete, which in this case means that the message has been received into the destination location and the destination location can be read.

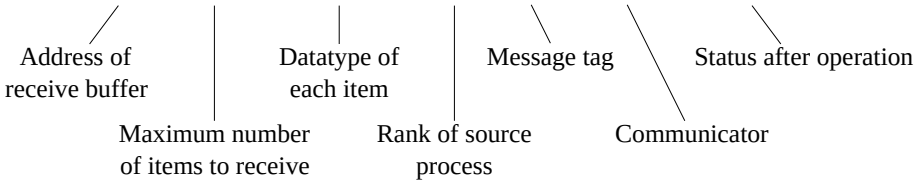
The general format of parameters of the blocking send is

```
MPI_Send(buf, count, datatype, dest, tag, comm)
```



The general format of parameters of the blocking receive is

```
MPI_Recv(buf, count, datatype, src, tag, comm, status)
```



Example

To send an integer *x* from process 0 to process 1,

```
MPI_Comm_rank(MPI_COMM_WORLD, &myrank);      /* find process rank */
if (myrank == 0) {
   int x;
   MPI_Send(&x, 1, MPI_INT, 1, msgtag, MPI_COMM_WORLD);
} else if (myrank == 1) {
   int x;
   MPI_Recv(&x, 1, MPI_INT, 0, msgtag, MPI_COMM_WORLD, status);
}
```

Nonblocking Routines

A nonblocking routine returns immediately; that is, allows the next statement to execute, whether or not the routine is locally complete.

Nonblocking send - `MPI_Isend()`, where `I` refers to the word *immediate*, will return even before the source location is safe to be altered.

Nonblocking receive - `MPI_Irecv()`, will return even if there is no message to accept.

Formats

```
MPI_Isend(buf, count, datatype, dest, tag, comm, request)
MPI_Irecv(buf, count, datatype, source, tag, comm, request)
```

Completion can be detected by separate routines, `MPI_Wait()` and `MPI_Test()`.

`MPI_Wait()` waits until the operation has actually completed and will return then.

`MPI_Test()` returns immediately with a flag set indicating whether the operation has completed at that time.

These routines need to know whether the particular operation has completed, which is determined by accessing the `request` parameter.

Example

To send an integer `x` from process 0 to process 1 and allow process 0 to continue,

```
MPI_Comm_rank(MPI_COMM_WORLD, &myrank);      /* find process rank */
if (myrank == 0) {
   int x;
   MPI_Isend(&x, 1, MPI_INT, 1, msgtag, MPI_COMM_WORLD, req1);
   compute();
   MPI_Wait(req1, status);
} else if (myrank == 1) {
   int x;
   MPI_Recv(&x, 0, MPI_INT, 1, msgtag, MPI_COMM_WORLD, status);
}
```

Send Communication Modes

Four communication modes that define the send/receive protocol.

Standard Mode Send,

It is not assumed that the corresponding receive routine has started. The amount of buffering, if any, is implementation dependent and not defined by MPI.

If buffering is provided, the send could complete before the receive is reached.

Buffered Mode

Send may start and return before a matching receive. It is necessary to provide specific buffer space in the application for this mode.

Buffer space is supplied to the system via the MPI routine `MPI_Buffer_attach()` and removed with `MPI_Buffer_detach()`.

Synchronous Mode

Send and receive can start before each other but can only complete together.

Ready Mode

Send can only start if the matching receive has already been reached, otherwise an error will occur. The ready mode must be used with care to avoid erroneous operation.

Each of the four modes can be applied to both blocking and nonblocking send routines.

Only the standard mode is available for the blocking and nonblocking receive routines.

Any type of send routine can be used with any type of receive routine.

Collective Communication

Involves a set of processes.

The processes are those defined by an intra-communicator.

Message tags are not present.

