COMP310
Multi-Agent Systems
Chapter 12 - Making Group Decisions

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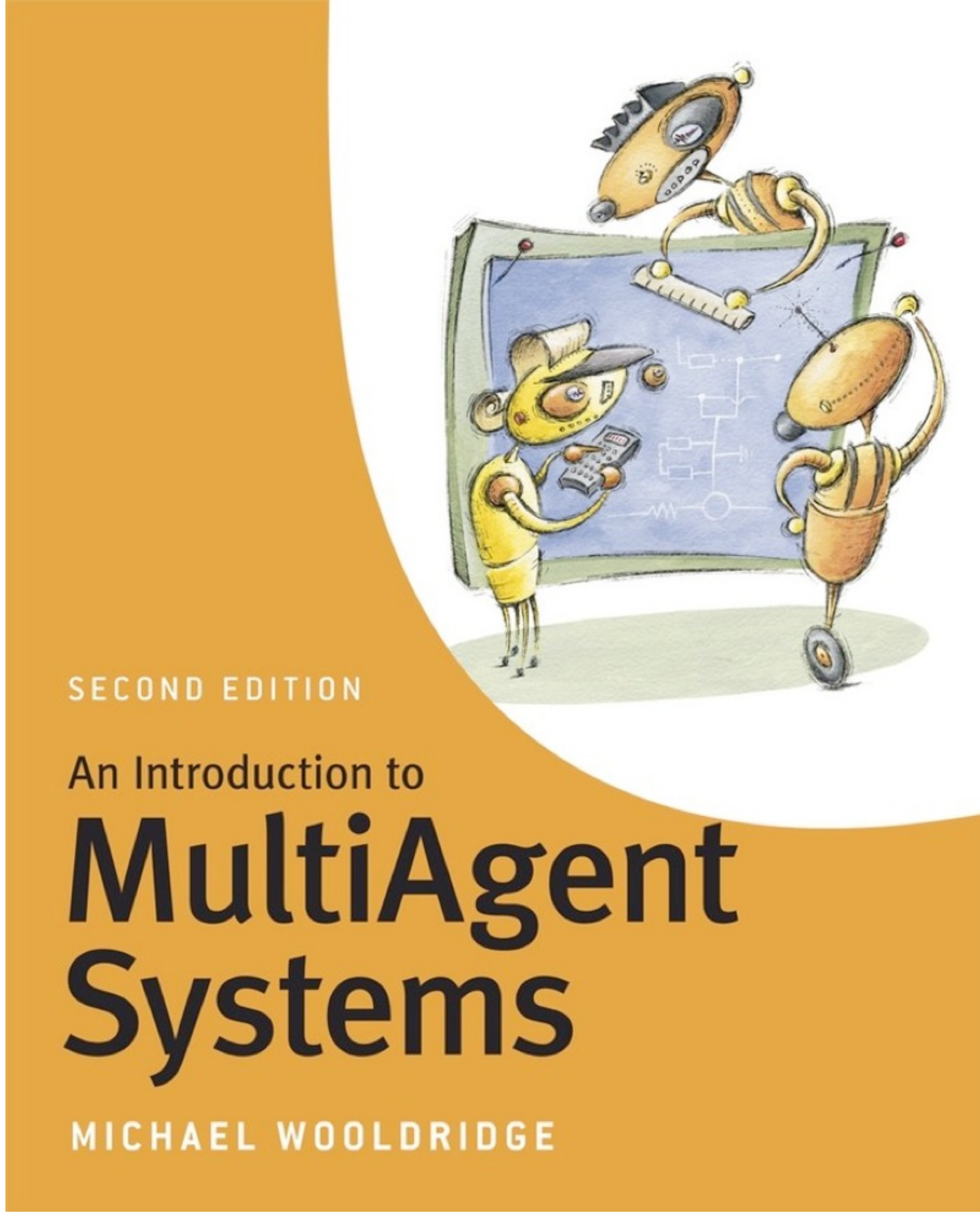


Social Choice

- We continue thinking in the same framework as the previous chapter:
  - multiagent encounters
  - game-like interactions
  - participants act strategically
- ***Social choice theory*** is concerned with group decision making.
  - Agents make decisions based on their preferences, but they are aware of other agents' preferences as well.
- Classic example of social choice theory: ***voting***
  - Formally, the issue is combining preferences to ***derive a social outcome***.

Components of a Social Choice Model

- Assume a set $Ag = \{1,\ldots,n\}$ of ***voters***.
  - These are entities who express preferences.
  - Voters make group decisions with respect to a set $\Omega = \{\omega_1, \omega_2, \ldots\}$ of ***outcomes***.
    - Think of these as the ***candidates***.
  - If $|\Omega| = 2$, we have a pairwise election.

- Each voter has preferences over Ω
  - An ordering over the set of possible outcomes Ω.
    - Sometimes we will want to pick one, most preferred candidate.
    - More generally, we may want to rank, or order these candidates.

Preference Order Example

Suppose

$\Omega = \{pear, plum, banana, orange\}$

then we might have agent i with preference order:

$(banana, plum, pear, orange)$

meaning

$banana >_i plum >_i pear >_i orange$

Preference Aggregation

- The fundamental problem of social choice theory is that…
  - ***…different voters typically have different preference orders!***

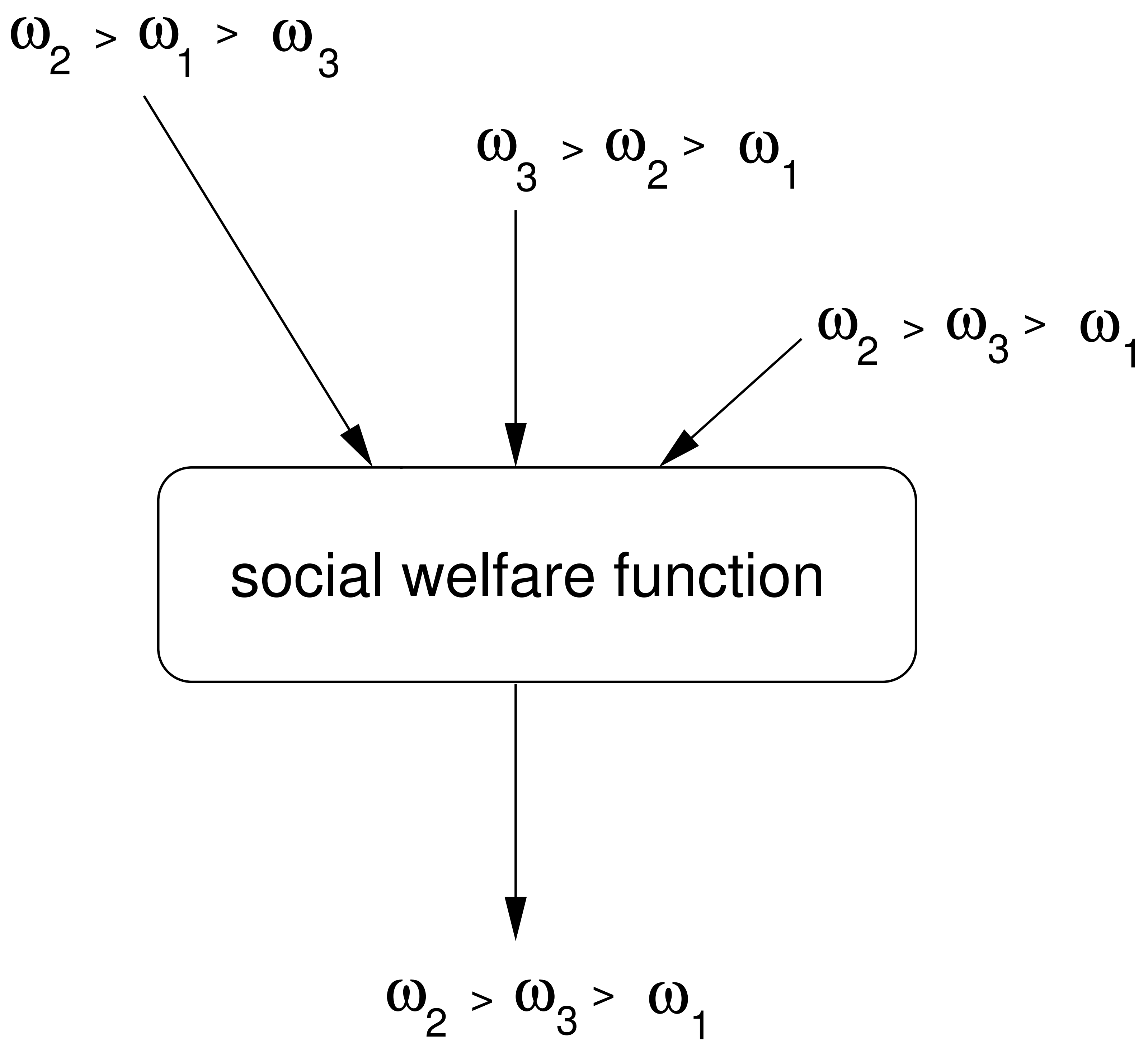
> "... given a collection of preference orders, one for each voter, how do we combine these to derive a group decision, that reflects as ***closely as possible*** the preferences of voters? ***..."***

