Monodic Temporal Resolution

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First-order temporal logic is a concise and powerful notation, with many potential applications in both Computer Science and Artificial Intelligence. While the full logic is incomplete [9], recent work on *monodic* first-order temporal logics has identified important enumerable and even decidable fragments [7].

Let $\mathcal{L}$ be a first-order temporal language with temporal operators $\Box$ (always), $\Diamond$ (sometime), $\bigcirc$ (in the next moment), and U (until), but without equality or functional symbols. A formula is called *monodic* if any its subformula of the form $\bigcirc\phi$, $\Box\phi$, $\Diamond\phi$, $\phi\,\mathsf{U}\,\psi$ contains at most one free variable. We consider the monodic fragment interpreted over both *constant* and *expanding* domains [6] with the flow of time isomorphic to $\mathbb{N}$, and provide a complete resolution calculus for this fragment, based on our work on clausal temporal resolution over a number of years [3, 5, 1].

We are interested in the satisfiability of *unconditional eventuality monodic temporal problems in Divided Separated Normal Form (DSNF)*. A divided separated normal form DSNF is a quadruple $\langle \mathcal{I}, \mathcal{U}, \mathcal{S}, \mathcal{E}\rangle$ where $\mathcal{I}$ and $\mathcal{U}$ are an *initial part* and a universal part, respectively, given by a set of arbitrary closed first-order formulae; $\mathcal{S}$ is a *step part* given by a set of *step clauses* (or *step rules*) and $\mathcal{E}$ is an *eventuality part* given by a set of eventuality clauses. Step and eventuality clauses are of the following forms:

$\forall x(P(x) \Rightarrow \bigcirc M(x))$	(a *non-ground* step clause),	$p \Rightarrow \bigcirc m$	(a *ground* step clause),
$\Diamond L(x)$	(a *non-ground* eventuality clause),	$\Diamond l$	(a *ground* eventuality clause),

where $P(x)$ is a unary predicate; $L(x)$ and $M(x)$ are unary literals; p is a propositional symbol; and l and m are propositional literals. We do not allow two different temporal step clauses with the same left-hand sides. With each DSNF we associate the formula $\mathcal{I} \wedge \Box\mathcal{U} \wedge \Box\forall x\mathcal{S} \wedge \Box\forall x\mathcal{E}$. Now, when we talk about particular properties of temporal problems (e.g., satisfiability, validity, logical consequences etc) we mean properties of the associated formula.

Arbitrary monodic FOTL-formulae can be transformed into DSNF in a satisfiability equivalence preserving way with at most a linear increase in the size of the formula using a renaming technique replacing non-atomic subformulae with new propositions and removing all occurrences of the U operator [4, 2].

Let P be a monodic temporal problem, and let

$$P_{i_1}(x) \Rightarrow \bigcirc M_{i_1}(x), \ldots, P_{i_k}(x) \Rightarrow \bigcirc M_{i_k}(x) \tag{1}$$

be a subset of the set of its original non-ground step clauses. Then formulae of the form

$$P_{i_j}(c) \Rightarrow \bigcirc M_{i_j}(c), \tag{2}$$

$$\exists x(P_{i_1}(x) \wedge \ldots \wedge P_{i_k}(x)) \Rightarrow \bigcirc\exists x(M_{i_1}(x) \wedge \ldots \wedge M_{i_k}(x)), \tag{3}$$

$$\forall x(P_{i_1}(x) \vee \ldots \vee P_{i_k}(x)) \Rightarrow \bigcirc\forall x(M_{i_1}(x) \vee \ldots \vee M_{i_k}(x)) \tag{4}$$

are called *derived* step clauses, where $c \in const(\mathsf{P})$. (Formulae of the form (2) and (3) are called *e-derived* step clauses.) Note that formulae of the form (2) and (3) are logical consequences of (1) in the *expanding domain* case; and formulae of the form (2), (3), *and* (4) are logical consequences of (1) in the *constant domain* case.

Let $\{\Phi_1 \Rightarrow \bigcirc\Psi_1, \ldots, \Phi_n \Rightarrow \bigcirc\Psi_n\}$ be a set of derived (e-derived) step clauses or original *ground* step clauses. Then $\bigwedge_{i=1}^{n} \Phi_i \Rightarrow \bigcirc \bigwedge_{i=1}^{n} \Psi_i$ is called a *merged derived step clause* (and *merged e-derived step clause*, resp.).

Let $\mathcal{A} \Rightarrow \bigcirc\mathcal{B}$ be a merged derived (e-derived) step clause, let $P_1(x) \Rightarrow \bigcirc M_1(x), \ldots, P_k(x) \Rightarrow \bigcirc M_k(x)$ be a subset of the original step clauses, and let $\mathcal{A}(x) \leftrightharpoons \mathcal{A} \wedge \bigwedge_{i=1}^{k} P_i(x)$, $\mathcal{B}(x) \leftrightharpoons \mathcal{B} \wedge \bigwedge_{i=1}^{k} M_i(x)$. Then $\forall x(\mathcal{A}(x) \Rightarrow \bigcirc\mathcal{B}(x))$ is called a *full merged step clause* (*full e-merged step clause*, resp.).