Broadcast and Scatter Routines

The principal collective operations operating upon data are

```
MPI_Bcast()         - Broadcast from root to all other processes
MPI_Gather()        - Gather values for group of processes
MPI_Scatter()       - Scatters buffer in parts to group of processes
MPI_Alltoall()      - Sends data from all processes to all processes
MPI_Reduce()        - Combine values on all processes to single value
MPI_Reduce_scatter()- Combine values and scatter results
MPI_Scan()          - Compute prefix reductions of data on processes
```

Example

To gather items from the group of processes into process 0, using dynamically allocated memory in the root process, we might use

```
int data[10];                    /*data to be gathered from processes*/
     .
MPI_Comm_rank(MPI_COMM_WORLD, &myrank);          /* find rank */
if (myrank == 0) {
   MPI_Comm_size(MPI_COMM_WORLD, &grp_size);      /*find group size*/
   buf = (int *)malloc(grp_size*10*sizeof(int));  /*allocate memory*/
}
MPI_Gather(data,10,MPI_INT,buf,grp_size*10,MPI_INT,0, MPI_COMM_WORLD);
```

Note that `MPI_Gather()` gathers from all processes, including the root.

Barrier

As in all message-passing systems, MPI provides a means of synchronizing processes by stopping each one until they all have reached a specific “barrier” call.

```
#include “mpi.h”
#include <stdio.h>
#include <math.h>
#define MAXSIZE 1000

void main(int argc, char *argv)
{
   int myid, numprocs;
   int data[MAXSIZE], i, x, low, high, myresult, result;
   char fn[255];
   char *fp;

   MPI_Init(&argc,&argv);
   MPI_Comm_size(MPI_COMM_WORLD,&numprocs);
   MPI_Comm_rank(MPI_COMM_WORLD,&myid);

   if (myid == 0) {           /* Open input file and initialize data */
      strcpy(fn,getenv(“HOME”));
      strcat(fn,”/MPI/rand_data.txt”);
      if ((fp = fopen(fn,”r”)) == NULL) {
         printf(“Can’t open the input file: %s\n\n”, fn);
         exit(1);
      }
      for(i = 0; i < MAXSIZE; i++) fscanf(fp,”%d”, &data[i]);
   }

   /* broadcast data */
   MPI_Bcast(data, MAXSIZE, MPI_INT, 0, MPI_COMM_WORLD);

/* Add my portion Of data */
   x = n/nproc;
   low = myid * x;
   high = low + x;
   for(i = low; i < high; i++)
      myresult += data[i];
   printf(“I got %d from %d\n”, myresult, myid);

/* Compute global sum */
   MPI_Reduce(&myresult, &result, 1, MPI_INT, MPI_SUM, 0, MPI_COMM_WORLD);
   if (myid == 0) printf(“The sum is %d.\n”, result);

   MPI_Finalize();
}
```

Figure 2.16 Sample MPI program.

Pseudocode Constructs

We shall use a pseudocode for describing algorithms. Our pseudocode will omit the clutter of parameters that are secondary to understanding the operation.

To send the message consisting of an integer `x` and a float `y`, from the process called `master` to the process called `slave`, assigning to `a` and `b`, we simply write in the master process

```
send(&x, &y, P_slave);
```

and in the slave process

```
recv(&a, &b, P_master);
```

where `x` and `a` are declared as integers and `y` and `b` are declared as floats. The integer `x` will be copied to `a`, and the float `y` copied to `b`.

Where appropriate, the *i*th process will be given the notation P_i, and a tag may be present that would follow the source or destination name; i.e.,

```
send(&x, P_2, data_tag);
```

sends `x` to process 2, with the message tag `data_tag`.

The most common form of basic message-passing routines needed in our pseudo- code is the locally blocking `send()` and `recv()`, which will be written as given.

In many instances, the locally blocking versions are sufficient.

Other forms will be differentiated with prefixes; i.e.,

```
ssend(&data1, P_destination);              /* Synchronous send */
```

Evaluating Parallel Programs

Parallel Execution Time

The parallel execution time, t_p, is composed of two parts: a computation part, say t_{comp}, and a communication part, say t_{comm}; i.e.,

$$t_p = t_{comp} + t_{comm}$$

The computation time can be estimated in a similar way to that of a sequential algorithm.

Communication Time

AS a first approximation, we will use

$$t_{comm} = t_{startup} + nt_{data}$$

where $t_{startup}$ is the *startup time*, sometimes called the *message latency.* This is essentially the time to send a message with no data. The startup time is assumed to be a constant.

The term t_{data} is the transmission time to send one data word, also assumed a constant, and there are n data words.