- We need a way to combine these opinions into on overall decision.
  - What social choice theory is about is finding a way to do this.
  - Two variants of preference aggregation:
    - *social welfare functions*
    - *social choice functions*

Social Welfare Function

- Let $\Pi(\Omega)$ be a set of preference orderings over Ω
  - A ***social welfare function*** takes voter preferences and produces a ***social preference order***.
    - That is it merges voter opinions and comes up with an order over the candidates.

$$f : \underbrace{\Pi(\Omega) \times \cdots \times \Pi(\Omega)}_{n \text{ times}} \mapsto \Pi(\Omega)$$

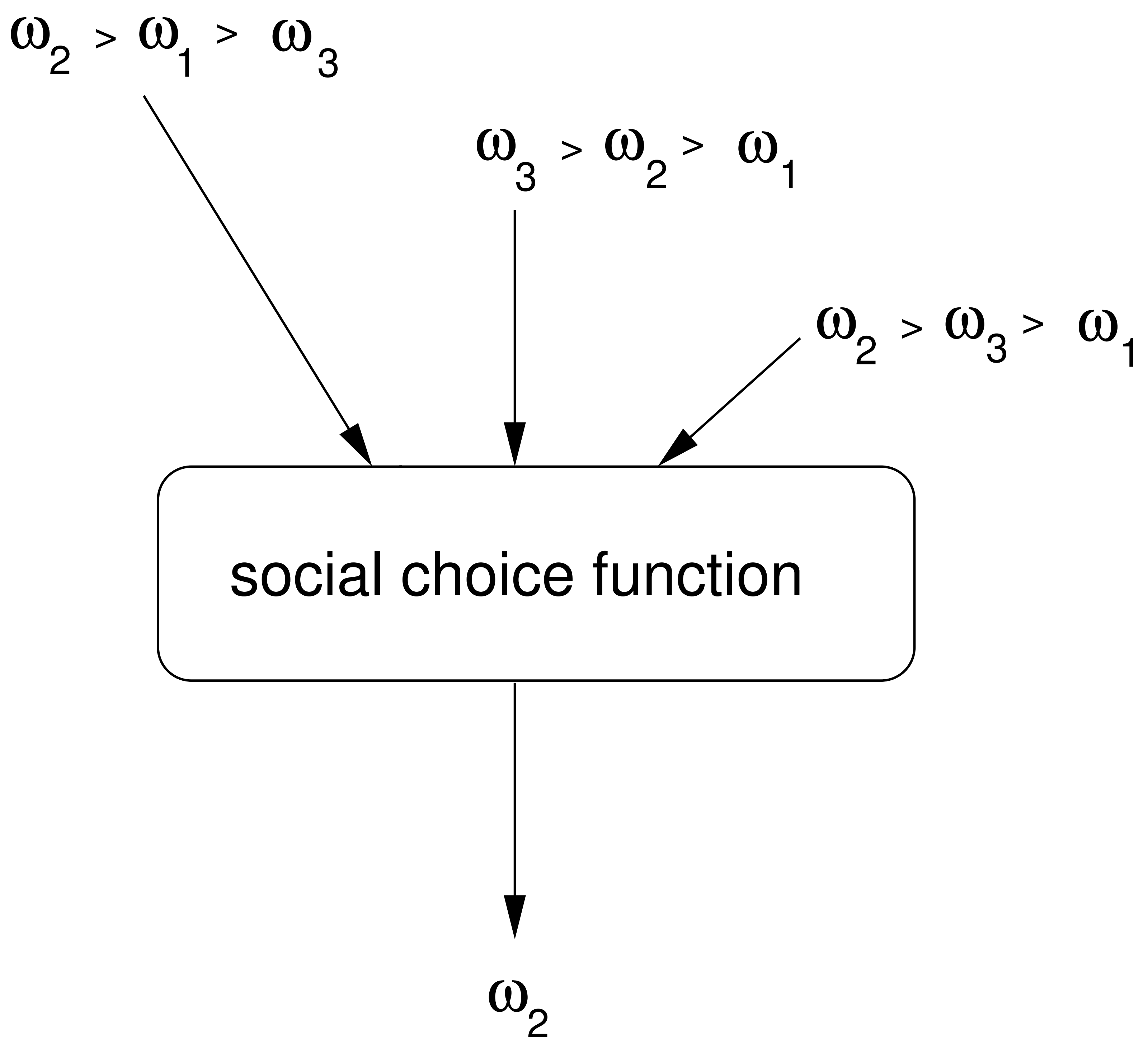


- We let $>_*$ denote to the outcome of a social welfare function: $\omega >_* \omega'$
  - which indicates that ω is ranked above ω' in the social ordering
    - Example: combining search engine results, collaborative filtering, collaborative planning, etc.

Social Choice Function

- Sometimes, we just one to select ***one*** of the possible candidates, rather than a social order.
  - This gives a ***social choice function*** (see opposite)
    - For example, a local by-election or presidential election
- In other words, we don't get an ordering out of a social choice function but, as its name suggests, we get ***a single choice***.
  - Of course, if we have a social welfare function, we also have a social choice function.
- For the rest of this chapter…
  - …we'll refer to both both social choice and social welfare functions as ***voting procedures***.

$$f : \underbrace{\Pi(\Omega) \times \cdots \times \Pi(\Omega)}_{n \text{ times}} \mapsto \Omega$$



Voting Procedures: Plurality

- Social choice function: selects a single outcome.
  - Each voter submits preferences.
  - Each candidate gets one point for every preference order that ranks them first.
- Winner is the one with largest number of points.
  - Also known in the UK as ***first past the post***, or ***relative majority***
    - Example: Political elections in UK.
- If we have only two candidates, then plurality is a ***simple majority election***

Anomalies with Plurality

Suppose:

$|Ag| = 100$ and $\Omega = \{\omega_1, \omega_2, \omega_3\}$

with:

40% voters voting for ω_1

30% of voters voting for ω_2

30% of voters voting for ω_3

With plurality, ω_1 gets elected even though a **clear majority** (60%) prefer another candidate!

Strategic Manipulation by Tactical Voting

- Suppose agent i wants ω_1 to win, but otherwise prefers $\boldsymbol{\omega_2}$ over ω_3
  - i.e. its preferences are: $\omega_1 >_i \omega_2 >_i \omega_3$

- However:
  - you believe 49% of voters have preferences: $\boldsymbol{\omega_2} > \omega_1 > \omega_3$
  - and you believe 49% have preferences: $\omega_3 > \boldsymbol{\omega_2} > \omega_1$

- You may do better voting for $\boldsymbol{\omega_2}$, ***even though this is not your true preference profile***.
  - This is ***tactical voting***: an example of ***strategic manipulation*** of the vote.

Condorcet's Paradox

Nicolas de Caritat, marquis de Condorcet (1743-1794)

- In a democracy, it seems inevitable that we can't choose an outcome that will make everyone happy.
- Condorcet's paradox tells us that in some situations, no matter which outcome we choose, a majority of voters will be unhappy with the outcome.

