PAPER CODE NO. **COMP310**	EXAMINER : Dr T. R. Payne DEPARTMENT : Computer Science	Tel. No. 54251

UNIVERSITY OF LIVERPOOL

PROBLEM SOLVING EXAMINATIONS

Multiagent Systems

TIME ALLOWED : Two and a Half Hours

INSTRUCTIONS TO CANDIDATES

This is a mock paper - solutions are available

If you attempt to answer more questions than the required number of questions (in any section), the marks awarded for the excess questions answered will be discarded (starting with your lowest mark).

1. Consider the environment $Env_1 = \langle E, e_0, \tau \rangle$ defined as follows:

$$E = \{e_0, e_1, e_2, e_3, e_4, e_5\} \qquad \begin{array}{l} \tau(e_0 \xrightarrow{\alpha_0}) = \{e_1, e_2\} \\ \tau(e_1 \xrightarrow{\alpha_1}) = \{e_3\} \end{array} \qquad \tau(e_2 \xrightarrow{\alpha_2}) = \{e_4, e_5\}$$

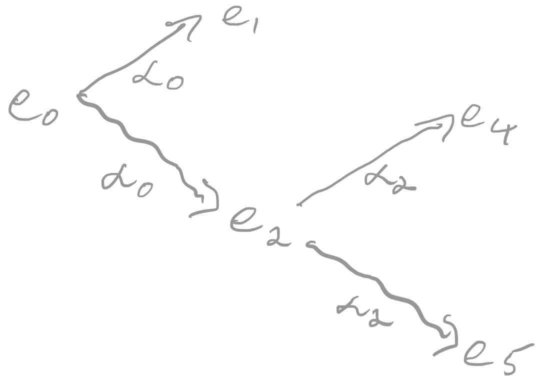
There are two agents, Ag_1 and Ag_2, with respect to this environment:

$$\begin{array}{l|l} Ag_1(e_0) = \alpha_0 & Ag_2(e_0) = \alpha_0 \\ Ag_1(e_1) = \alpha_1 & Ag_2(e_2) = \alpha_2 \end{array}$$

(a) Given these definitions draw a graph of the possible runs for the two agents Ag_1 and Ag_2 with respect to Env_1. **(4 marks)**

Agent Ag_1

Agent Ag_2



Given the environment in the previous question, assume that the utility function and probabilities of the various runs are defined as follows:

$$P(e_0 \xrightarrow{\alpha_0} e_1 \mid Ag_1, Env_1) = 0.5 \qquad P(e_0 \xrightarrow{\alpha_0} e_1 \mid Ag_2, Env_1) = 0.1$$
$$P(e_0 \xrightarrow{\alpha_0} e_2 \mid Ag_1, Env_1) = 0.5 \qquad P(e_0 \xrightarrow{\alpha_0} e_2 \mid Ag_2, Env_1) = 0.9$$
$$P(e_1 \xrightarrow{\alpha_1} e_3 \mid Ag_1, Env_1) = 1.0 \qquad P(e_2 \xrightarrow{\alpha_2} e_4 \mid Ag_2, Env_1) = 0.4$$
$$P(e_2 \xrightarrow{\alpha_2} e_5 \mid Ag_2, Env_1) = 0.6$$

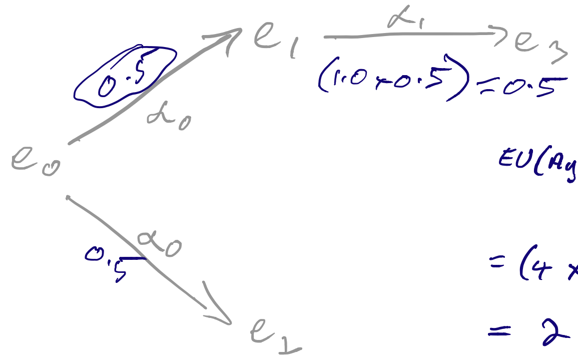
Assume the utility function u_1 is defined as follows:

$$u_1(e_0 \xrightarrow{\alpha_0} e_1) = 4 \qquad u_1(e_0 \xrightarrow{\alpha_0} e_2) = 3$$
$$u_1(e_1 \xrightarrow{\alpha_1} e_3) = 7$$
$$u_1(e_2 \xrightarrow{\alpha_2} e_4) = 3 \qquad u_1(e_2 \xrightarrow{\alpha_2} e_5) = 2$$

(b) Determine the expected utility of both agents, and explain which agent is optimal with respect to Env_1 and u_1. Include an explanation of your calculations in your solution.

