Final report: 'Analysis and Mechanisation of Decidable First-Order Temporal Logics'

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1 Background

First-order temporal logic (FOTL) has long been regarded by many as a perfect formalism for program specification and verification, temporal databases, synthesis of programs, model checking, temporal knowledge representation and reasoning, etc. The fatal problem was that mechanisation seemed out of the question, because only 'negative' results (undecidability, non-recursive enumerability) were known. The starting point of this project was the discovery in [HWZ00] of *decidable and yet rather expressive* **'monodic'** fragments of FOTL, which opened new and exciting opportunities for using FOTL in various areas of computer science and artificial intelligence.

The project was jointly undertaken by the University of Liverpool, King's College London, and Imperial College London. Its principal objectives were

1. to develop practical proof algorithms, based on decidable monodic FOTL and its extensions, using both tableau and resolution techniques;
2. to carry out a detailed analysis of logical and computational properties of monodic FOTL;
3. to extend current results concerning axiomatisable and decidable classes beyond the monodic case;
4. to implement some of the tableau/resolution systems developed in (1); and
5. to evaluate the systems developed, and to apply them to a range of verification case studies.

Although we envisaged considerable collaboration between sites, the London side was to be responsible for developing tableaux in (1), as well as (2) and (3), and also planned to contribute to (5) by applying the one-variable fragment of FOTL to spatio-temporal representation and reasoning. The team in Liverpool, while being involved with much of the theoretical work on the project, were to be particularly concerned with the development and implementation of provers for monodic first-order temporal logics, and their application to the problem of verification, in particular of security protocols and infinite-state systems.

2 Achievement

All the objectives were achieved. The work done on each of them is outlined below, following the workpackage structure from the original project proposal.

WP1: Tableau algorithms for monodic FOTL

The aim of WP1 was to provide general criteria under which a tableau decision procedure for a fragment of first-order logic can be combined in a *modular way* with standard tableau systems for propositional temporal logic given in [Wol85] in order to construct tableau-based decision procedures for decidable fragments of monodic FOTL. A general framework for such combinations (with both expanding and constant domains) over the natural numbers flow of time was developed in [KLWZ04]. Our approach was based on the following ideas:

- finite 'quasimodel' representations of temporal models with potentially infinite first-order domains developed in [WZ98, WZ00c, HWZ00]—elements indistinguishable by subformulas (of a given formula) with at most one free variable are represented by types;

- the minimal types technique of [LSWZ02, LSWZ01] for dealing with constant domains in temporal models.

[KLWZ04] presents a number of instantiations of the proposed framework: e.g., tableau decision algorithms for the one-variable fragment of FOTL and the monodic temporalisations of modal logic $\mathbf{S4}_u$ (used for spatial representation and reasoning; see WP6). The success in designing modular tableau procedures for monodic FOTL led to the idea of using a similar approach to some other kinds of temporal representation and reasoning, in particular, for modelling complex systems evolving over time in a modular fashion. In [End03, EG03], a temporal logic was constructed that can be regarded as the result of extending propositional linear temporal logic by a second dimension that allows us to 'zoom' into states and thereby to further refine the specification of events associated with these states. In a sense, this logic combines features from both point-based temporal logics and modal interval logics. We gave both a semantic and an axiomatic characterisation of the logic, investigated its expressive power, and proved its decidability.

Deliverables: [AFWZ02, End03, EG03, KLWZ04, LSWZ02, LSWZ01]

WP2: Clausal resolution for monodic FOTL

As an alternative to tableau, we have extended and adapted the clausal temporal resolution method [FDP01] for use with monodic FOTL. The basic clausal resolution technique has been extended [DFK02, DFK03b, DFK04] and refined for practical use. Work has also been carried out extending the applicability of the resolution method beyond the straightforward monodic fragment, for example to other decidable classes and classes involving equality [DFK03a].

In [KDD$^+$03, KDD$^+$05], we focused on an important subclass of temporal models with a wide range of applications, for example in spatio-temporal logics [WZ02b, GKK$^+$03] and temporal description logics [AFWZ02] — namely, models with *expanding domains*. In such models, the domains over which first-order terms range can increase at each temporal step. The focus on this class of models allowed us to produce a simplified clausal resolution calculus, termed the *fine-grained* resolution calculus, which is more amenable to efficient implementation [KDD$^+$03, KDD$^+$05].