- Suppose $Ag = \{1,2,3\}$ and $\Omega = \{\omega_1,\omega_2,\omega_3\}$ with:

$$\omega_1 \succ_1 \omega_2 \succ_1 \omega_3$$

$$\omega_3 \succ_2 \omega_1 \succ_2 \omega_2$$

$$\omega_2 \succ_3 \omega_3 \succ_3 \omega_1$$

- For every possible candidate, there is another candidate that is preferred by a majority of voters!
  - If we pick ω_1, two thirds of the voters prefer ω_3 to ω_1.
  - If we pick ω_3, two thirds of the voters prefer ω_2.
  - If we pick ω_2, it is still the case that two thirds of the voters prefer a different candidate, in this case ω_1 to the candidate we picked.
- This is Condorcet's paradox: there are situations in which:
  - ***no matter which outcome we choose, a majority of voters will be unhappy with the outcome chosen.***

Sequential Majority Elections

- One way to improve on plurality voting is to reduce a general voting scenario to a series of pairwise voting scenarios.

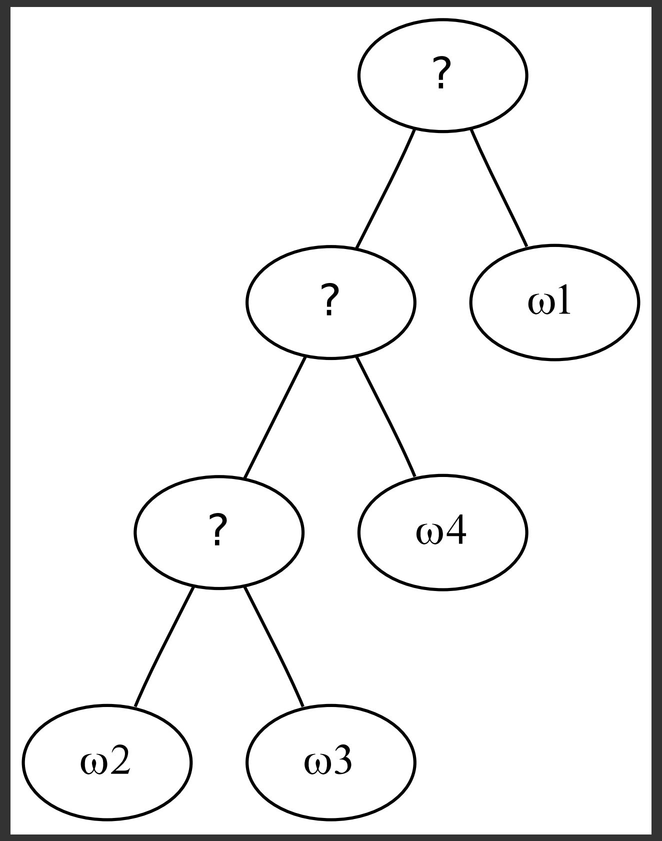
Linear Sequential Pairwise Elections

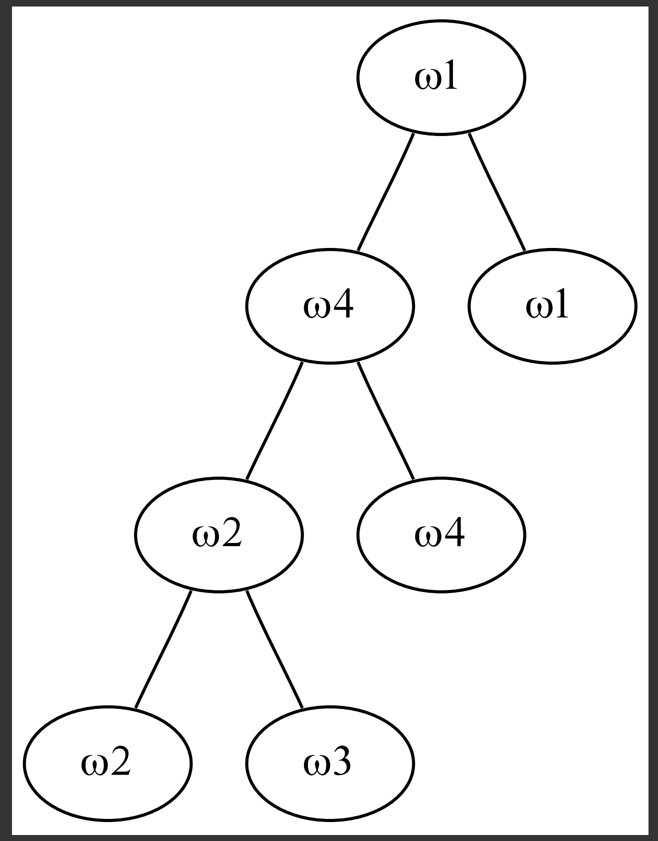
One agenda for the election between $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$ is $\omega_2, \omega_3, \omega_4, \omega_1$

First we have an election between ω_2 and ω_3.

The winner enters an election with ω_4.

The winner of that faces ω_1.



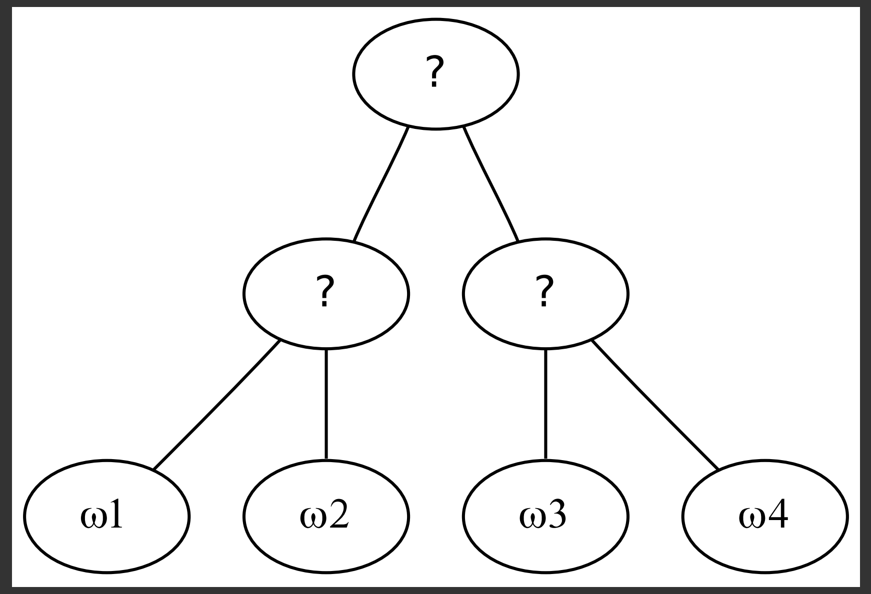


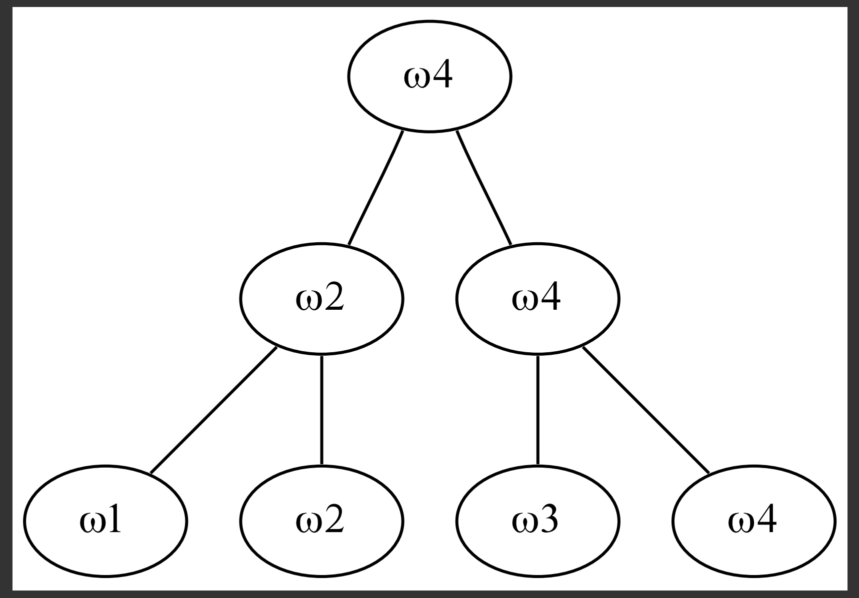
Balanced Binary Tree

We can also organise this as a balanced binary tree.