(6 marks)

Agent Ag_1

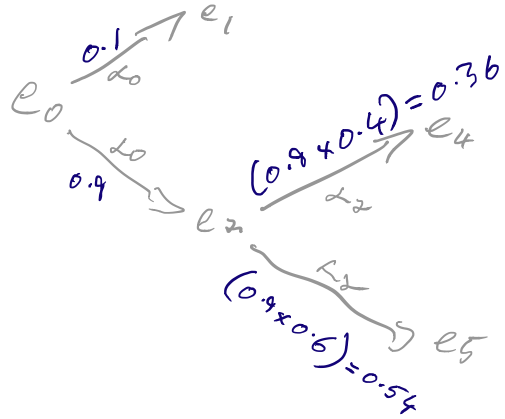


$EU(Ag_1, Env) = \sum_{r \in R(Ag, Env)} u(r) P(r \mid Ag, Env)$

$= (4 \times 0.5) + (3 \times 0.5) + (7 \times 0.5)$

$= 2 + 1.5 + 3.5 = \underline{\underline{7}}$

Agent Ag_2



$EU(Ag_2, Env_1) =$

$= (4 \times 0.1) + (3 \times 0.9) + (3 \times 0.36) + (2 \times 0.54)$

$= 0.4 + 2.7 + 1.08 + 1.08$

$= 5.26$

$\therefore$ $Agent_1$ is optimal as its expected utility (7) is greater than that of $Agent_2$ (5.26)

2. In the context of cooperative games, consider the following marginal contribution net:

$$\begin{array}{rl} a \wedge b \rightarrow 6 & \text{Rule 1} \\ b \rightarrow 4 & \text{Rule 2} \\ c \rightarrow 5 & \text{Rule 3} \\ b \wedge \neg c \rightarrow 3 & \text{Rule 4} \end{array}$$

Let ν be the characteristic function defined by these rules. Give the values of the following, and in each case, justify your answer with respect to the rule or rules of the above marginal contribution net:

a) $\nu(\{\varnothing\})$ No rules, $\therefore$ $\nu = 0$ **(2 marks)**

b) $\nu(\{a\})$ No rules, $\therefore$ $\nu = 0$ **(2 marks)**

c) $\nu(\{a, b\})$ **(2 marks)**

Rules 1
Rule 2 $b \in \{a, b\}$
Rule 4 $c \notin \{a, b\}$
$\therefore 6 + 4 + 3 = 13$

d) $\nu(\{b, c\})$ **(2 marks)**

Rules 2 & 3
$\therefore$ $\nu = 4 + 5 = 9$

e) $\nu(\{a, b, c\})$ Rules 1, 2, 3 $= 6 + 4 + 5 = 15$ **(2 marks)**

Consider the coalition game with agents $Ag = \{a, b, c\}$ and characteristic function ν defined by:

$$\begin{aligned} \nu\{\varnothing\} &= 0 \\ \nu\{a\} &= 12 \\ \nu\{b\} &= 18 \\ \nu\{c\} &= 6 \\ \nu\{a,b\} &= 60 \\ u\{b,c\} &= 48 \\ \nu\{a,c\} &= 72 \\ \nu\{a,b,c\} &= 120 \end{aligned}$$

f) Compute the Shapley values for the agents a, b, and c. You should show the relevant steps in your answer that are used to derive the answer. (**9 marks, 3 for each agent**)

Let $\delta_i(S)$ be the marginal contribution that agent i adds to S.
Such that $\nu(S \cup \{i\}) - \nu(S)$

The Shapley value for i $\varphi_i = \dfrac{\sum_{r \in R} \delta_i(S_i(r))}{|Ag|!}$

For agent a

$\delta_a(\phi) = \nu(\{a\}) - \nu(\phi) = 12 - 0 = 12$ $\{a,b,c\}; \{a,c,b\}$

$\delta_a(\{b\}) = \nu(\{a,b\}) - \nu(\{b\}) = 60 - 18 = 42$ $\{b,a,c\}$

$\delta_a(\{c\}) = \nu(\{a,c\}) - \nu(\{c\}) = 72 - 6 = 66$ $\{c,a,b\}$

$\delta_a(\{b,c\}) = \nu(\{a,b,c\}) - \nu(\{b,c\}) = 120 - 48 = 72$ $\{b,c,a\}, \{c,b,a\}$

$\varphi_a = \dfrac{12 + 12 + 42 + 66 + 72 + 72}{3!} = \dfrac{276}{6} = 46$

For agent b

$\delta_b(\phi) = \nu(\{b\}) - \nu(\phi) = 18 - 0 = 18$ bac, bca

$\delta_b(\{a\}) = \nu(\{a,b\}) - \nu(\{a\}) = 60 - 12 = 48$ abc

$\delta_b(\{c\}) = \nu(\{b,c\}) - \nu(\{c\}) = 48 - 6 = 42$ cba

$\delta_b(\{a,c\}) = \nu(\{a,b,c\}) - \nu(\{a,c\}) = 120 - 72 = 48$ acb, cab

$\therefore \varphi_b = \dfrac{18 + 18 + 48 + 42 + 48 + 48}{3!} = \dfrac{222}{6} = 37$

