Monodic ASMs and Temporal Verification^{*}

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Abstract. In this paper, we pursue the goal of automatic deductive verification for certain classes of ASM. In particular, we base our work on a translation of general ASMs to full first-order temporal logic. While such a logic is, in general, not finitely axiomatisable, recent work has identified a fragment, termed the *monodic* fragment, that is finitely axiomatisable and many of its subfragments are decidable. Thus, in this paper, we define a class of monodic ASMs whose semantics in terms of temporal logic fits within the monodic fragment. This, together with recent work on clausal resolution methods for monodic fragments, allows us to carry out temporal verification of monodic ASMs. The approach is illustrated by the deductive verification of FloodSet algorithm for Consensus problem, and Synapse N+1 cache coherence protocol; both are specified by monodic ASMs.

1 Introduction

The underlying idea which provides Abstract State Machines (ASMs) with much of their flexibility, is that states of an arbitrary computation are represented by general *many-sorted first-order structures*. The vocabularies of these structures may be freely chosen in order to model any particular algorithm at an appropriate level of abstraction [16]. (This flexibility contrasts with many standard models, such as Turing machines or finite state automata, where the alphabets used are fixed.) The process of computation is then modelled by the evolution of states according to given rules; on being ‘fired’, these rules change the structure produced by re-interpreting its basic functions.

Since its introduction, the ASM approach has proved to be very successful in specifying a variety of different types of software and hardware, including abstract algorithms, realistic programming languages, and distributed computations [1]. This success provides empirical evidence supporting the Abstract State Machine Thesis [16], which asserts that *every* algorithm is behaviourally equivalent to an ASM and so may be simulated, step-by-step, by that machine. For example, in [17] and [2], this thesis was established for very general formalisations of the notions of *sequential* and *parallel* algorithms, respectively. The combination of generality, varying levels of abstractness and strong formal foundations has led to the ASM approach beginning to be applied in significant industrial applications [18–20].

However, while specification and implementation techniques using ASMs have been studied in depth, there has been relatively little work on the verification of ASMs.

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Research in this area has, so far, mainly concerned either human-assisted proofs of correctness [36, 15], dealing with ASM representations in a relatively manual way, or algorithmic verification of ASMs [3, 9, 38, 39], based on model checking techniques [5].

In this paper, we pursue the goal of automatic deductive verification for certain classes of ASM. In particular, we base our work on a translation of general ASMs to full first-order temporal logic with natural numbers as a time flow [11]. While such a logic is, in general, not finitely axiomatisable, recent work has identified a fragment, termed the *monodic* fragment, that is finitely axiomatisable [23], thus making a semi-decision procedure possible. Furthermore, some of its natural (e.g two variables or monadic) subfragments, are decidable. Thus, in this paper, we define a class of *monodic ASMs* whose semantics in terms of temporal logic fits within the monodic fragment. This, together with recent work on clausal resolution methods [12] for monodic fragments [6, 28, 7], allows us to carry out temporal verification of properties of monodic ASMs. The approach is illustrated by the deductive verification of the correctness of both a FloodSet algorithm for the Consensus problem (in the presence of crash failures) and a cache coherence protocol, both specified using monodic ASMs.

We begin, in Sections 2 and 3 respectively, with descriptions of the ASM approach and the variety of temporal logic we use. In Section 4, we define how general ASMs can be translated into formulae in first-order temporal logic, and establish formal properties of such a translation. Given that a translation of general ASMs to temporal logic is possible, we next consider, in Section 5, what class of ASMs translate to *monodic* first-order temporal logic. We define the class of *monodic ASMs* that capture this property, thus allowing systems specified in this class to be translated into a temporal fragment amenable to automatic (or at least semi-automatic) verification. In Section 6, we consider two applications: one concerning a distributed consensus problem; the other concerning cache coherence protocols. In Section 7 we compare our work with related work of A.Nowack. Finally, in Section 8, we provide concluding remarks and discuss future work.

2 Abstract State Machines

Definition 1

An Abstract State Machine, $\mathcal{M}$, can be defined as a tuple $\mathcal{M} = \langle \sigma, TR, IC, \Psi_0 \rangle$ where:

- σ is a *many-sorted signature*, i.e. a finite collection of function names, together with their sorts; σ is a disjoint union of three (possibly empty) parts: $\sigma = \sigma_s \cup \sigma_d \cup \sigma_e$, which consist of *static*, *dynamic* and *external* names, respectively. We assume the static part, σ_s , includes constants for all elements of any finite sort. We will identify boolean-valued terms with predicates whenever convenient.
- TR is a finite set of *transition rules*, i.e. the *program* of $\mathcal{M}$, where a *transition rule* is an expression of the form

if $g(\bar{x})$ **then** $U_1(\bar{x}), \dots, U_m(\bar{x})$

where $g(\bar{x})$ is a first-order formula (a *guard*) of the given signature over variables $\bar{x}$ and $U_i(\bar{x})$ is an expression of the form $f(t_1(\bar{x}), \dots, t_k(\bar{x})) = t(\bar{x})$ where $f \in \sigma_d$ and $\bar{x}$ are all free variables of the update, although any term $t_1, \dots, t_k, t$, or guard $g(\bar{x})$ is not required to contain all of $\bar{x}$;

- IC is a set of *integrity constraints*, with each being a first-order sentence within $\sigma_s \cup \sigma_e$;
- Ψ_0 is a first-order sentence in the vocabulary σ describing initial condition.