- *An election between ω_1 and ω_2.*
- *An election between ω_3 and ω_4.*
- *An election between the two winners.*

Rather like the Final Four





Linear Sequential Pairwise Elections

- Here, we pick an ordering of the outcomes – ***the agenda*** – which determines who plays against who.
  - For example, if the agenda is:

$$\omega_2, \omega_3, \omega_4, \omega_1$$

  - then the first election is between ω_2 and ω_3...
  - ... and the winner goes on to the second election with ω_4 ...
  - ... and the winner of this election goes in the final election with ω_1.

Anomalies with Sequential Pairwise Elections

Majority Graphs

A directed graph with:

- *vertices = candidates*
- *an edge (i, j) if i would beat j is a simple majority election.*

A compact representation of voter preferences. With an odd number of voters (no ties) the majority graph is such that:

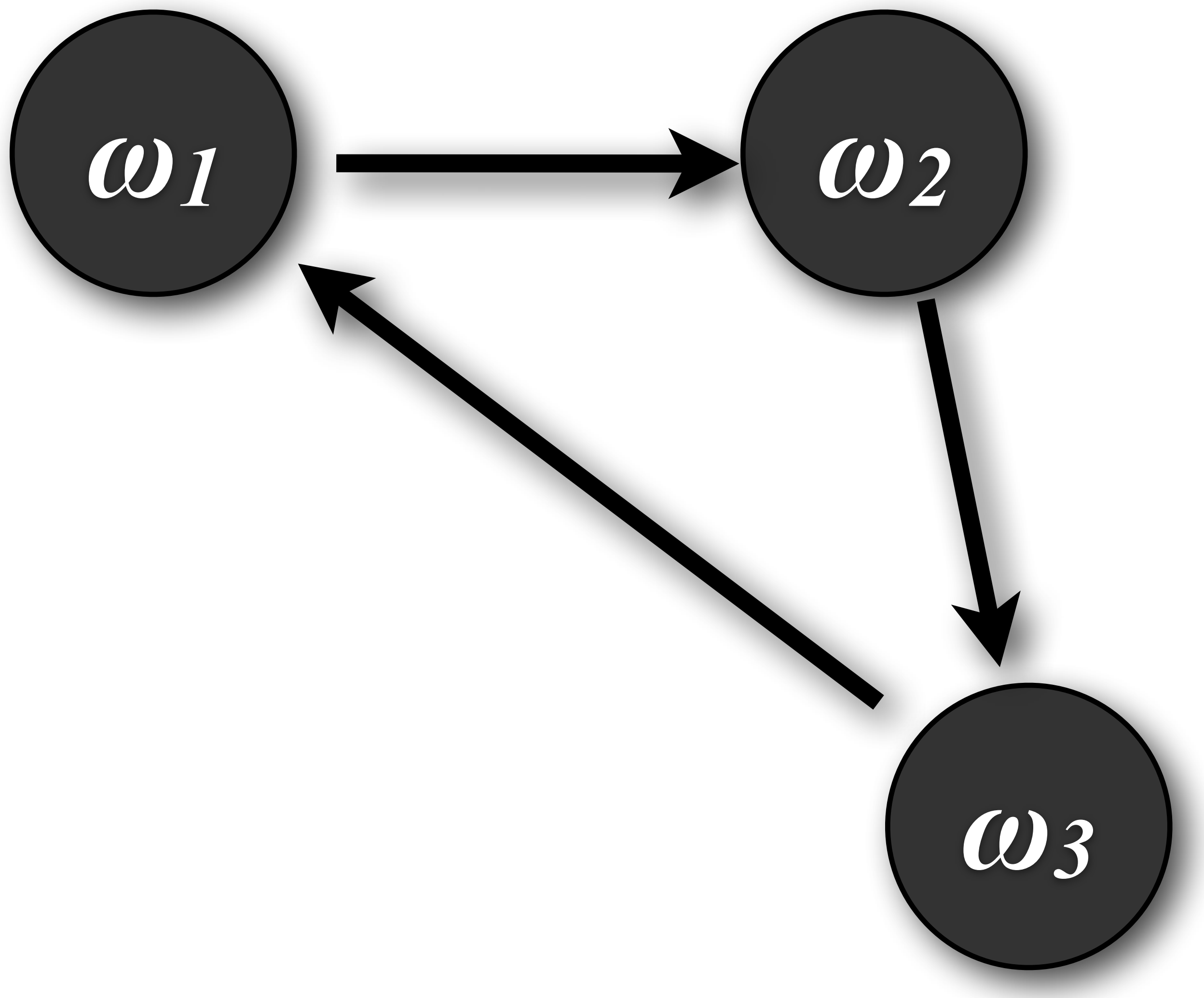
- The graph is complete.
- The graph is asymmetric.
- The graph is irreflexive.

Such a graph is called a ***tournament***, a nice summarisation of information about voter preferences.

- Suppose:
  - 33 voters have preferences: $\omega_1 >^* \omega_2 >^* \omega_3$
  - 33 voters have preferences: $\omega_3 >^* \omega_1 >^* \omega_2$
  - 33 voters have preferences $\omega_2 >^* \omega_3 >^* \omega_1$
- Then for every candidate, ***we can fix an agenda*** for that candidate to win in a sequential pairwise election!
- This idea is easiest to illustrate using a ***majority graph***.

Majority Graph Example

- Given the previous example:
  - with agenda (ω_3, ω_2, ω_1), ω_1 wins
    - i.e. the winner of ω_3 vs ω_2 is ω_2, which is beaten by ω_1
  - with agenda (ω_1, ω_3, ω_2), ω_2 wins
    - i.e. the winner of ω_1 vs ω_3 is ω_3, which is beaten by ω_2
  - with agenda (ω_1, ω_2, ω_3), ω_3 wins
    - i.e. the winner of ω_1 vs ω_2 is ω_1, which is beaten by ω_3



- Since the graph contains a cycle, it turns out that we can fix whatever result we want.
  - All we have to do is to pick the right order of the elections.

Agendas and Majority Graphs

- This is another example of a majority graph in which every outcome is a possible winner