Equation is illustrated below:

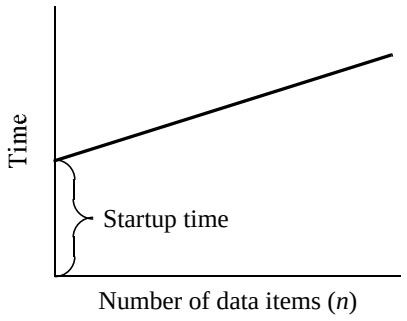


Figure 2.17 Theoretical communication time.

Important Note on Interpretation of Equations

Many assumptions in the analysis (see textbook). Only intended to give a starting point to how an algorithm might perform in practice.

The parallel execution time, t_p, will be normalized to be measured in units of an arithmetic operation, which of course will depend upon the computer system.

We might find that the computation requires m computational steps so that

$$t_{comp} = m$$

Since we are measuring time in units of computational steps, the communication time has to be measured in the same way.

We will not differentiate between sending an integer and sending a real number, or other formats. All are assumed to require the same time.

Suppose q messages are sent, each containing n data items. We have

$$t_{comm} = q(t_{startup} + nt_{data})$$

Both the startup and data transmission times, $t_{startup}$ and t_{data}, are measured in computational steps, so that we can add t_{comp} and t_{comm} together to obtain the parallel execution time, t_p.

Latency Hiding

A way to ameliorate the situation of significant message communication times is to overlap the communication with subsequent computations.

The nonblocking send routines are provided particularly to enable latency hiding.

Latency hiding can also be achieved by mapping multiple processes on a processor and use a time-sharing facility that switches for one process to another when the first process is stalled because of incomplete message passing or otherwise.

Relies upon an efficient method of switching from one process to another. *Threads* offer an efficient mechanism.

Time Complexity

As with sequential computations, a parallel algorithm can be evaluated through the use of time complexity (notably the O notation — "order of magnitude," big-oh).

Start with an estimate of the number of the computational steps, considering all arithmetic and logical operations to be equal and ignoring other aspects of the computation such as computational tests.

An expression of the number of computational steps is derived, often in terms of the number of data items being handled by the algorithm.

Example

Suppose an algorithm, A1, requires $4x^2 + 2x + 12$ computational steps for x data items.

As we increase the number of data items, the total number of operations will depend more and more upon the term $4x^2$. The first term will "dominate" the other terms, and eventually the other terms will be insignificant. The growth of the function in this example is *polynomial*.

Another algorithm, A2, for the same problem might require $5 \log x + 200$ computational steps.In the function $5 \log x + 200$, the first term will eventually dominate the other term, which can be ignored, and we only need to compare the dominating terms. The growth of function $\log x$ is *logarithmic*.

For a sufficiently large x, logarithmic growth will be less than polynomial growth.

We can capture growth patterns in the O notation (big-oh). Algorithm A1 has a big-oh of $O(x^2)$. Algorithm A2 has a big-oh of $O(\log x)$.

Formal Definition

The O notation

$f(x) = O(g(x))$ if and only if there exists positive constants, c and x_0, such that $0 \leq f(x) \leq cg(x)$ for all $x \geq x_0$

where $f(x)$ and $g(x)$ are functions of x.

For example, if $f(x) = 4x^2 + 2x + 12$, the constant $c = 6$ would work with the formal definition to establish that $f(x) = O(x^2)$, since $0 < 4x^2 + 2x + 12 \leq 6x^2$ for $x \geq 3$.

Unfortunately, the formal definition also leads to alternative functions for $g(x)$ that will also satisfy the definition. Normally, we would use the function that grows the least for $g(x)$.

Θ notation - upper bound

$f(x) = \Theta(g(x))$ if and only if there exists positive constants c_1, c_2, and x_0 such that $0 \leq c_1 g(x) \leq f(x) \leq c_2 g(x)$ for all $x \geq x_0$.

If $f(x) = \Theta(g(x))$, it is clear that $f(x) = O(g(x))$ is also true.

Ω notation - lower bound

$f(x) = \Omega(g(x))$ if and only if there exists positive constants c and x_0 such that $0 \leq cg(x) \leq f(x)$ for all $x \geq x_0$.

It follows from this definition that $f(x) = 4x^2 + 2x + 12 = \Omega(x^2)$

We can read O() as "grows at most as fast as" and Ω() as "grows at least as fast as."