The idea underlying the separation of σ into three parts is as follows. The static names, i.e. those from σ_s are assumed to not change their interpretations during the computation. The interpretations of dynamic names (σ_d) may change during the computation by the application of transition rules from TR . External names (σ_e) serve to model the interaction with the environment and their interpretations are not subject to change by transition rule applications, but nevertheless their interpretations may be different in different states. It is natural to consider the evaluation of an external function as being carried out by an oracle (environment).

Note 1. Our definition is somewhat more general than the common one, where instead of the sentence Ψ_0 an initial first-order structure S_0 is assumed.

Note 2. We do not allow dynamic functions within integrity constraints. There are two reasons for this. First, this would make the definition of the operational semantics more complicated. Second, one can model any possible effects of such constraints by appropriate extension of the transition rules. We feel, however, that in some cases constraints including dynamic functions are more natural and we are plan to consider these elsewhere.

Definition 2 A *state* of an ASM $\mathcal{M} = \langle \sigma, TR, IC, \Psi_0 \rangle$ is a (many-sorted) first-order structure in a signature σ

Definition 3 A *computation (run)* of an ASM $\mathcal{M}$ is a sequence of its states $S_0, \dots, S_k, \dots$ such that

- $S_0 \models \Psi_0$, i.e. the initial state satisfies the initial condition.
- the domain of each S_i coincides with that of S_0 ;
- the interpretations of all functional symbols from σ_s in each S_i coincide with those in S_0 ;
- the interpretation of all functional symbols from σ_d in S_{i+1} is obtained from the state S_i by applying of *all* transition rules, subject to the restrictions that every S_i satisfies all the integrity constraints from IC and the updates are consistent in S_i . The *application* of a transition rule within a given state S_i of $\mathcal{M}$ means simultaneously executing the updates $U_1(\bar{x}), \dots, U_m(\bar{x})$ over the state S_{i+1} whenever the guard, $g(\bar{x})$ is satisfied. Thus, if an update $f(t_1(\bar{x}), \dots, t_k(\bar{x})) = t(\bar{x})$ is executed in S_i then $f^{S_{i+1}}(t_1^{S_i}(\bar{x}), \dots, t_k^{S_i}(\bar{x})) = t^{S_i}(\bar{x})$ holds for all $\bar{x}$ such that $S_i \models g(\bar{x})$. The *consistency* of updates in a state S means that no two different updates are to update *differently* the interpretation of same function on the same argument.
- the interpretation of all functional symbols from σ_e may be arbitrary, provided every S_i satisfies all the integrity constraints from IC .

Definition 4 If, at some state S , the updates are inconsistent, or execution of updates leads to the state violating some integrity constraint, it is said that $\mathcal{M}$ *crashes* in S . For example, the rules **if** b **then** $f(3) = 5$ **and** **if** b **then** $f(3) = 7$ would lead to the crash of the machine in a state S such that $S \models b$.

Note 3. According to the Definition 3, any run is either infinite or a finite crashing one. This is essential in our approach since we will be using first-order temporal logic over an infinite flow of time isomorphic to ω to model an execution of ASMs. However, it does not prevent one dealing with finite runs of ASMs, ending by reaching some end condition (not by crashing) within this approach: one may formally extend the finite runs into infinite ones by adding obvious rules to ASM forcing the machine to loop over last state forever.

Note 4. We consider basic and restricted forms of ASM. The most notable omission is *import* instructions, allowing new elements to be imported to the domain during of computation [16]. In addition, we do not consider other extensions of ASM such as *non-deterministic choice*, *object-orientation*, or *distribution* mechanisms [1].

Definition 5 An ASM rule is said to be normal if, and only if, it consists of at most one update in its right-hand side. An ASM program is normal if, and only if, it consists of only normal rules.

Proposition 1. *Any ASM program can be transformed into an equivalent normal ASM program.*

Proof. Simply replace any rule **if** $g(\bar{x})$ **then** $U_1(\bar{x}), \dots, U_m(\bar{x})$ with the set of rules $\{ \text{if } g(\bar{x}) \text{ then } U_i(\bar{x}) \mid i = 1 \dots m \}$.

3 Temporal Logic

The representation of dynamic activity via temporal formalisms is used in a wide variety of areas within Computer Science and Artificial Intelligence, for example temporal databases, program specification, system verification, agent-based systems, robotics, simulation, planning, knowledge representation, and many more [31, 32, 5, 40, 27].

3.1 First-Order Temporal Logic (FOTL)

Here, we concentrate on one very popular variety, namely discrete linear temporal logic, which has an underlying model of time isomorphic to the Natural Numbers (i.e. an infinite sequence with distinguished initial point) and is also linear, with each moment in time having at most one successor. The infinite and linear constraints ensure that each moment in time has *exactly* one successor.

The language $\mathcal{TL}$ of the first order temporal logic over the Natural Numbers is constructed in the standard way from a classical (non-temporal) first order language $\mathcal{L}$ and a set of future-time temporal operators ' $\Diamond$ ' (*sometime*), ' $\Box$ ' (*always*), ' $\bigcirc$ ' (*in the next moment*), ' $\mathcal{U}$ ' (*until*).