For agent c

$\delta_c(\phi) = \nu(\{c\}) - \nu(\phi) = 6 - 0 = 6$ abc, bac

$\delta_c(\{a\}) = \nu(\{a,c\}) - \nu(\{a\}) = 72 - 12 = 60$ acb

$\delta_c(\{b\}) = \nu(\{b,c\}) - \nu(\{b\}) = 48 - 18 = 30$ bca

$\delta_c(\{a,b\}) = \nu(\{a,b,c\}) - \nu(\{a,b\}) = 120 - 60 = 60$ abc, bac

$\therefore \varphi_c = \dfrac{6 + 6 + 60 + 30 + 60 + 60}{6} = \dfrac{222}{6} = 37$

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3. Several friends made plans to go to see a movie, and each voted on a genre. The preference schedule is shown below:

Votes	3	2	5	3
First Choice	action	romance	comedy	drama
Second Choice	drama	drama	action	romance
Third Choice	comedy	comedy	drama	action
Forth Choice	romance	action	romance	comedy

Given this preference schedule, calculate the winner (and in each case show the working) using:

a) Plurality voting First Choice **(4 marks)**

Action = 3
Romance = 2
Comedy = 5
Drama = 3

$\therefore$ Comedy has 5 votes

b) Borda count **(4 marks)**

If $k = |\Omega|$ outcomes
First choices $k-1$
Second choice $k-2$

Action $= (3\times3) + 0 + (5\times2) + (3\times1) = 22$

Romance $= (0) + (2\times3) + (0) + (3\times2) = 0+6+0+6 = 12$

Comedy $= (3\times1) + (2\times1) + (5\times3) + 0 = 3+2+15+0 = 20$

Drama $= (3\times2) + (2\times2) + (5\times1) + (3\times3) = 6+4+5+9 = 24$

Drama wins (24)

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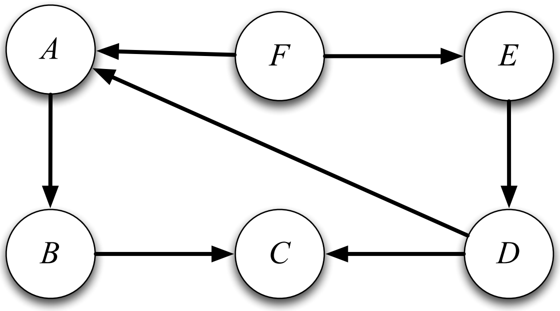
4. The following figure shows an *Abstract Argumentation* system.



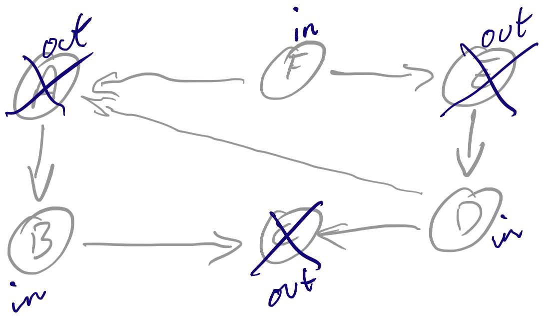
(a) Calculate the *Admissible* sets of this argumentation system. **(4 marks)**

Conflict Free			Mutually Defensive	Admissible
{a}	{a,c}	{b,d}	{f}	{f}
{b}	{a,e}	{b,e}	{d,f}	{d,f}
{c}	{a,c,e}	{b,f}	{b,f}	{b,f}
{d}				
{e}	{c,e}	{b,d,f}	{b,d,f}	{b,d,f}
{f}	{c,f}	{d,f}	∅	
∅				

(b) Determine the *Preferred Extensions* of this argumentation system. **(2 marks)**

{b,d,f}

(c) Determine the *Grounded Extensions* of this argumentation system. **(2 marks)**



{b,d,f}

5. (a) Identify with explanation the pure strategy Nash Equilibrium outcome(s) in the game of chicken, defined by the following payoff matrix: (**5 marks**)

		i	
		defect	coop
j	defect	1 1	2 4
	coop	4 2	3 3

There are 2 NE: (D, C) & (C, D)

If i knows that j cooperates, then i can do no better than to defect
Likewise if j knows that i will defect then j can do no better than to cooperate

(b) Give an example of a game which has no pure strategy Nash equilibria, but has a mixed strategy Nash equilibrium. (**5 marks**)

		i	
		defect	coop
j	defect	1 -1	-1 1
	coop	-1 1	1 -1

Defect (heads)
Coop (tails)

50% heads
50% tails

(c) Define and give an example of *dominant strategy equilibrium*. (**5 marks**)

		i	
		defect	coop
j	defect	6 6	4 6
	coop	6 4	4 4

A dominant strategy for one agent results in a better outcome irrespective of the strategy of the other agent

Eg D dominating C for both agents