1) ω_1 wins with agenda $(\omega_3, \omega_4, \omega_2, \omega_1)$

ω_3 vs ω_4
$\to \omega_3$ vs. ω_2
$\to \omega_3$ vs. ω_1
$\to \omega_1$

2) ω_2 wins with agenda $(\omega_1, \omega_3, \omega_4, \omega_2)$

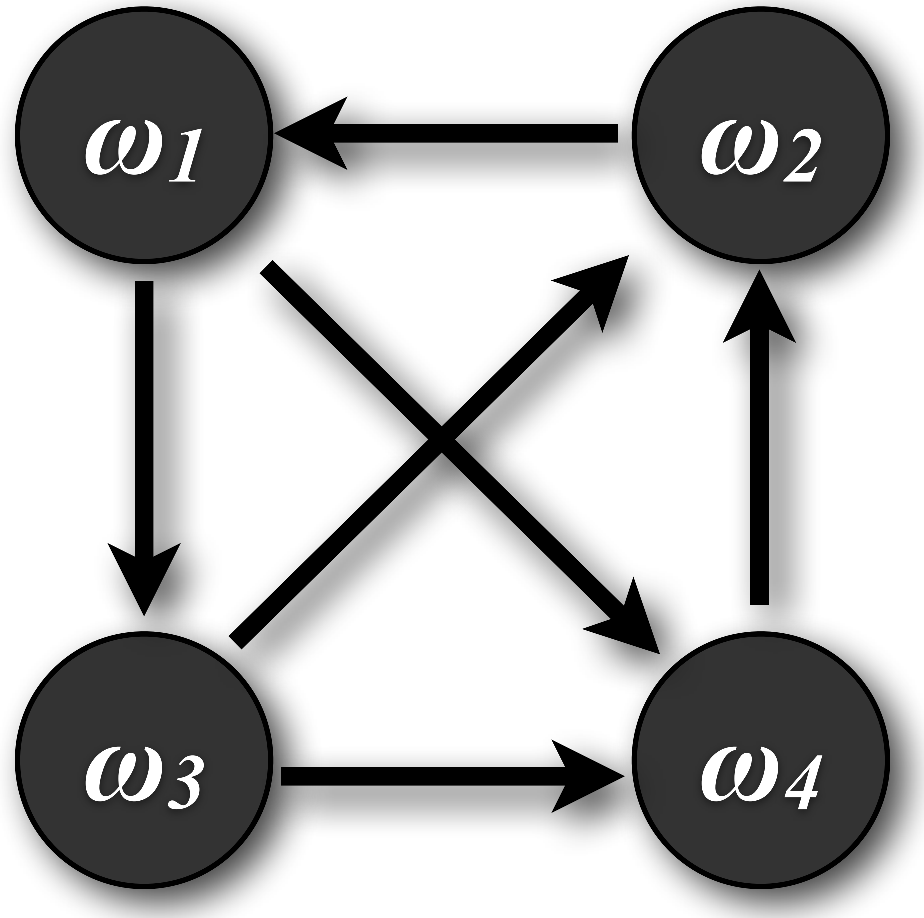
ω_1 vs ω_3
$\to \omega_1$ vs. ω_4
$\to \omega_1$ vs. ω_2
$\to \omega_2$

3) ω_3 wins with agenda $(\omega_1, \omega_4, \omega_2, \omega_3)$

ω_1 vs ω_4
$\to \omega_1$ vs. ω_2
$\to \omega_2$ vs. ω_3
$\to \omega_3$

4) ω_4 wins with agenda $(\omega_1, \omega_3, \omega_2, \omega_4)$

ω_1 vs ω_3
$\to \omega_1$ vs. ω_2
$\to \omega_2$ vs. ω_4
$\to \omega_4$



- Note, that there may be multiple agendas that result in the same winner:
  - ω_1 also wins with agenda $(\omega_4, \omega_2, \omega_3, \omega_1)$

Condorcet Winners

- Now, we say that a result is a ***possible winner*** if there is an agenda that will result in it winning overall.
  - The majority graph helps us determine this.

...

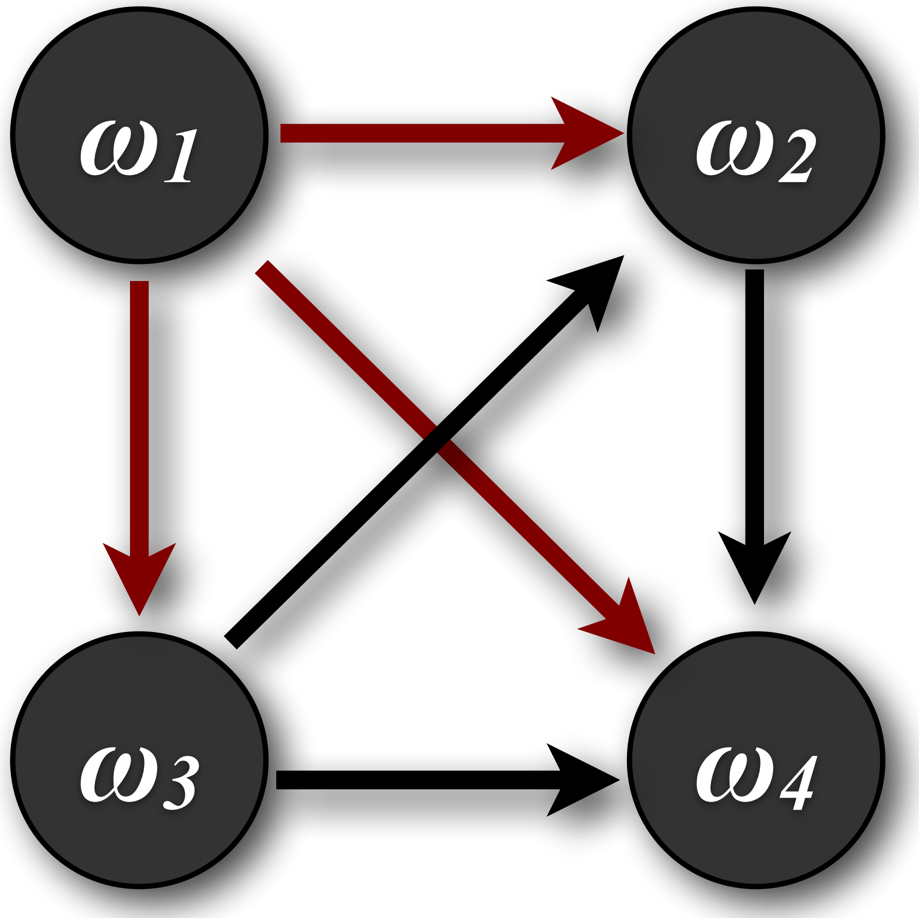
2) ω_1 wins with agenda $(\omega_1, \omega_3, \omega_4, \omega_2)$

ω_1 vs ω_3
$\rightarrow\omega_1$ vs. ω_4
$\rightarrow\omega_1$ vs. ω_2
$\rightarrow\omega_1$

3) ω_1 wins with agenda $(\omega_1, \omega_4, \omega_2, \omega_3)$

ω_1 vs ω_4
$\rightarrow\omega_1$ vs. ω_2
$\rightarrow\omega_1$ vs. ω_3
$\rightarrow\omega_1$

etc...



- To determine if ω_i is a possible winner, we have to find, for every other ω_j, if there is a path from ω_i to ω_j in the majority graph.
  - This is computationally easy to do.

Not examined in 2017-2018

The Slater Ranking

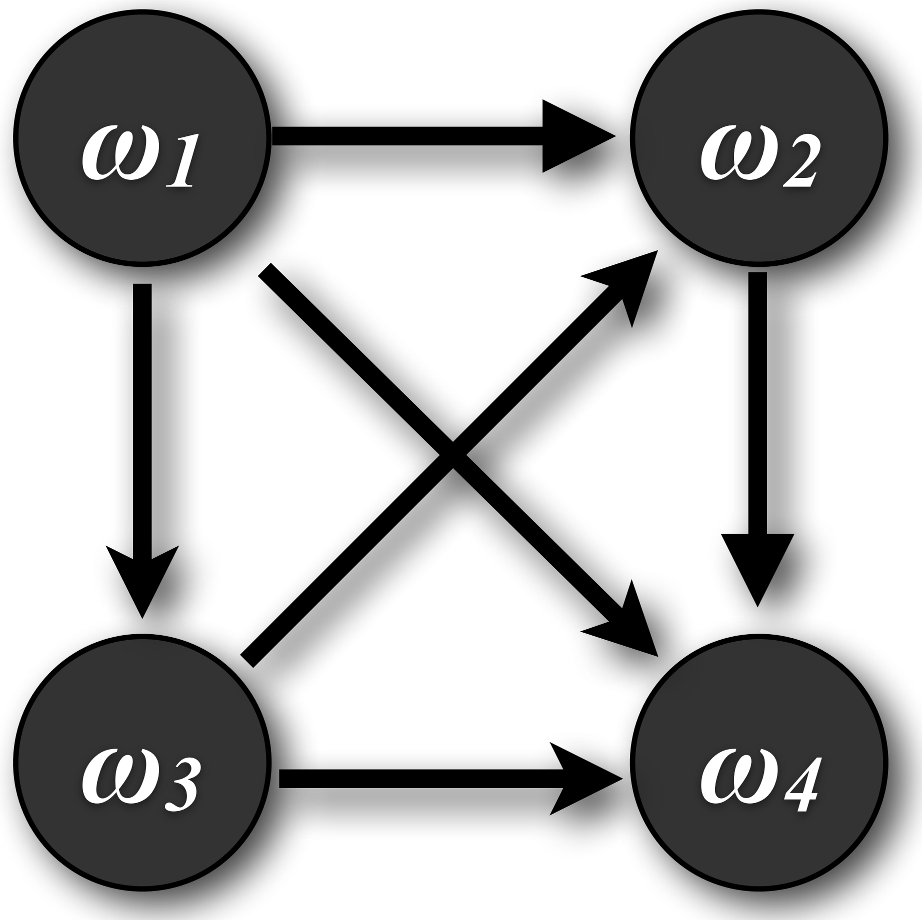
- The Slater rule is interesting because it considers:
  - the question “of which social ranking should be selected”, as
  - “the question of trying to find a consistent ranking that is as close to the majority graph as possible”
    - i.e. one that does not contain cycles
- Think of it as:
  - If we reversed some edges in a graph, which ordering minimises this inconsistency measure