Formulae in $\mathcal{TL}$ are interpreted in *first-order temporal structures* of the form $\mathfrak{M} = \langle D, I \rangle$, where D is a non-empty set, the *domain* of $\mathfrak{M}$, and I is a function associating with every moment of time $n \in \mathbb{N}$ an interpretation of predicate, function and constant symbols of $\mathcal{L}$ over D . First-order (nontemporal) structures corresponding to each point of time will be denoted $\mathfrak{M}_n = \langle D, I(n) \rangle$. Intuitively, the interpretations of $\mathcal{TL}$ -formulae

are sequences of first-order structures, or *states* of $\mathfrak{M}$, such as $\mathfrak{M}_0, \mathfrak{M}_1, \dots, \mathfrak{M}_n \dots$. An *assignment* in D is a function α from the set $\mathcal{L}_v$ of individual variables of $\mathcal{L}$ to D . If P is a predicate symbol then $P^{I(n)}$ (or simply P^n if I is understood) is the interpretation of P in the state $\mathfrak{M}_n$. We require that (individual) variables and constants of $\mathcal{TL}$ are *rigid*, that is neither assignments nor interpretations of constants depend on the state in which they are evaluated. On the other hand we allow both predicates and functional symbols to be *flexible*. Flexibility of functions, which is not a very common assumption in the semantics of FOTL, is very convenient for direct modelling of ASM rules.

The *truth-relation* $\mathfrak{M}_n \models^\alpha \varphi$ (or simply $n \models^\alpha \varphi$, if $\mathfrak{M}$ is understood) in the structure $\mathfrak{M}$ for the assignment α is defined inductively in the usual way under the following semantics of temporal operators:

$$\begin{aligned} n \models^\alpha \bigcirc \varphi &\text{ iff } n+1 \models^\alpha \varphi; \\ n \models^\alpha \Diamond \varphi &\text{ iff there is } m \geq n \text{ such that } m \models^\alpha \varphi; \\ n \models^\alpha \Box \varphi &\text{ iff } m \models^\alpha \varphi \text{ for all } m \geq n; \\ n \models^\alpha \varphi \mathcal{U} \psi &\text{ iff there is } m \geq n \text{ such that } m \models^\alpha \psi \text{ and} \\ &\quad k \models^\alpha \varphi \text{ for every } n \leq k < m; \end{aligned}$$

A formula φ is said to be *satisfiable* if there is a first-order structure $\mathfrak{M}$ and an assignment α such that $\mathfrak{M}_0 \models^\alpha \varphi$. If $\mathfrak{M}_0 \models^\alpha \varphi$ for every structure $\mathfrak{M}$ and for all assignments then φ is said to be *valid*. For a closed formula φ denote by $\text{Mod}(\varphi)$ the set of all its models, i.e. first-order temporal structures $\mathfrak{M}$ such that $\mathfrak{M}_0 \models \varphi$.

3.2 Monodic FOTL

While *first-order temporal logic* (FOTL) is very expressive and, in general, highly undecidable, recent results have identified recursively enumerable fragments of FOTLs [23].

Definition 6 A first-order temporal formula is said to be **monodic** if, and only if, any subformula with its main connective being a temporal operator has at most one free variable.

The monodic fragment of FOTL has appealing properties. It is axiomatizable [41] and many of its subfragments, such as the two-variable or monadic cases, are decidable. Decidability of these fragments holds also for the case of semantics where only temporal structures over *finite domains* are allowed. However, these good properties hold mainly for the case when equality is not allowed. Adding equality makes even very restricted fragments non-axiomatizable [8]. A notable exception is the packed monodic fragment with equality, which is decidable [22].

3.3 Temporal Tools

The widespread use of temporal logics has led to the development of key techniques for utilising temporal specifications, such as model checking and deductive verification [30, 12]. While the former has restrictions in terms of the size of models considered, the latter has problems in terms of worst-case complexity. For propositional (discrete, linear) temporal logics, the decision procedure is already PSPACE [37]. In spite of this, a range

of useful tools have been developed, with our clausal resolution approach [12] having been shown to be particularly effective [26].

For general (again, discrete and linear) FOTL, incompleteness is a problem. While there have been formalisations of proof mechanisms, for example infinitary sequent systems, proof in such complex logics will require significant user interaction. We have developed a prover, based on the λ Clam proof planning system [35] which incorporates techniques from the clausal resolution method in order to aid proof search [4].

The breakthrough concerning monodic FOTL is now leading to the development of improved proof methods. We have extended our clausal resolution approach to this fragment [6, 28, 7], and have also provided usable implementations [24]. We believe this to be a most productive route for temporal verification in the future.

4 From ASM to FOTL

In this paper, FOTL provides the key tool for formalising the execution of ASMs. In particular, as the operational semantics of an ASM is defined in terms of runs (sequences, or partially-ordered sets, of states), these can naturally be seen as the models of FOTL.