The Ω notation can be used to indicate the best case situation.

For example, the execution time of a sorting algorithm often depends upon the original order of the numbers to be sorted. It may be that it requires at least $n \log n$ steps, but could require n^2 steps for n numbers depending upon the order of the numbers. This would be indicated by a time complexity of $\Omega(n \log n)$ and $O(n^2)$.

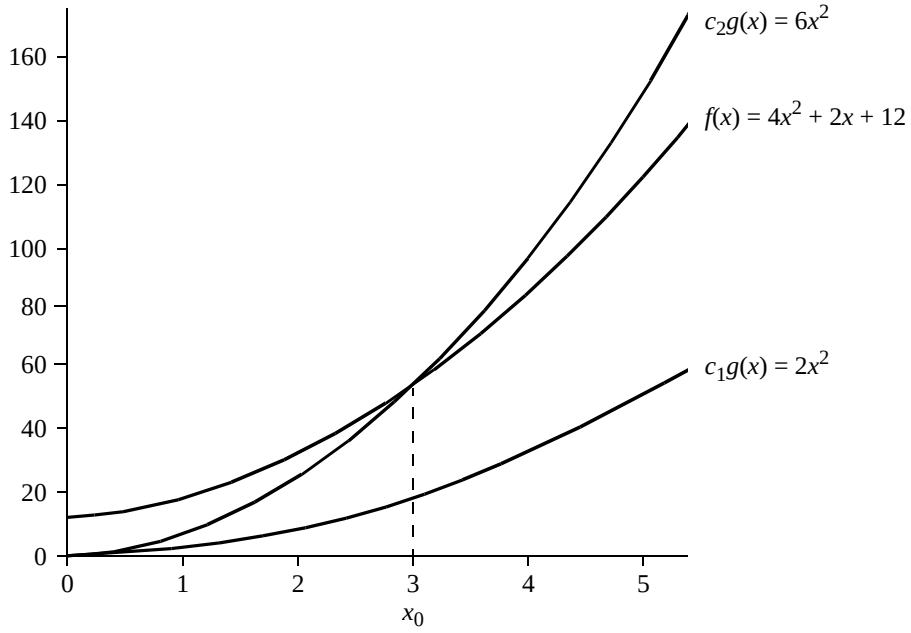


Figure 2.18 Growth of function $f(x) = 4x^2 + 2x + 12$.

Time Complexity of a Parallel Algorithm

I

If we use time complexity analysis, which hides lower terms, t_{comm} will have a time complexity of $O(n)$.

The time complexity of t_p will be the sum of the complexity of the computation and the communication.

Example

Suppose we were to add n numbers on two computers, where each computer adds $n/2$ numbers together, and the numbers are initially all held by the first computer. The second computer submits its result to the first computer for adding the two partial sums together. This problem has several phases:

1. Computer 1 sends $n/2$ numbers to computer 2.
2. Both computers add $n/2$ numbers simultaneously.
3. Computer 2 sends its partial result back to computer 1.
4. Computer 1 adds the partial sums to produce the final result.

As in most parallel algorithms, there is computation and communication, which we will generally consider separately:

Computation (for steps 2 and 4):

$$t_{comp} = n/2 + 1$$

Communication (for steps 1 and 3):

$$t_{comm} = (t_{startup} + n/2t_{data}) + (t_{startup} + t_{data}) = 2t_{startup} + (n/2 + 1)t_{data}$$

The computational complexity is $O(n)$. The communication complexity is $O(n)$. The overall time complexity is $O(n)$.

Cost-Optimal Algorithms

A*cost-optimal* (or *work-efficient* or *processor-time optimality*) algorithm is one in which the cost to solve a problem is proportional to the execution time on a single processor system (using the fastest known sequential algorithm); i.e.,

$$\text{Cost} = t_p \times n = k \times t_s$$

where k is a constant.

Given time complexity analysis, we can say that a parallel algorithm is cost-optimal algorithm if

(Parallel time complexity) × (number of processors) = sequential time complexity

Example

Suppose the best known sequential algorithm for a problem has time complexity of $O(n \log n)$. A parallel algorithm for the same problem that uses n processes and has a time complexity of $O(\log n)$ is cost optimal, whereas a parallel algorithm that uses n^2 processors and has time complexity of $O(1)$ is not cost optimal.