Deliverables: [DFK02, DFK03b, DFK03a, KDD$^+$03, KDD$^+$05, DFK04]

WP3: Computational properties of monodic FOTL

The main aims of WP3 were to determine the computational complexity of decidable monodic fragments, investigate monodic fragments based on alternative models of time (in particular, branching flows) and on decidable first-order fragments defined in terms of the quantifier structure (in particular, Maslov's class), and to axiomatise the monodic fragments over various linear flows of time.

Computational complexity. The computational complexity of monodic fragments of FOTL was investigated in [HKK$^+$03, Hod04, GKWZ03]. In [HKK$^+$03], we showed that over a wide range of flows of time, the one-variable fragment—even with sole temporal operator 'sometime in the future'—is EXPSPACE-hard (this solves some open problems of [HV89] and [Rey97]). We also established matching EXPSPACE upper bounds for the one-variable, two-variable and monadic monodic fragments over the natural numbers flow of time with the 'next-time', 'until' and 'since' temporal operators. These fragments are EXPSPACE-complete even if they are interpreted in models with finite first-order domains. The packed (as well as guarded and loosely guarded) monodic fragment turns out to be as complex as its pure first-order part—2EXPTIME-complete. The cases of other linear flows of time, e.g. arbitrary linear orders and rational numbers time as well as the reals (the latter with finite first-order domains only), were considered in [Hod04], where it was proved that all the guarded monodic fragments are 2EXPTIME-complete. A proof that the one-variable, two-variable and monadic monodic fragments are decidable in 2EXPTIME was also outlined. The proofs use quasimodels and mosaic-based work of Reynolds on complexity of propositional temporal logic with 'until' and 'since' over linear and real time [Rey03].

Branching time. Monodic first-order temporal logics over branching time were investigated in [HWZ02, BHWZ02, BHWZ04]. In [HWZ02], we analysed the decision problem for fragments of first-order extensions of branching-time temporal logics such as computational tree logics CTL and CTL^* or Prior's Okhamist logic of historical necessity. To our (and other temporal logicians') surprise, it turned out that the one-variable fragments of logics like first-order CTL^* are undecidable (both 'bundled' and 'unbundled' versions, and even with sole temporal operator 'now or sometime in the future'). The proof used algebraic logic. On the other hand, it was proved that decidable fragments can be obtained by restricting applications of first-order quantifiers to state (i.e., path-independent) formulas, and restricting applications of temporal operators and path quantifiers to formulas with at most one free variable (as in monodic FOTL). The same arguments show decidability of 'non-local' propositional CTL^*, in which truth values of propositional atoms depend on the history as well as the current time. [BHWZ02] and [BHWZ04] continued this investigation and showed decidability of other kinds of fragments of first-order CTL^*: the so-called weak one-variable and weak monodic fragments, where quantifiers are not restricted to state formulas, but the next-time operator may only be applied to formulas with at most one free variable and all other temporal operators and path quantifiers are applicable only to sentences. Very complex quasimodels are pivotal in the decidability proofs.

Gödel's and Maslov's classes. In [DFK04, HKS05], we applied monodic temporal resolution to establish decidability of monodic fragments: in particular, those defined by imposing restrictions on quantifier alternation and interaction between quantifiers and temporal operators within formulas. We treated two special classes: the *temporalised Gödel class* and the *temporalised Maslov class* (based on classical first-order Gödel and Maslov classes, respectively [BGG97]).

Axiomatisation. In [GKWZ03, WZ02a], we constructed Hilbert-style axiomatic systems for a number of monodic fragments—in particular, full monodic FOTL over the natural numbers flow of time, and the one-variable fragment.

Deliverables: [BHWZ02, BHWZ04, DFK03b, DFK04, GKWZ03, HKS05, HWZ02, HKK$^+$03, Hod04, WZ02a].

WP4: Resolution and tableaux implementation

Liverpool developed and implemented practical algorithms for monodic temporal reasoning. While there was work on the implementation of monodic tableau [Gue05], the main effort concerned resolution-based algorithms for monodic FOTL. We have implemented two resolution-based theorem provers: TRP++ is a resolution based theorem prover for propositional linear-time temporal logics [HK03]; and TeMP is a monodic first order theorem prover [HKRV04]. TRP++ has been shown to perform well in comparison with other implemented decision procedures for this logic while TeMP is being used for practical verification (see WP6). As described in WP2 above, the foundation for these methods was given in [KDD$^+$03, KDD$^+$05], thus providing the first practical tool for handling monodic FOTL.