$\llbracket \mathcal{M} \rrbracket$	$= \llbracket \Psi_0 \rrbracket_{initial}^\sigma \wedge \llbracket \sigma \rrbracket_{sorts} \wedge \llbracket \sigma \rrbracket_{static} \wedge \llbracket TR \rrbracket_{rules}^\sigma \wedge$ $\wedge \llbracket IC \rrbracket_{integrity}^\sigma \wedge \llbracket TR \rrbracket_{frame}^\sigma$
$\llbracket \Psi_0 \rrbracket_{initial}^\sigma$	$= \llbracket \Psi_0 \rrbracket_{fo}^\sigma$
$\llbracket \sigma \rrbracket_{static}$	$= \Box \bigwedge_{f \in \sigma_s} \forall \bar{x}, y \text{ Persists}(f(\bar{x}) = y)$
$\llbracket \{r_1, r_2, \dots, r_n\} \rrbracket_{rules}^\sigma$	$= \bigwedge_{i=1}^n \llbracket r_i \rrbracket_{rule}^\sigma$
$\llbracket \{c_1, c_2, \dots, c_n\} \rrbracket_{integrity}^\sigma$	$= \Box \bigwedge_{i=1}^n \llbracket c_i \rrbracket_{fo}^\sigma$

Fig. 1. ASM $\rightarrow$ FOTL Translation Functions

4.1 Translation

We assume in this section that an ASM program is given in a normal form. In Fig. 1, we present translation functions which effectively provide a semantics in terms of first-order temporal logic for an ASM, ‘ $\mathcal{M}$ ’. In the Figure and below we use the notation $Persists(\phi(\bar{x}))$ to denote $\phi(\bar{x}) \rightarrow \bigcirc \phi(\bar{x})$. Four semantic functions remain undefined and require further explanation.

First-Order Translation. The $\llbracket \cdot \rrbracket_{fo}^\sigma$ function simply takes the many-sorted first order description given as an argument and produces a first-order description to be used in the temporal semantics. In the case of general ASMs considered here, this is simply the identity function. Later, however, we will see that some modification of the form of

the first-order sentences can be useful when translating from ASM notation to FOTL formulae.

Sort Constraints. The $\llbracket \sigma \rrbracket_{sorts}$ function takes the vocabulary σ and produces ($\Box$ -closed) one-sorted first-order description of all sort constraints assumed for many-sorted functional symbols. For finite sorts, it also contains sentences enumerating explicitly the elements of these sorts.

Translation of Rules. Before we present the general translation of rules, we will consider an example. Translating the ASM transition rule, **if** $b(x)$ **then** $f(3, k(x)) = 2$, we have to take into account that the term $k(x)$ and the update $f(3, k(x)) = 2$ according to the semantics of ASM rules, should be evaluated in different time slices. So, a straightforward translation $\Box \forall x (b(x) \rightarrow \bigcirc f(3, k(x)) = 2)$ would be incorrect. The correct translation is

$$\Box \forall x \forall v [(b(x) \wedge (v = k(x)) \rightarrow \bigcirc f(3, v) = 2)]$$

In general, given any rule r : **if** $g(\bar{x})$ **then** $f(t_1(\bar{x}), \dots, t_k(\bar{x})) = t(\bar{x})$, we define the formula

$$\Phi_r(\bar{x}, v_1, \dots, v_k) \leftrightarrow g(\bar{x}) \wedge \bigwedge_{i=1 \dots k} (v_i = t_i(\bar{x})) \quad (1)$$

where all v_i are fresh variables. Then we define the translation $\llbracket r \rrbracket_{rule}^\sigma$ of r as

$$\Box [\forall \bar{x}, \bar{v}, z (\Phi_r(\bar{x}, \bar{v}) \wedge z = t(\bar{x}) \rightarrow \bigcirc f(\bar{v}) = z)]$$

where z is a fresh variable.

Frame Conditions. In addition to the transition rules and integrity constraints that are explicitly part of the ASM notation, there are implicit *frame conditions*. In essence, these conditions aim to ensure that any function retains its previous value on any argument if not affected by an update (or integrity constraint). Thus, the formula produced by $\llbracket TR \rrbracket_{frame}^\sigma$ attempts to capture this persistence. The approach we follow in generating this formula is based on the simple idea provided by Reiter [34], namely that all the potential updates and guards within transition rules are examined and new formulae are added to describe the behaviour if one or more of the guards are *not* satisfied.

Consider as an example the ASM program with the following transition rules:

if $b(x)$ **then** $f(3, k(x)) = 2$
if $c(x)$ **then** $f(x, 5) = 7$
if d **then** $f(4, m(x)) = 3$

Given some state s we are interested in identifying those elements v and w of the domain of s , for which $f(v, w)$ will not change its value after executing the above rules in s . It is clear that this is the case precisely when $\langle v, w \rangle$ is not a pair of values of arguments of f in the left-hand side of any of the updates under substitution which makes the corresponding guard true. Now, for the above transition rules, we have the following frame condition (where variables v , w , and z are universally quantified and the whole formula is assumed to be under “always” operator):

$$\left(\begin{array}{c} \neg \exists x (v = 3 \wedge w = k(x) \wedge b(x)) \\ \wedge \\ \neg \exists x (x = v \wedge w = 5 \wedge c(x)) \\ \wedge \\ \neg \exists x (v = 4 \wedge w = m(x) \wedge d) \end{array} \right) \Rightarrow Persists(f(v, w) = z)$$

So, more generally, we define $\llbracket TR \rrbracket_{frame}^\sigma$ as follows. For a function symbol f define $I_f = \{i | r_i \in TR \text{ and } r_i \text{ updates } f\}$. Then, let $\llbracket TR \rrbracket_{frame}^{\sigma, f}$ be

$$\Box \forall \bar{v}, z \left[\left(\bigwedge_{i \in I_f} \neg \exists \bar{x} \Phi_{r_i}(\bar{x}, \bar{v}) \right) \rightarrow Persists(f(\bar{v}) = z) \right]$$

where Φ_r is defined above (1). Finally, we define $\llbracket TR \rrbracket_{frame}^\sigma$ as $\bigwedge_{f \in \sigma_d} \llbracket TR \rrbracket_{frame}^{\sigma, f}$

4.2 Correctness of the Translation

Here, we give the general statement of correctness of the translation of ASMs into full (i.e. including equality) FOTL. This is based on the strong correspondence between the execution sequences of ASMs, and the models of FOTL. The operational semantics of sequential ASMs is defined in terms of runs i.e. sequences of first-order structures, which allows to consider the *infinite* runs as the models for discrete, linear first-order logic. The restriction to infinite runs is essential since we consider FOTL over an infinite flow of time isomorphic to ω . (See also Note 3.)