Not examined in 2017-2018

Inconsistency Measure

- Consider this majority graph (upper)
  - No cycles, therefore the ranking $\omega_1 >^* \omega_3 >^* \omega_2 >^* \omega_4$ is acceptable:
  - The graph is ***consistent***

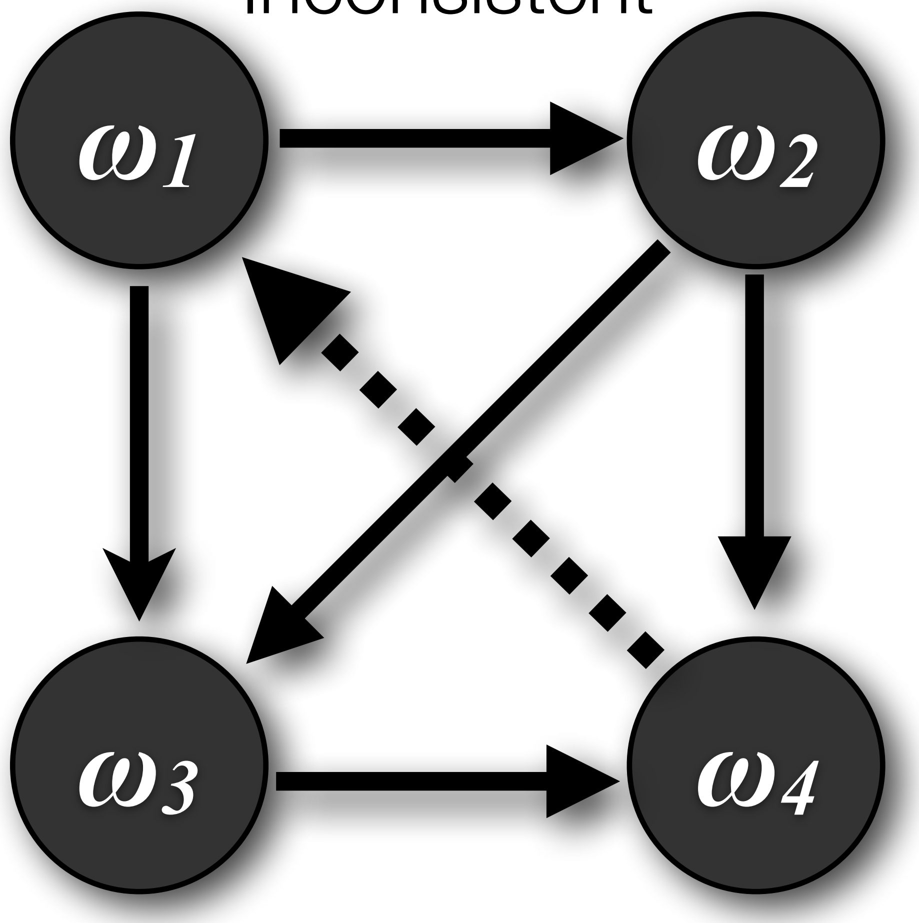
Consistent



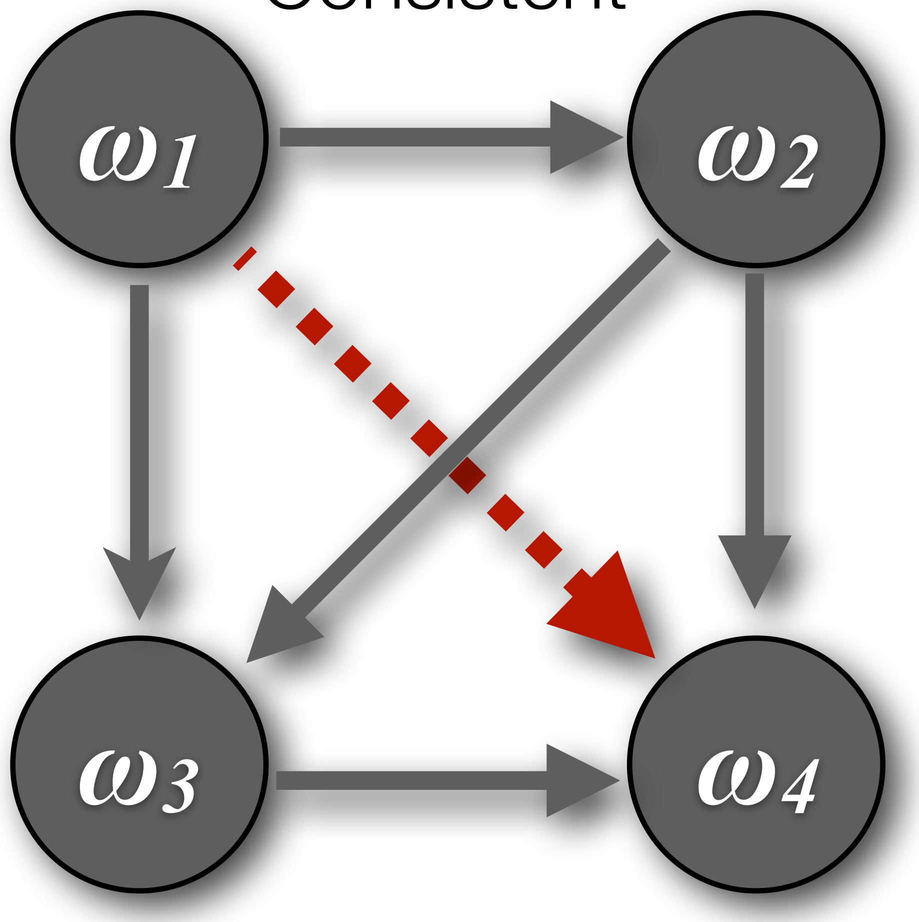
- This majority graph (lower) has cycles
  - We can have a ranking where one candidate beats another, although it would loose in a pairwise election
  - $\omega_1 >^* \omega_2 >^* \omega_3 >^* \omega_4$ even though ω_4 beats ω_1 in a pairwise election
  - By flipping the edge (ω_4, ω_1) we would have a consistent graph

- As this is the only edge we would need to flip, we say ***the cost of this order*** is 1.