Deliverables: [HK03, KDD$^+$03, HKRV04, KDD$^+$05]

Software: `http://www.csc.liv.ac.uk/~konev/TeMP`

WP5: Beyond monodic FOTL

The main aim of WP5 was to investigate the possibility of extending monodic FOTL by adding equality, and by relaxing the monodicity requirement—i.e., by allowing, in certain cases, applications of temporal operators to formulas with more than one free variable.

Adding equality. It was shown in [DFL02] that adding equality to the two-variable monodic FOTL results in a non-recursively enumerable logic. However, [Hod02] showed that the packed and guarded monodic fragments are decidable even with equality—2EXPTIME-complete, to be more precise [Hod04]. Moreover, it was proved in [DFK03a] that the resolution-based decision procedure developed in [KDD$^+$03] can be extended to handle equality in the guarded and loosely guarded monodic fragments. This was done by combining the constructions from [Hod02] and the superposition-based decision procedure for the guarded fragment with equality given in [GN99].

Relaxing monodicity. In [DFK04], we considered an extension of the monodic fragment that allows a 'local' *next-time* operator to be applied to formulas with more than one free variable. Such an extension is motivated by possible applications in transaction protocol verification and temporal databases where changes are often local: e.g., at the next moment of time. We showed that temporal resolution can be used to check satisfiability of formulas from this extension, by translating them (with only a linear growth in size) into monodic temporal problems.

Deliverables: [DFL02, DFK03a, DFK04, Hod02, Hod04].

WP6: Case studies and empirical analysis

The results we obtained for monodic FOTL were applied to analyse the computational behaviour of spatio-temporal logics obtained by combining standard propositional temporal logic (over different flows of time) with various spatial formalisms such as RCC-8, its extension BRCC-8 with Boolean region terms [WZ00a], or other logics embeddable in the bimodal logic $\mathbf{S4}_u$ interpreted in topological spaces.

The research was launched in [WZ00b], which introduced a hierarchy of spatio-temporal logics based on propositional temporal logic **PTL**, RCC-8, and BRCC-8. We extended the results in [BCWZ02] and proposed to consider the product $\mathbf{PTL} \times \mathbf{S4}_u$ as a unifying framework for the constructed hierarchy (all of its logics are indeed fragments of the product). However, as turned out later, the product logic is undecidable and thus gives us little 'positive' information about the computational behaviour of the hierarchy.

This 'negative' result stimulated an in-depth investigation of the complexity of spatio-temporal hybrids and led to [GKK$^+$03, GKK$^+$05], where we demonstrated how different ways of combining spatial and temporal formalisms give rise to spatio-temporal logics with computational complexity ranging from NP and PSPACE to EXPSPACE and 2EXPSPACE. In particular, we showed that some of the obtained logics can be regarded as fragments of one-variable FOTL. Although not all of them can, we still were able to use techniques developed for monodic FOTL (such as quasimodels and encoding of tiling problems) to prove decidability and establish complexity bounds for the spatio-temporal hybrids.

The temporal resolution tools developed in WP4 have also been applied to key case studies, in particular:

- **Security verification.** Here security problems are described in a temporal logic of knowledge, and this specification is translated to monodic FOTL. The TeMP resolution prover [HKRV04] is then used to verify correctness of the security protocol [DGFH04].

- **Program verification.** Abstract State Machines (ASMs) are a high-level specification notation for general programs. However, verifying properties of ASMs is notoriously difficult. In [FL04], we identified a fragment of ASM specifications, namly *monodic ASMs*, and translated the problem of verifying a monodic ASM system into the problem of analyzing a monodic FOTL problem.

- **Verifying infinite numbers of identical processes.** Automated verification has traditionally concerned finite-state problems. Recently, however, attention has turned to infinite state systems with decidable verification problems. A key example of such a class is that of arbitrary numbers of identical finite state machines, communicating via broadcast message-passing. In [FKL05], we showed how the TeMP prover could be used to automatically verify many verification problems in this area.

- **Artificial Intelligence.** Recent work has used temporal proof in knowledge games [Dix05] and in the analysis of robotic systems [WSG+05].

In [FHD+05], we provided a summary of our experiences in using clausal temporal resolution on such applications.

Deliverables: [BCWZ02, DGFH04, GKK+03, GKK+05, FKL05, HWZ01, FHD+05].