Theorem 1 (Correctness of Translation) The set $Runs(\mathcal{M})$ of infinite runs of an ASM, $\mathcal{M}$, is exactly the set of models $Mod(\llbracket \mathcal{M} \rrbracket)$ of the formula $\llbracket \mathcal{M} \rrbracket$, as defined in Fig. 1 and Section 4.1

Proof [OUTLINE]. Given $\mathcal{M} = \langle \sigma, TR, IC, \Psi_0 \rangle$, then Fig. 1 defines

$$\llbracket \mathcal{M} \rrbracket = \llbracket \Psi_0 \rrbracket_{initial}^\sigma \wedge \llbracket \sigma \rrbracket_{static} \wedge \llbracket TR \rrbracket_{rules}^\sigma \wedge \llbracket IC \rrbracket_{integrity}^\sigma \wedge \llbracket TR \rrbracket_{frame}^\sigma$$

Consider an arbitrary infinite sequence $\gamma = S_0, S_0, \dots S_k, \dots$ of first-order structures in the vocabulary σ . Then straightforward check of the definitions of run and translation shows that $\gamma \in Runs(\mathcal{M})$ if, and only if, $\gamma \in Mod(\llbracket \mathcal{M} \rrbracket)$. See more details in [14]. $\square$

4.3 Verification of ASM Translations

Given an ASM program $\mathcal{M}$ one may use the above translation for verification of this program as follows: express correctness conditions by a first-order temporal formula ψ and then check whether the implication $\llbracket \mathcal{M} \rrbracket \rightarrow \psi$ is a valid FOTL formula. In general, of course we have no decision, or even semi-decision procedure to check validity (or satisfiability) of such formulae. But, if the resulting formula turns out to be in a ‘good’ (e.g. monodic) fragment one can apply decision procedures, or theorem proving, for verification. Otherwise, one can apply heuristic methods to show its validity. Some such methods, derived from the resolution procedures for monodic fragments, and incomplete in general are presented in [4]. In what follows we describe restrictions on ASM programs that ensure their temporal translations are in monodic fragments.

5 From ASM to Monodic FOTL: Monodic ASMs

In this section we are looking for restrictions of ASM programs which ensure that their temporal translations fall within “good” monodic fragments. A quick glance at the defined translations reveals that non-monodic formulae may appear both in the translations of transition rules and in frame conditions.

Definition 7 Any term which is not a constant or variable is said to be *complex*, otherwise it is *simple*.

Definition 8 A functional symbol f of a sort $S_1 \times \dots \times S_k \rightarrow S$ within the ASM vocabulary is said to be *finitely valued* if, and only if, the sort S is finite.

Definition 9 A normal transition rule, **if** $g(\bar{x})$ **then** $f(t_1(\bar{x}), \dots, t_k(\bar{x})) = t(\bar{x})$ is called *monodic* if, and only if,

1. At most one free variable ranging over an infinite sort occurs both in the guard and in the update (i.e. the free variable is shared);
2. The main functional symbol f of an update has at most one infinite input sort;
3. An immediate subterm $t_i(\bar{x})$ of $f(t_1(\bar{x}), \dots, t_k(\bar{x}))$ of an infinite sort, is a variable (not a constant or complex term);
4. All functional symbols in the rule are finitely valued;

Definition 10 An ASM program is called *monodic* if, and only if,

- it consists of only normal monodic rules, and
- static part Σ_s of the vocabulary consists of only finitely valued functional symbols of arities at most 1, and
- neither a guard of any rule, nor the initial condition of ASM nor integrity constraints contain equality.

Examples. Let S_i be an infinite sort, S_f be a finite sort and a be a constant of sort S_f .

1. Assuming $g : S_i \times S_i \rightarrow S_f$, then the rule **if** true **then** $f(x, x) = a$ is not monodic (condition 2 does not hold)
2. Assume now that $g : S_i \times S_f \rightarrow S_f$, $k : S_i \rightarrow S_f$.
Then the rule **if** true **then** $g(x, k(x)) = a$ is monodic.

5.1 Translation of a Monodic ASM Program

Before we formulate a general statement on the translation of monodic ASM programs into temporal logic, it is instructive to consider an example of such a translation. Take the transition rule **if** true **then** $f(x, k(x)) = a$ of the previous example and consider an ASM program $\mathcal{M}$ which contains this as its only rule. Assume also $S_f = \{a, b\}$. Then the translation $\llbracket TR \rrbracket_{rules}^\sigma$ of the rule is $\Box \forall x, v (x = v \rightarrow \bigcirc (f(v, k(v)) = a))$ and frame condition $\llbracket TR \rrbracket_{frame}$ is $\forall v, w, z (\neg \exists x (v = x \wedge w = k(x)) \rightarrow Persists(f(v, w) = z))$ which, after obvious simplification, become $\Box \forall x \bigcirc (f(x, k(x)) = a)$ and

$$\forall v, w, z (w \neq k(v) \rightarrow \text{Persists}(f(v, w) = z))$$

Notice that the formula expressing the frame condition is not monodic: *Persists* is applied to the subformula with *three* free variables, which destroys monodicity. Further, both above translations contain functional symbols and equality, which also makes it problematic to be in any decidable, or axiomatizable, fragment of FOTL. However, replacing functional symbols with corresponding predicates and, further, using the finiteness conditions imposed by definition of a monodic ASM program, makes it possible to translate these formulae to the good monodic fragment, preserving the “meaning” of the original ASM program.

