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MODAL LOGIC AND PHILOSOPHY

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Modal logic is one of philosophy's many children. As a mature adult it has moved out of the parental home and is nowadays straying far from its parent. But the ties are still there: philosophy is important for modal logic, modal logic is important for philosophy. Or, at least, this is a thesis we try to defend in this chapter. Limitations of space have ruled out any attempt at writing a survey of all the work going on in our field — a book would be needed for that. Instead, we have tried to select material that is of interest in its own right or exemplifies noteworthy features in interesting ways. Here are some themes which have guided us throughout the writing:

- *The back-and-forth between philosophy and modal logic.* There has been a good deal of give-and-take in the past. Carnap tried to use his modal logic to throw light on old philosophical questions, thereby inspiring others to continue his work and still others to criticise it. He certainly provoked Quine, who in his turn provided — and continues to provide — a healthy challenge to modal logicians. And Kripke's and David Lewis's philosophies are connected, in interesting ways, with their modal logic. Analytic philosophy would have been a lot different without modal logic!

- *The interpretation problem.* The problem of providing a certain modal logic with an intuitive interpretation should not be conflated with the problem of providing a formal system with a model-theoretic semantics. An intuitively appealing model-theoretic semantics may be an important step towards solving the interpretation problem, but only a step. One may compare this situation with that in probability theory, where definitions of concepts like 'outcome space' and 'random variable' are orthogonal to questions about "interpretations" of the concept of probability.

- *The value of formalisation.* Modal logic sets standards of precision, which are a challenge to — and sometimes a model for — philosophy. Classical philosophical questions can be sharpened and seen from a new perspective when formulated in a framework of modal logic. On the other hand, representing old questions in a formal garb has its dangers, such as simplification and distortion.

- *Why modal logic rather than classical (first or higher order) logic?* The idioms of modal logic — today there are many! — seem better to correspond to human ways of thinking than ordinary extensional logic. (Cf. Chomsky's conjecture that the NP + VP pattern is wired into the human brain.)

In his *An Essay in Modal Logic* [107] von Wright distinguished between four kinds of modalities: *alethic* (modes of truth: necessity, possibility and impossibility), *epistemic* (modes of being known: known to be true, known to be false, undecided), *deontic* (modes of obligation: obligatory, permitted, forbidden) and *existential* (modes of existence: universality, existence, emptiness). The existential modalities are not usually counted as modalities, but the other three categories are exemplified in three sections into which this chapter is divided. Section 1 is devoted to alethic modal logic and reviews some main themes at the heart of philosophical modal logic. Sections 2 and 3 deal with topics in epistemic logic and deontic logic, respectively, and are meant to illustrate two different uses that modal logic or indeed any logic can have: it may be applied to already existing (non-logical) theory, or it can be used to develop new theory.

1 ALETHIC MODAL LOGIC

In this part we consider the challenge that Quine posed in 1947 to the advocates of modal logic to provide an account of modal notions that is intuitively clear, allows "quantifying in", and does not presuppose intensional entities. The modal notions that Quine and his contemporaries were primarily concerned with in the 1940's were, broadly speaking, the logical modalities rather than the metaphysical ones that have since come to prevail. In the 1950's modal logicians responded to Quine's challenge by providing quantified modal logic with model-theoretic semantics of various types. In doing so they also, explicitly or implicitly, addressed Quine's interpretation problem. Here we shall consider the approaches developed by Carnap in the late 1940's, and by Kanger, Hintikka, Montague, and Kripke in the 1950's and early 1960's, and discuss to what extent these approaches were successful in meeting Quine's doubts about the intelligibility of quantified modal logic.

It is useful to divide the reactions to Quine's challenge into two periods. During the first period modal logicians provided modal logic with formal semantics as just mentioned. In the second period philosophers — inspired by the success of possible worlds semantics — came to take the notion of a possible world seriously as a tool for philosophical analysis. Philosophical analyses in terms of possible worlds were provided for many concepts of central philosophical importance: propositional attitudes [42, 43, 45], metaphysical necessity, identity, and naming [69, 70], "intensional entities" like propositions, properties and events [84, 61, 102, 103], counterfactual conditionals and causality [77, 78], supervenience [62]. At the same time the notion of a possible world itself came in for philosophical analysis. The problems of giving a satisfactory analysis of this notion indicates that Quine's interpretational challenge is still alive. The basic philosophical questions surrounding the notions of alethic necessity and possibility are as puzzling as ever! We end this section by discussing the relationship between the logical and metaphysical interpretation of the alethic modalities.

1.1 The search for the intended interpretation

Starting with the work of C. I. Lewis, an immense number of formal systems of modal logic have been constructed based on classical propositional or predicate logic. The originators of modern modal logic, however, were not very clear about the intuitive meaning of the symbols $\Box$ and $\Diamond$, except to say that these should stand for some kind of necessity and possibility, respectively. For instance, in *Symbolic Logic* [72], Lewis and Langford write:

> It should be noted that the words "possible", "impossible" and "necessary" are highly ambiguous in ordinary discourse. The meaning here assigned to $\Diamond p$ is a *wide* meaning of "possibility" — namely, logical conceivability or the absence of self-contradiction. (160–61)

This situation led to a search for more rigorous interpretations of modal notions. Gödel [35] suggested interpreting the necessity operator $\Box$ as standing for provability (*informal provability* or, alternatively, *formal provability* in a fixed formal system), a suggestion that subsequently led to the modern *provability interpretations* of Solovay, Boolos and others.[1]

After Tarski [105, 106] had developed rigorous notions of satisfaction, truth and logical consequence for classical extensional languages, the question arose whether the same methods could be applied to the languages of modal logic and related systems. One natural idea, that occurred to Carnap in the 1940's, was to let $\Box\varphi$ be true of precisely those formulæ φ that are *logically valid* (or logically true) according to the standard semantic definition of logical validity. This idea led him to the following semantic clause for the operator of logical necessity:

$\Box\varphi$ is true in an interpretation $\mathcal{I}$ iff φ is true in every interpretation $\mathcal{I}'$.

This kind of approach, which we may call the *validity interpretation*, was pursued by Carnap, using so-called state descriptions, and subsequently also by Kanger [53, 54] and Montague [83], using Tarski-style model-theoretic interpretations rather than state descriptions. In Hintikka's and Kanger's early work on modal semantics other interpretations of $\Box$ were also considered, especially, epistemic ('It is known that φ') and deontic ones ('It ought to be the case that φ'). In order to study these and other non-logical modalities, the introduction by Hintikka and Kanger of *accessibility relations* between

[1] Cf. [101] and [13].