Inconsistent



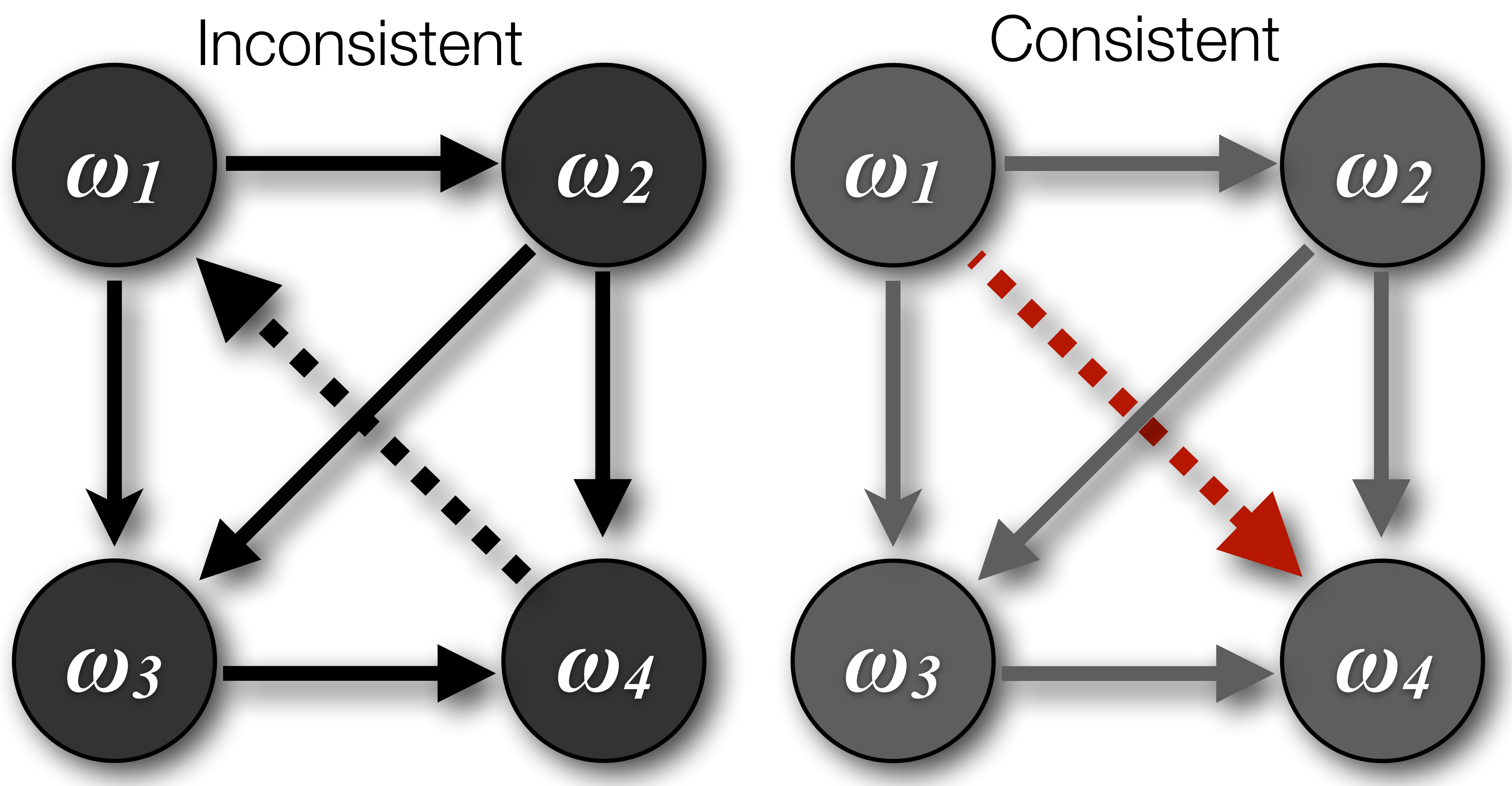
Consistent



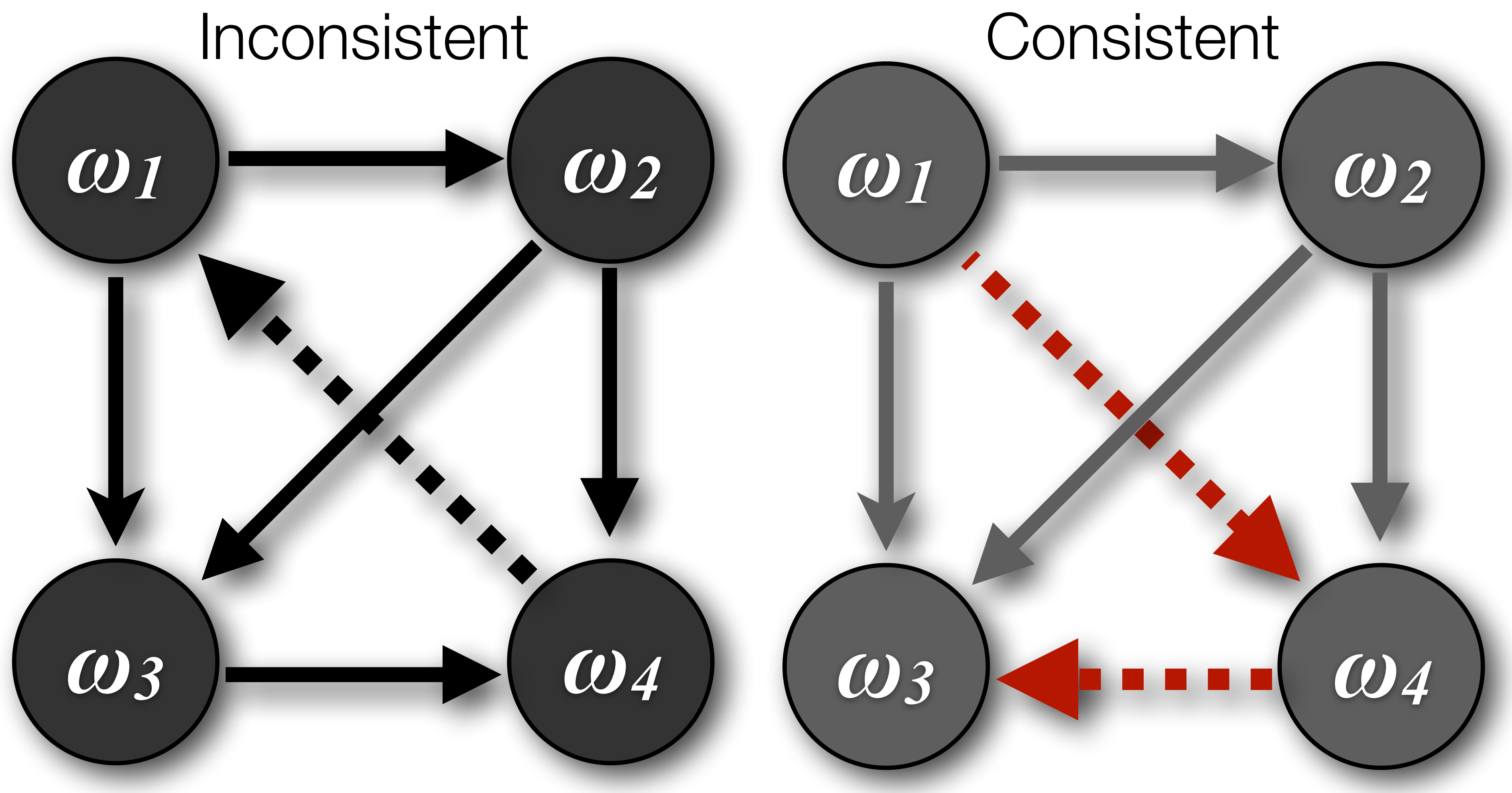
Not examined in 2017-2018

The Slater Ranking

- Remember that the following ranking has a ***cost of 1***
  - $\omega_1 >^* \omega_2 >^* \omega_3 >^* \omega_4$
    - By flipping the single edge (ω_4, ω_1) we would have a consistent graph.



- Consider the alternate ranking:
  - $\omega_1 >^* \omega_2 >^* \omega_4 >^* \omega_3$
    - In this case, we would have to flip two edges (ω_4, ω_1) and (ω_3, ω_4) giving a ***cost of 2*** giving



Not examined in 2017-2018

The Slater Ranking

- The Slater ranking is the one with minimal cost
  - i.e. calculate the cost of each ordering and find the one with the minimal cost
  - Computing the ordering with minimal Slater ranking is NP-hard

Borda Count

- One reason plurality has so many anomalies is that it ignores most of a voter's preference orders: it only looks at the top ranked candidate.
  - The Borda count takes whole preference order into account.
- Suppose we have k candidates - i.e. $k = |\Omega|$
  - For each candidate, we have a variable, counting the strength of opinion in favour of this candidate.
    - If ω_i appears first in a preference order, then we increment the count for ωi by $k - 1$;
    - we then increment the count for the next outcome in the preference order by $k - 2$,
    - . . . , until the final candidate in the preference order has its total incremented by 0.
- After we have done this for all voters, then the totals give the ranking.