Time Complexity of Broadcast/Gather

Broadcast on a Hypercube Network

Consider a three-dimensional hypercube.

To broadcast from node 000 to every other node, 001, 010, 011, 100, 101, 110 and 111, an efficient algorithm is

	Node		Node
1st step:	000	→	001
2nd step:	000	→	010
	001	→	011
3rd step:	000	→	100
	001	→	101
	010	→	110
	011	→	111

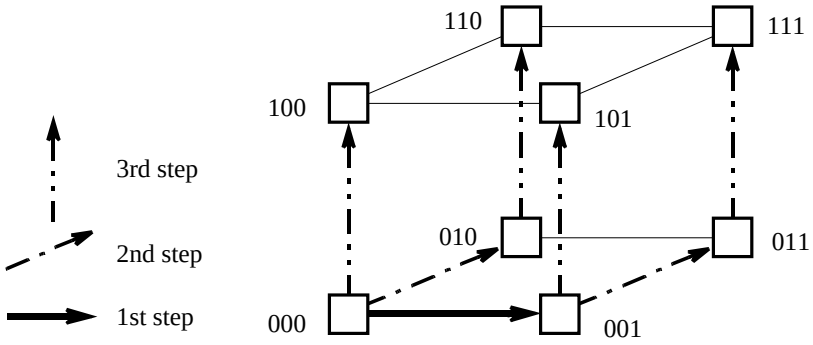


Figure 2.19 Broadcast in a three-dimensional hypercube.

The time complexity for a hypercube system will be O(log *n*), using this algorithm, which is optimal because the *diameter* of a hypercube network is log *n*. It is necessary at least to use this number of links in the broadcast to reach the furthest node.

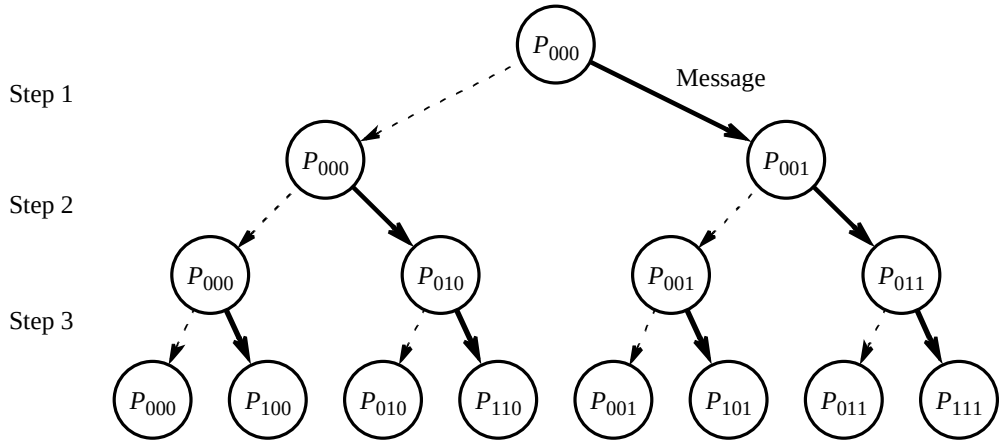


Figure 2.20 Broadcast as a tree construction.

Gather on a Hypercube Network

The reverse algorithm can be used to gather data from all nodes to, say, node 000; i.e., for a three-dimensional hypercube,

	Node		Node
1st step:	100	→	000
	101	→	001
	110	→	010
	111	→	011
2nd step:	010	→	000
	011	→	001
3rd step:	001	→	000

In the case of gather, the messages become longer as the data is gathered, and hence the time complexity is increased over $O(\log n)$.

Broadcast on a Mesh Network

Send message across the top row and down each column as the message reaches the top node of that column.

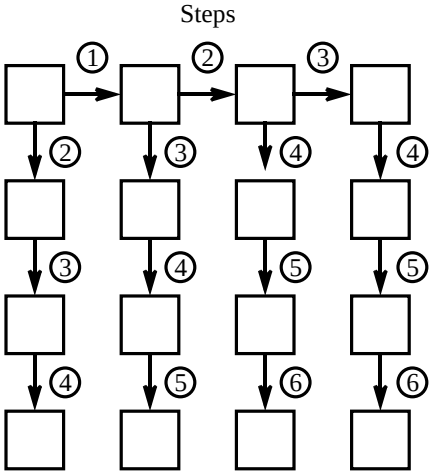


Figure 2.21 Broadcast in a mesh.