3 Project plan review

The original plan was to employ a highly qualified expert in the field, namely, Dr Ágnes Kurucz, as the project's London RA. However, by the time the project began, Kurucz had accepted a permanent position at King's College London. We were remarkably fortunate that Mr Roman Kontchakov, at that time a PhD student at Moscow State University doing very interesting research in temporal logic and its application, agreed to come to King's and work on this project. We were also fortunate in being able to employ Mr David Gabelaia, a postgraduate RA on another EPSRC project at King's, as a half time researcher, and Mr Ulrich Endriss joined the team for four months to investigate temporal logics of ordered trees. In the end, this 'calculated risk' proved to be extremely successful—witness the list of publications below, and the fact that all three RAs have now been awarded their PhD degrees. In spite of losing our planned RA at the outset, we were able to bring the project to a successful conclusion on time and within budget, and its contribution to training future researchers has been far greater than anticipated.

Dr Anatoli Degtyarev started as a co-investigator of the Liverpool team; he became a co-investigator of the London side after moving to King's in 2002. Prof. Frank Wolter was added as a co-investigator in 2003 and his PhD student, Christian Guensel, worked on developing tableaux for monodic FOTL.

Dr. Boris Konev began as RA at Liverpool but, once he took up a lecturing post in Liverpool, a new RA was enlisted. This involved a delay in the middle of the project for Liverpool (which explains why the Liverpool project lasted until April 2005). However, we were fortunate to be able to employ Dr. Mari-Carmen Fernandez Gago for the last 15 months of the project.

4 Collaboration

As well as collaboration between London and Liverpool, members of the project have collaborated closely on aspects related to the project with: Alessandro Artale and Enrico Franconi (Italy), Carsten Lutz and Holger Sturm (Germany), Brandon Bennett and Tony Cohn (Leeds, UK), Ágnes Kurucz (King's) and other members of the London Logic Forum, and Mark Reynolds (Australia). The papers [HKK+03, Hod04] were also announced in the final report of EPSRC grant GR/S19905/01.

5 Dissemination

Publications:

- Book (research monograph) [GKWZ03];

- 13 journal articles [BHWZ04], [BCWZ02], [DFL02], [DFK04], [Dix05], [FHD+05], [GKK+05], [Hod02], [Hod04], [KDD+05], [KLWZ04], [LSWZ02], [WZ02a];

- 20 chapters in books and conference proceedings [AFWZ02], [BHWZ02], [DFK02], [DFK03a], [DFK03b], [DGFH04], [EG03], [FKL05], [GKK+03], [HKS05], [HKK+03], [HWZ01], [HWZ02], [HK03], [HKRV04] [KDD+03], [KZ03], [LSWZ01], [WSG+05], [WZ02b]

- 4 PhD theses [End03], [Kon04] [Gue05] (plus Gabelaia).

- Software: TeMP prover, see `http://www.csc.liv.ac.uk/~konev/TeMP`

Conferences and workshops: As well as our journal and conference publications, we have attended and participated in many major international conferences and workshops: IJCAI 2001, 2003, IJCAR 2001, LICS 2002, KR 2002, JELIA 2002, TIME 2002, ESSLLI 2002, LPAR 2003, FLAIRS 2003, TIME-ICTL 2003, CADE 2003, Discrete Models in Control Systems Theory V (Moscow, 2003), TIME 2004, 4th European Congress of Mathematics, 2004. We have taken part in the following UK conferences and workshops: ARW'02, BCTSC 18 (Bristol), COST274/TARSKI-Workshop 2003 (Leeds), BCTCS 19 (Leicester) BLC 2003 (St. Andrews, Scotland), ARW'03, BCTCS 2004 (Pitlochry, Scotland), ARW04, VISSAS'05.

6 Conclusion

This project laid the theoretical foundations for the use of decidable monodic first-order temporal logics. We conducted a comprehensive analysis of the computational behaviour of such logics, determined their complexity, and developed implementable tableau-based and resolution-based decision procedures. We investigated possible extensions of monodic fragments by adding equality and by relaxing the monodicity condition, and discovered new decidable fragments of FOTL with complex quantifier structures.

Having started with a promising idea of monodicity and some encouraging results, we finish the project with a fully-fledged theory which provides a solid basis for both future theoretical work and applications. The work on the case studies at the end of the project has shown the applicability of the monodic approach and its importance in the verification of complex systems.

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