Public-Key Encryption

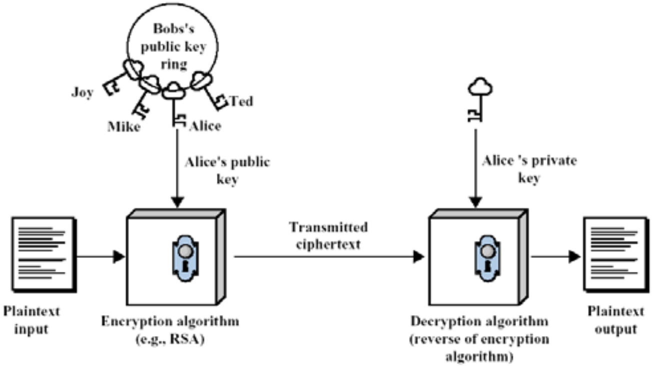
Public-key, or asymmetric encryption

Public-key encryption techniques. It is particular and most important kind of

Asymmetric encryption (or asymmetric key encryption):

- **One key** is used for encryption (usually publicly known, *public key*);
- **Another key** is used for decryption (usually *private*, or *secret* key)

Public-key encryption



(a) Encryption

Components of public-key encryption

- Plaintext
- Encryption algorithm
- Public and private key
- Ciphertext
- Decryption algorithm

Essential steps in communications using public-key encryption

- Each user generates a **pair** of keys;
- Each users makes one of the key publicly accessible (public key). The other key of the pair is kept private;
- If B wishes to send a private message to A, B **encrypts** the message using **A's public key;**
- When A receives the message, A **decrypts** it using **A's private key**. No other recipient can decrypt the message – nobody else knows A's private key.

Public-key encryption

Advantages

- All keys (public and private) are generated locally;
- No need in distribution of the keys;
- Moreover, each user can change his own pair of public/private key at any time;

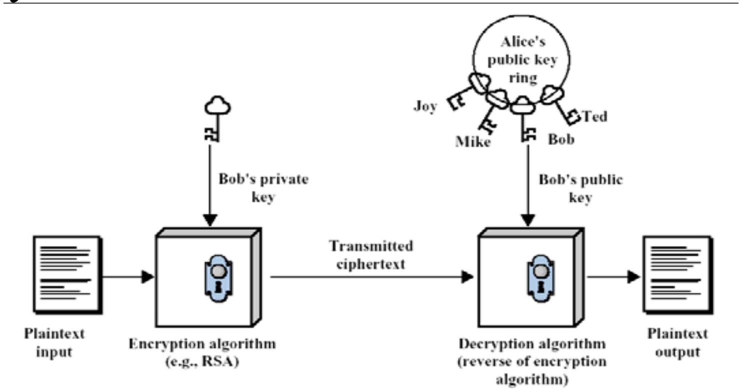
Disadvantages

- It is more computationally expensive.

Applications of Public-Key Cryptosystems

- **Encryption/decryption:** the sender encrypts a message with the recipient's public key.
- **Digital signature (authentication):** the sender "signs" the message with its private key; a receiver can verify the identity of the sender using sender's public key.
- **Key exchange:** both sender and receiver cooperate to exchange a (session) key.

Authentication using public-key systems



(b) Authentication

Requirements for Public-Key Cryptography

Diffie and Hellman conditions

"Easy part"

- It is computationally easy for a party B to generate a pair (public key , private key).
- It is computationally easy for a sender A, knowing the public key of B and the message M to generate a ciphertext:
- It is computationally easy for the receiver B to decrypt the resulting ciphertext using his private key

Requirements for Public-Key Cryptography

"**Difficult part"**

- It is computationally infeasible for anyone, knowing the public key, to determine the private key,

Additional useful requirement (not always necessary)

- Either of the two related keys can be used for encryption, with the other used for decryption.

Public-key cryptography and number theory

- Many public-key cryptosystems use non-trivial number theory;
- Security of most known RSA public-key cryptosystem is based on the hardness of factoring big numbers;
- We will overview basic notions of divisors, prime numbers, modular arithmetic

Divisors and prime numbers

Divisors

Let **a** and **b** are integers and **b** is not equal to **0;**

then we say **b** is a divisor of **a** if there is an integer **m** such that **a** = **mb**;

Prime numbers

An integer **p** is a *prime number* if its only divisors are **1, -1, p, -p**

gsd and relatively prime numbers

- **gcd(a,b)** is a greatest common divisor of **a** and **b**
  Examples: gcd(12, 15) = 3; gcd(49,14) = 7.

- **a** and **b** are **relatively prime** if gcd(a,b) = 1.
  Example: gcd (9,14) = 1.

Modular arithmetic

- If ***a*** is an integer and ***n*** is a positive integer, we define ***a mod n*** to be the remainder when *a* is divided by *n:*
  a = qn+r,
  Here ***q*** is a quotient and ***r = a mod n***
- If ***(a mod n) = (b mod n)*** then ***a*** and ***b*** are ***congruent modulo n***;
- It is easy to see, that ***(a mod n) = (b mod n)*** iff ***n*** is a divisor of ***a-b***.

Modular arithmetic. Properties

- ***[(a mod n) + (b mod n)] mod n = (a+b) mod n***
- ***[(a mod n) – (b mod n)] mod n = (a-b) mod n***
- ***[(a mod n) x (b mod n)] mod n = (a x b) mod n***

Example: 3 mod 5 x 4 mod 5 = 12 mod 5 = 2 mod 5