To replace functional symbols, we introduce new relational symbols (i.e. boolean valued functional symbols)

$$val_f : S_i \times S_f \times S_f \rightarrow Bool;$$

$$val_k : S_i \times S_f \rightarrow Bool$$

Then we can rewrite $\llbracket TR \rrbracket_{rules}^\sigma$ into $\forall x \Box \bigcirc (\exists u (val_k(x, u) \wedge val_f(x, u, a)))$ which is now monodic and without functional symbols. Further, using the finiteness of the sort S_f we can remove the existensial quantifier:

$$\llbracket TR \rrbracket_{rules}^* = \forall x \Box \bigcirc \left[\begin{array}{l} ((val_k(x, a) \wedge val_f(x, a, a)) \vee \\ (val_k(x, b) \wedge val_f(x, b, a)) \end{array} \right]$$

Now, rewrite the frame condition into

$$\forall v, w, z (\neg val_k(v, w) \rightarrow \text{Persists}(val_f(v, w, z)))$$

Notice, that this is still a non-monodic formula. Denote it by $\forall v, w \Psi(v, w)$. Further, using finiteness of S_f we can remove variables v and w and quantifiers upon them, making the resulting formula monodic:

$$\llbracket TR \rrbracket_{frame}^* = \Psi(a, a) \wedge \Psi(a, b) \wedge \Psi(b, a) \wedge \Psi(b, b)$$

To finish this translation we have to specify the *functionality* property of newly introduced relational symbols. The corresponding formula $Fun_{k,f}$:

$$\forall x, y ((val_k(x, a) | val_k(x, b)) \wedge (val_f(x, y, a) | val_f(x, y, b)))$$

has to be added to the integrity constraints $\llbracket IC \rrbracket_{integrity}$. Here $|$ stands for “exclusive OR”.

Notice that $\llbracket TR \rrbracket_{rules}^* \wedge \llbracket TR \rrbracket_{frame}^* \wedge Fun_{k,f}$ is a monodic formula without functional symbols and equality. Notice also that it models closely the behaviour of the initial translation $\llbracket TR \rrbracket_{rules} \wedge \llbracket TR \rrbracket_{frame}$: the sets of their temporal models are the same modulo renaming $f \mapsto val_f$ and $k \mapsto val_k$. Now we can formulate a general statement

Theorem 2 Let $\mathcal{M}$ be monodic ASM program. Then its temporal translation $\llbracket \mathcal{M} \rrbracket$ can be further translated into monodic temporal formula $\llbracket \mathcal{M} \rrbracket^*$ which:

- does not contain functional symbols;
- does not contain equality;
- has the same set of temporal models as $\llbracket \mathcal{M} \rrbracket$ modulo some renaming of functional symbols into relational ones.

Proof [OUTLINE]. Given a translation $\llbracket \mathcal{M} \rrbracket$ of an ASM program $\mathcal{M} = \langle \sigma, TR, IC, S_0 \rangle$ we perform further translation as follows. For every functional symbol f of the sort $S_1 \times \dots \times S_k \rightarrow S$ introduce a corresponding predicate symbol val_f of the sort $S_1 \times \dots \times S_k \times S \rightarrow Bool$. For all functional symbols $f_1, \dots, f_s$ add $Fun_{f_1, \dots, f_s}$, the formulae expressing functionality of all $val_{f_1}, \dots, val_{f_s}$, to $\llbracket IC \rrbracket_{integrity}^\sigma$. Because all of $f_1, \dots, f_k$ are finitely valued and there are constants for all elements of finite sorts, $Fun_{f_1, \dots, f_s}$ can be expressed without equality. For a term $f(t_1, \dots, t_k)$ (an atomic formula $Q(t_1, \dots, t_k)$) introduce notations $f[t_{i_1}, \dots, t_{i_m}; \bar{\theta}]$ ($Q[t_{i_1}, \dots, t_{i_m}; \bar{\theta}]$), where $t_{i_1}, \dots, t_{i_m}$ are *all complex* immediate subterms and $\bar{\theta}$ is a sequence of *all simple* immediate subterms (i.e variables or constants) of $f(t_1, \dots, t_k)$ (of $Q(t_1, \dots, t_k)$). Then for any complex term $f[t_{i_1}, \dots, t_{i_m}; \bar{\theta}]$ define a formula $val_{f[t_{i_1}, \dots, t_{i_m}; \bar{\theta}]}(v)$ as follows. If all immediate subterms are simple then $val_{f[t_{i_1}, \dots, t_{i_m}; \bar{\theta}]}(v) \equiv val_f(\bar{\theta}, v)$, where v is a fresh variable. Otherwise define it as $val_{f[t_{i_1}, \dots, t_{i_m}; \bar{\theta}]}(v) \equiv \exists v_1 \dots v_m (val_{t_1}(v_1) \wedge \dots \wedge val_{t_m}(v_m) \wedge val_f(v_1, \dots, v_m, \bar{\theta}, v))$, where v is a fresh variable.