Requires $2(n - 1)$ steps or $O(n)$ on an $n \times n$ mesh, again an optimal algorithm in terms of number of steps because the diameter of a mesh without wraparound is given by $2(n - 1$

Broadcast on a Workstation Cluster

Broadcast on a single Ethernet connection can be done using a single message that is read by all the destinations on the network simultaneously

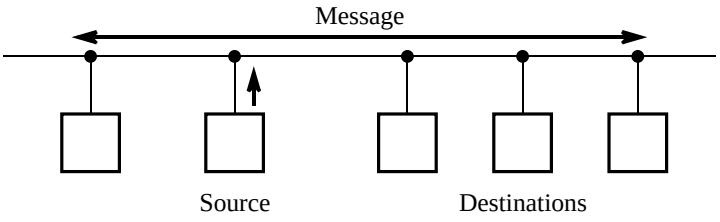


Figure 2.22 Broadcast on an Ethernet network.

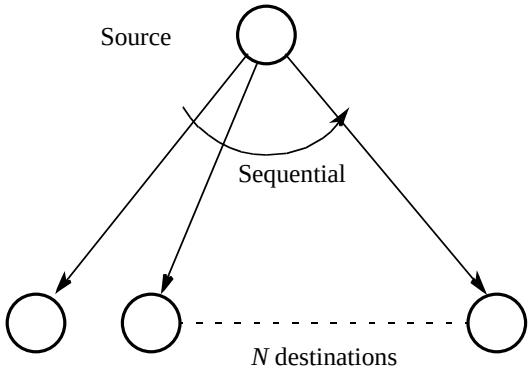


Figure 2.23 1-to-*N* fan-out broadcast.

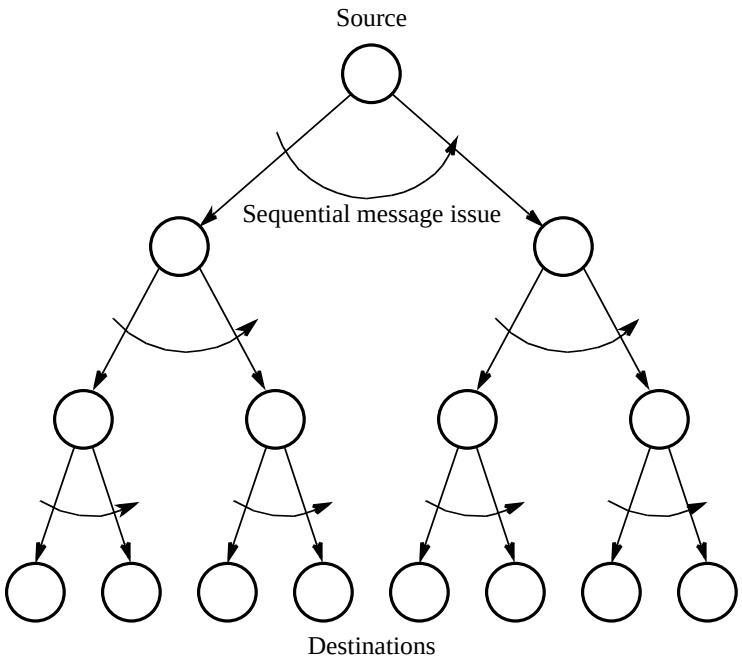


Figure 2.24 1-to-*N* fan-out broadcast on a tree structure.

Debugging and Evaluating Parallel Programs

Writing a parallel program or, more accurately, getting a parallel program to work properly can be a significant intellectual challenge.