Example of Borda Count

Assume we have three voters with preferences:

$$\omega_2 >_1 \omega_1 >_1 \omega_3$$
$$\omega_3 >_2 \omega_2 >_2 \omega_1$$
$$\omega_1 >_3 \omega_2 >_3 \omega_3$$

The Borda count of ω_2 is 4:

2 from the first place vote of voter 1.

1 each from the second place votes of voters 2 and 3.

What are the Borda counts of the other candidates?

Alternative Vote (AV)

- A ***social choice*** voting method
  - Also known as Instant Runoff Voting (IRV)
  - Results in a single winner
- Unlike Plurality voting, voters in IRV rank the candidates in order of preference.
  - ***Counting proceeds in rounds***, with the last place candidate being eliminated, until there is a majority vote
- Offers a solution to Condorcet's paradox

William Robert Ware

(1832-1915)

- Used in national elections in several countries, including:
  - Members of the Australian House of Representatives and most Australian state legislatures
  - The President of India, and members of legislative councils in India
  - The President of Ireland

Alternative Vote (AV)

Round 1

Votes	1st choice	2nd choice	3rd choice	4th choice
7	**action**	horror	comedy	drama
5	**comedy**	action	horror	drama
2	**drama**	horror	comedy	action
5	**comedy**	drama	action	horror
4	**horror**	action	drama	comedy

23 voters chose their favourite movie genres.

Majority (i.e. >50%) will be 12 or more votes

	Round 1	*Round 2*	*Round 3*
action	*7*		
comedy	*5+5=10*		
drama	*2*		
horror	*4*		

In the first round, we consider all of the 1st choice votes

As ***drama*** received the fewest votes, we eliminate this and reallocate the overall votes.

Alternative Vote (AV)

Round 2

Votes	1st choice	2nd choice	3rd choice	4th choice
7	**action**	horror	comedy	~~*drama*~~
5	**comedy**	action	horror	~~*drama*~~
2	~~*drama*~~	**horror**	comedy	action
5	**comedy**	~~*drama*~~	action	horror
4	**horror**	action	~~*drama*~~	comedy

23 voters chose their favourite movie genres.

Majority (i.e. >50%) will be 12 or more votes

	Round 1	*Round 2*	*Round 3*
action	*7*	*7*	
comedy	*5+5=10*	*10*	
drama	*2*	——	
horror	*4*	*4+2=6*	

In the second round, we allocate the 2 votes for ***drama*** to the next choice, which is horror

However, ***horror*** now has the fewest votes, and is eliminated

Alternative Vote (AV)

Round 3

Votes	1st choice	2nd choice	3rd choice	4th choice
7	**action**	~~horror~~	comedy	~~drama~~
5	**comedy**	action	~~horror~~	~~drama~~
2	~~drama~~	~~horror~~	**comedy**	action
5	**comedy**	~~drama~~	action	~~horror~~
4	~~horror~~	**action**	~~drama~~	comedy

23 voters chose their favourite movie genres.

Majority (i.e. >50%) will be 12 or more votes

	Round 1	*Round 2*	*Round 3*
action	*7*	*7*	*7+4=11*
comedy	*5+5=10*	*5+5=10*	*5+2+5=12*
drama	*2*	——	——
horror	*4*	*4+2=6*	——

In the third round, we allocate the 6 votes for ***horror*** to the next choices: 2 votes to ***comedy***, and 4 to ***action***

Comedy now has the majority votes

Desirable Properties of Voting Procedures

- Can we classify the properties we want of a “good” voting procedure?
- Three key properties:
  - The Pareto property;
  - The Condorcet Winner condition;
  - Independence of Irrelevant Alternatives (IIA).
- We should also avoid dictatorships!

The Pareto Property

If everybody prefers ω_i over ω_j, then ω_i should be ranked over ω_j in the social outcome.

Condorcet Winner

If ω_i is a condorcet winner, then ω_i should always be ranked first.

Independence of Irrelevant Alternatives (IIA)

Whether ω_i is ranked above ω_j in the social outcome should depend only on the relative orderings of ω_i and ω_j in voters profiles.

The Pareto Condition

- Recall the notion of Pareto efficiency from the previous lecture.
  - An outcome is Pareto efficient if ***there is no other outcome that makes one agent better off*** without making another worse off.
  - In voting terms, if every voter ranks ω_i above ω_j then $\omega_i \succ_* \omega_j$.

- Satisfied by plurality and Borda but not by sequential majority.

The Condorcet winner condition

- Recall that the Condorcet winner is an outcome that would beat every other outcome in a pairwise election.
  - A Condorcet winner is a strongly preferred outcome.
- The Condorcet winner condition says that if there is a Condorcet winner, then it should be ranked first.
  - Seems obvious.
- However, of the ones we've seen, only sequential majority satisfies it.

Independence of irrelevant alternatives

- Suppose there are a number of candidates including ω_i and ω_j and voter preferences make $\omega_i \succ_* \omega_j$.
  - Now assume one voter k changes preferences, but still ranks $\omega_i \succ_k \omega_j$
  - The independence of irrelevant alternatives (IIA) condition says that however $\succ_*$ changes, $\omega_i \succ_* \omega_j$ still.
  - In other words, the social ranking of ω_i and ω_j should depend only on the way they are ranked in the $\succ$ relations of the voters.
- Plurality, Borda and sequential majority do not satisfy IIA.

Dictatorship

- Not a desirable property, but a useful notion to define.
- A social welfare function f is a dictatorship if for some agent i:

$$f(\varpi_1, \varpi_2, \ldots \varpi_n) = \varpi_i$$

$\varpi_1, \varpi_2, \ldots \varpi_n$ denotes the preference orders of agents $1, \ldots, n$

  - In other words the output is exactly the preference order of the single "dictator" agent i.

- Plurality and the Borda count are not dictatorships.
  - But, dictatorships satisfy the Pareto condition and IIA.

Theoretical Results

- We have now explored several social choice functions
  - Do any of these satisfy our desirable properties (i.e. Pareto, etc)?
    - **No** - according to Arrow’s Theorem
- Furthermore, voters can benefit by strategically misrepresenting their preferences, i.e., lying – tactical voting
  - Are there any voting methods which are non-manipulable, in the sense that voters can never benefit from misrepresenting preferences?
    - **No** - according to the Gibbard-Satterthwaite Theorem

Theoretical Results

- Arrows Theorem
  - For elections with more than 2 candidates the ***only voting procedure satisfying the Pareto condition and IIA is a dictatorship***
    - in which the social outcome is in fact simply selected by one of the voters.
  - This is a negative result: there are fundamental limits to democratic decision making!

- The Gibbard-Satterthwaite Theorem
  - The ***only non-manipulable voting method*** satisfying the Pareto property for elections with more than 2 candidates is a ***dictatorship***.
  - In other words, every "realistic" voting method is prey to strategic manipulation…

Computational Complexity to the Rescue

- However…
  - Gibbard-Satterthwaite only tells us that manipulation is possible ***in principle***.
    - It does not give any indication of how to misrepresent preferences.
  - Bartholdi, Tovey, and Trick showed:
    - that there are elections that are prone to manipulation in principle, but where manipulation was ***computationally complex***.
    - “Single Transferable Vote” is NP-hard to manipulate!

Summary

- In this lecture we have looked at mechanisms for group decision making.
  - This has been a bit stylised — we looked at how, if a group of agents ranks a set of outcomes, we might create a consensus ranking.
    - This does have a real application in voting systems.
    - Social choice mechanisms are increasingly used in real systems as a way to reach consensus.
  - We looked at the behaviour of some existing voting systems and some theoretical results for voting systems in general.
    - most of these results were pretty negative.
- Lots we didn't have time to cover— another area with lots of active research.

Class Reading (Chapter 12):

"The computational difficulty of manipulating an election", J.J. Bartholdi, C.A.Tovey and M.A.Trick. Social Choice and Welfare. Vol. 6 227-241, 1989.

This is the article that prompted the current interest in computational aspects of voting. It is a technical scientific article, but the main thrust of the article is perfectly understandable without a technical detailed background.