Translate $\llbracket \mathcal{M} \rrbracket$ into the formula in the new vocabulary, where all functional symbols f are replaced by predicates val_f , as follows. Replace any atomic formula $Q[t_1, \dots, t_m; \bar{\theta}]$ with $\exists v_1, \dots, v_m (val_{t_1}(v_1) \wedge \dots \wedge val_{t_m}(v_m) \wedge Q(v_1, \dots, v_m, \bar{\theta}))$. Replace any atomic formula $t = s$, where t, s are complex terms, with $\exists v (val_t(v) \wedge val_s(v))$. Replace any atomic formula $t = r$, where t is complex and r is simple term, with $val_t(v) \mid_{v \mapsto r}$ (i.e r is substituted into v in $val_s(v)$).

Given that $\llbracket TR \rrbracket_{frame}$ and $\llbracket TR \rrbracket_{rules}$ are now in the relational vocabulary, further translation is performed as follows: replace all quantifiers over finite sorts with conjunctions and disjunctions over elements of the sort. Then, the first and second conditions of the definition of monodic rules ensure that, in the resulting formula, we have no more than one free variable in subformulae under the $\bigcirc$ operator in $\llbracket TR \rrbracket_{rules}$ and in the *Persists* subformulae in $\llbracket TR \rrbracket_{frame}$. It follows that the resulting formulae $\llbracket TR \rrbracket_{rules}^*$ and $\llbracket TR \rrbracket_{frame}^*$ are monodic.

Consider subformulae of the form $v_i = t_i(\bar{x})$ in the Φ_f subformulae of $\llbracket TR \rrbracket_{frame}$ and $\llbracket TR \rrbracket_{rules}$. During the above translation into relational vocabulary all equalities in such subformulae were eliminated, except the case, when $t_i(\bar{x})$ is a constant, or a variable. If it is a constant, then according to the condition 3 of the definition of monodic rules, it can be only of a finite sort and the quantification of v_i over this sort can be eliminated together with equality. If it is a variable, then subformula $v_i = x$ of $\llbracket TR \rrbracket_{rules}$ is replaced with *true*, the quantifier over v_i is removed and all occurrences of v_i are replaced by x . If $v_i = x$ is a subformula of Φ_f of $\llbracket TR \rrbracket_{frame}$ then it is replaced with *true* and is used to remove all occurrences of x in Φ_f . To illustrate last point consider as an example a formula, describing a frame condition:

$$\Box \forall v, w, z (\neg \exists x (v = 3 \wedge w = x \wedge b(x))) \Rightarrow Persists(val_f(v, w, z))$$

One can eliminate $w = x$ getting an equivalent formula:

$$\Box \forall v, w, z ((v \neq 3 \vee \neg b(w))) \Rightarrow Persists(val_f(v, w, z))$$

Thus after elimination of all such subformulae, we have final translation $\llbracket \mathcal{M} \rrbracket^*$ of the initial $\llbracket \mathcal{M} \rrbracket$ which is monodic and contains no equality. Finally, straightforward

but long argument shows that all stages of above translation preserve the “meaning” of temporal formulae and $\llbracket \mathcal{M} \rrbracket^*$ and $\llbracket \mathcal{M} \rrbracket$ have the same set of temporal models modulo renaming of functional symbols into relational ones. $\square$

6 Where Are the Monodic ASMs?

The notion of monodic ASM presented in the previous section is fairly restricted. The area of its applicability still needs to be understood better. In this section we present two examples of realistic distributed algorithms (protocols) which can be specified and verified via approach we have presented.

6.1 FloodSet Algorithm

We consider here a variant of the *FloodSet algorithm with alternative decision rule* (in terms of [29], p.105) designed for solution of the Consensus problem in the presence of crash (or fail-stop) failures.

The setting is as follows. There are n processes, each having an *input bit* and an *output bit*. The processes work synchronously, run the same algorithm and use *broadcast* for communication. Some processes may fail and, from that point onward, such processes do not send any further messages. Note, however, that the messages sent by a process *in the moment of failure* may be delivered to an *arbitrary subset* of the processes. Crucially, there is a bound, f , on the number of processes that may fail.

The goal of the algorithm is to eventually reach an agreement, i.e. to produce an output bit, which would be the same for all non-faulty processes. It is required also that if all processes have the same input bit, that bit should be produced as an output bit.

This protocol (adapted from [29]¹) is as follows.

- At the first round of computations, every process broadcasts its input bit.
- At every later round, a process broadcasts any value *the first time it sees it*.
- At every round the (tentative) output bit is set to the minimum value ever seen so far.

The correctness criteria for this protocol is that, eventually (actually, no later than in $f + 2$ rounds) the output bits of all non-faulty processes will be the same.

Claim. The above FloodSet algorithm can be specified (naturally) as a monodic ASM $\mathcal{M}$. Its temporal translation $\llbracket \mathcal{M} \rrbracket$ and correctness condition are in the decidable two-variable monodic (without equality) fragment of FOTL under finite domain semantics.