Let P be a monodic temporal problem, $\mathsf{P}^c = \mathsf{P} \cup \{\Diamond L(c) \mid \Diamond L(x) \in \mathcal{E}, c \in const(\mathsf{P})\}$ is the *constant flooded form* of P. Evidently, P^c is satisfiability equivalent to P.

We present now two calculi, $\mathfrak{I}_c$ and $\mathfrak{I}_e$, aimed at the constant and expanding domain cases, respectively. The inference rules of these calculi coincide; the only difference is in the merging operation. The calculus $\mathfrak{I}_c$ utilises merged derived and full merged step clauses; whereas $\mathfrak{I}_e$ utilises merged e-derived and full e-merged step clauses.

Inference Rules.[3]

- *Step resolution rule w.r.t. $\mathcal{U}$:* $\dfrac{\mathcal{A} \Rightarrow \bigcirc \mathcal{B}}{\neg \mathcal{A}}\ (\bigcirc^{\mathcal{U}}_{res})$, where $\mathcal{U} \cup \{\mathcal{B}\} \vdash \bot$.
- *Initial termination rule w.r.t. $\mathcal{U}$:* The contradiction $\bot$ is derived and the derivation is (successfully) terminated if $\mathcal{U} \cup I \vdash \bot$.
- *Eventuality resolution rule w.r.t. $\mathcal{U}$:*
$$\frac{\forall x(\mathcal{A}_1(x) \Rightarrow \bigcirc(\mathcal{B}_1(x)))\ldots\forall x(\mathcal{A}_n(x) \Rightarrow \bigcirc(\mathcal{B}_n(x))) \qquad \Diamond L(x)}{\forall x \bigwedge_{i=1}^{n} \neg \mathcal{A}_i(x)}\ (\Diamond^{\mathcal{U}}_{res}),$$
where $\forall x(\mathcal{A}_i(x) \Rightarrow \bigcirc \mathcal{B}_i(x))$ are full merged (full e-merged) step clauses such that for all $i \in \{1,\ldots,n\}$, the *loop* side conditions $\forall x(\mathcal{U} \wedge \mathcal{B}_i(x) \Rightarrow \neg L(x))$ and $\forall x(\mathcal{U} \wedge \mathcal{B}_i(x) \Rightarrow \bigvee_{j=1}^{n} (\mathcal{A}_j(x)))$ are both valid[4].
- *Ground eventuality resolution rule w.r.t. $\mathcal{U}$:* The rule repeats the eventuality resolution rule with the only difference that *ground* eventualities and merged derived (e-derived) step clauses are used instead of non-ground eventualities and full merged (full e-merged) step clauses.

A *derivation* in the described inference systems is a sequence of universal parts, extended bit by bit by the conclusions of the inference rules. Successful termination means that the given problem is unsatisfiable. The I, $\mathcal{S}$ and $\mathcal{E}$ parts of the temporal problem are not changed in a derivation.

Theorem 1. *The rules of $\mathfrak{I}_c$ preserve satisfiability over constant domains. If a monodic temporal problem P is unsatisfiable over constant domains, then there exists a successfully terminating derivation in $\mathfrak{I}_c$ from P^c.*

Theorem 2. *The rules of $\mathfrak{I}_e$ preserve satisfiability over expanding domains. If a monodic temporal problem P is unsatisfiable over expanding domains, then there exists a successfully terminating derivation in $\mathfrak{I}_e$ from P^c.*

The notion of a (full) merged step clause is quite involved and the search for appropriate merging of simpler clauses is computationally hard. Finding *sets* of such full merged step clauses needed for the eventuality resolution rule is even more difficult. We are working now on the efficient implementation of the inference rules. Work in this direction for the expanding domain case can be found in [8].

References

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[3] $\mathcal{A} \Rightarrow \bigcirc\mathcal{B}$ and $\mathcal{A}_i \Rightarrow \bigcirc\mathcal{B}_i$ denote here merged derived (e-derived) step clauses, $\forall x(\mathcal{A}(x) \Rightarrow \bigcirc(\mathcal{B}(x)))$ and $\forall x(\mathcal{A}_i(x) \Rightarrow \bigcirc(\mathcal{B}_i(x)))$ denote full merged (full e-merged) step clauses, and $\mathcal{U}$ denotes the (current) universal part of the problem.

[4] In the case $\mathcal{U} \vdash \forall x \neg L(x)$, the *degenerate clause*, **true** $\Rightarrow \bigcirc$**true**, can be considered as a premise of this rule; the conclusion of the rule is then $\neg$**true** and the derivation successfully terminates.