Visualization Tools

Programs can be watched as they are executed in a *space-time diagram* (or *process-time diagram*):

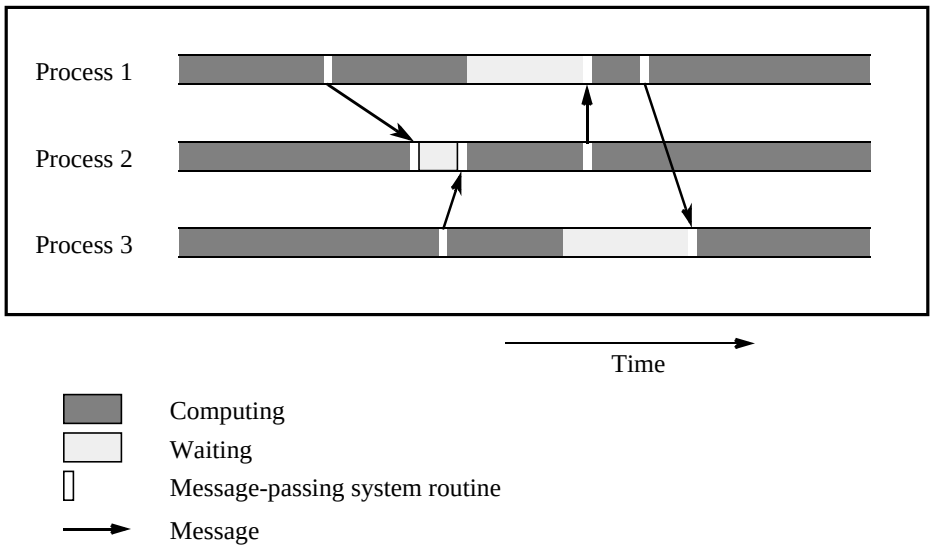


Figure 2.25 Space-time diagram of a parallel program.

PVM has a visualization tool called XPVM.

Implementations of visualization tools are available for MPI. An example is the Upshot program visualization system.

Debugging Strategies

Geist et al. (1994a) suggest a three-step approach to debugging message-passing programs:

1. If possible, run the program as a single process and debug as a normal sequential program.
2. Execute the program using two to four multitasked processes on a single computer. Now examine actions such as checking that messages are indeed being sent to the correct places. It is very common to make mistakes with message tags and have messages sent to the wrong places.
3. Execute the program using the same two to four processes but now across several computers. This step helps find problems that are caused by network delays related to synchronization and timing.

Evaluating Programs Empirically

Measuring Execution Time

To measure the execution time between point `L1` and point `L2` in the code, we might have a construction such as

```
        .
L1:  time(&t1);                         /* start timer */
        .
        .
L2:  time(&t2);                         /* stop timer */
        .
     elapsed_time = difftime(t2, t1);   /* elapsed_time = t2 - t1 */
     printf("Elapsed time = %5.2f seconds", elapsed_time);
```

MPI provides the routine `MPI_Wtime()` for returning time (in seconds).

Communication Time by the Ping-Pong Method

One process, say P_0, is made to send a message to another process, say P_1. Immediately upon receiving the message, P_1 sends the message back to P_0.

P_0

```
        .
L1:  time(&t1);
     send(&x, P1);
     recv(&x, P1);
L2:  time(&t2);
     elapsed_time = 0.5 * difftime(t2, t1);
     printf("Elapsed time = %5.2f seconds", elapsed_time);
        .
```

P_1

```
        .
     recv(&x, P0);
     send(&x, P0);
        .
```

Profiling

A *profile* of a program is a histogram or graph showing the time spent on different parts of the program:

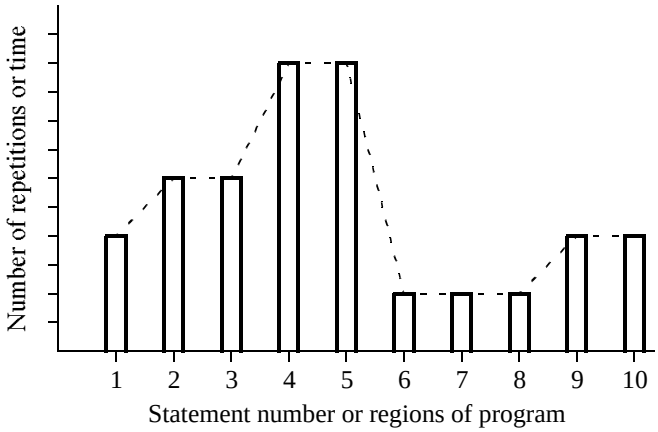


Figure 2.26 Program profile.