The detailed justification of this claim can be found in [14]. Here we present only one rule and its corresponding frame condition as examples. The vocabulary of ASMs here consists of (among others) unary predicate symbols *Normal* (dynamic), *Failure* (external) and *Faulty* (dynamic). Intuitively $Failure(x) = true$ at some moment means “processor denoted by x fails” at that moment. Similarly for $Faulty(x)$. $Normal(x)$ means x

¹ The difference being that, in [29], every process *knows* the bound f in advance and stops the execution of the protocol after $f + 2$ rounds, producing the appropriate output bit. For generality, we consider the version where the processes do not know f in advance and produce a *tentative output bit* at every round.

is not faulty nor in a failure state. One of the transition rules is **if** $Normal(x) \wedge Failure(x)$ **then** $Faulty(x) \wedge \neg Normal(x)$.

Our translation gives, for frame conditions on $Faulty$ (after simplification):

1. $\Box \forall x, z \neg (Normal(x) \wedge Failure(x)) \rightarrow Persists(Faulty(x))$ and
2. $\Box \forall x, z \neg (Normal(x) \wedge Failure(x)) \rightarrow Persists(\neg Faulty(x))$

6.2 Cache Coherence Protocols

Another class of protocols which can be specified via monodic ASMs, and for which correctness can be reduced to the validity of monodic temporal formulae, is the class of *cache coherence protocols* [21]. These are designed to provide data consistency between caches of different processors in shared-memory multiprocessor systems, i.e. to ensure that any copies of the same memory block in the caches of different processors are identical. Abstracting from the low-level implementation details of *read*, *write* and *synchronisation* primitives, cache coherence protocols [21, 10] can be seen as describing the families of *identical* finite state machines (each being cache controller of a processor) together with a simple form of communication: when one machine makes a transition (an *action*) a , the the other machines are required to do complementary transition (a *re-action*) $\bar{a}$. Correctness can then be expressed in terms of co-occurences some states in which, for example, the processors are allowed to change the content of their caches.

In [13] we have shown that theorem proving in monodic temporal fragments can be used for the verification of cache coherence protocols for an arbitrary number of processors. Here, we illustrate the point that *natural* ASM specifications of finite state based models of cache coherence protocols can be translated into temporal monodic specifications by the above translation procedure. As an example we consider “Synapse N+1” protocol which is particularly simple, consisting of only 3 states, 2 actions (transitions), *Read* and *Write* and two re-actions (complementary transitions) $\overline{Read}$ and $\overline{Write}$. The full ASM specification of this protocol (for an arbitrary number of processors) together with comments on temporal translation and verification can be found in [14]. For the moment, we simply note that, in the ASM specification of that protocol, we assume

- the elements of the domain represent cache controllers of different processors;
- states of cache controllers are represented by unary predicates holding or not on the corresponding elements of the domain, e.g. $P(x)$ means “automaton x is in state P ”;
- use of only two (external) predicates *Read* and *Write* for actions, while reactions $\overline{Read}$ and $\overline{Write}$ are expressed in terms of these two.

The ASM specification is monodic, except for one integrity constraint that uses an equality, expressing the fact that no more than one processor can act at every moment of time. The correctness conditions have a form

- χ_1 : $\Box \forall x \forall y (\neg D(x) \vee \neg V(y))$
(no two controllers can be in states D (“dirty”) and V (“valid”) respectively).
- χ_2 : $\Box \forall x \forall y (D(x) \wedge D(y) \rightarrow x = y)$
(no more than one controller can be in state D).

In general, since the obtained temporal formulae are monodic, we must consider the effect of the use of the equality predicate. It is known that even severely restricted classes of monodic formulae with equality may have a highly undecidable validity checking problem [8]. However, as is shown in [13] the proof methods for the fragment without equality [4] may be used for the proof of the above correctness condition χ_1 (and such like). As to the χ_2 , the equality may be abstracted away, by renaming the entire formula by a propositional letter and adding a corresponding $\Box$ -closed equivalence to the specification. After that the same procedure proves the validity.

7 Related Work

We have learned from one of the referees that in [33] Antje Nowack has suggested to use monodic fragments of FOTL to verify ASMs. In particular, he proposed the definition of *guarded* ASM and has shown that verification of the properties of such ASMs expressed in the guarded monodic fragment of FOTL is decidable. A reduction to the finite satisfiability problem of guarded monodic fragment is used.

It turns out that our work (done independently) is very much in spirit of [33]. We formulated restrictions on ASM which ensure that temporal translation falls into monodic fragment, which is, in general, undecidable, but finitely axiomatizable and hence semi-decidable. Technically, our definition of monodic ASM is less restrictive than that of guarded ASMs. On the other hand we can only guarantee existence of semi-decision procedure for monodic ASM (and suitable correctness conditions) as opposed to decision procedure for guarded ASMs (and suitable correctness conditions). However, checking whether the result of our temporal translation is in a known decidable fragment of FOTL is straightforward, so one can apply a decision procedure if it is the case. If the translation is outside known decidable fragment one can apply incomplete methods for verification. We have illustrated both these cases with the examples.

8 Conclusion

We have shown how to faithfully translate ASM specifications into the first-order temporal logic (FOTL) and have defined restrictions on ASM specifications (programs) which ensure that temporal translation is in a ‘good’ (monodic) fragment of FOTL. This allows us to use temporal translations for (semi-)automatic verifications of restricted ASM specifications either by decision procedures [7], or by theorem proving for restricted fragments of FOTL [4].

Design of effective automatic algorithms, where possible, is an important future development, as is an in-depth analysis of the applicability of monodic ASMs.

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