An Extension of Propositional Temporal Resolution

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We propose an extention of propositional temporal resolution presented in [Fis91, Fis92, DF00, FDP01] to solve the satisfiability problem for some subclass of the monodic fragment of the first-order temporal logic [HWZ00]. Knowledge of all basic notions related to propositional temporal resolution is assumed.

Let $\mathcal{L}$ be a first-order temporal language with some (excessive) set of future-time temporal connectives '$\Diamond$' (*sometime*), '$\Box$' (*always*), '$\bigcirc$' (*in the next moment*), '$\mathcal{U}$'(*until*), and '$\mathcal{W}$' (*unless*, or *weak until*), but without equality or functional symbols. We do not include past-time operators since our models have a finite past, therefore such operators add no extra expressive power. It is supposed that the semantics of $\mathcal{L}$-formulas is the same as in [Fis91, Fis92]. Following [HWZ00] we consider the set of all $\mathcal{L}$-formulas φ such that any subformula of φ of the form $\Diamond\psi$, $\Box\psi$, $\bigcirc\psi$, $\psi_1\,\mathcal{U}\,\psi_2$, $\psi_1\,\mathcal{W}\,\psi_2$ has at most one free variable. Such formulas are called *monodic*, and the set of monodic $\mathcal{L}$-formulas is denoted by $\mathcal{L}_1$. Decidability of $\mathcal{L}_1$ was proved in [HWZ00], however no deduction-based decision procedure has been proposed.

A temporal $\mathcal{L}$-problem, which satisfiability we are interested in, is represented in a normal form called *separated normal form* ($\mathcal{SNF}$) which uses a restricted set of temporal operators (namely, $\Diamond, \Box$ and $\bigcirc$). Here $\mathcal{SNF}$ is a refinement of the Separated Normal Form (SNF_f) introduced in [Fis92], and now the term 'separated' has two meanings. It expresses our intention, firstly, following [Fis92] to separate present and future-time components of formulas, and secondly, to separate the temporal and (ordinary) first-order parts of the problem in such way to get the temporal part as simple as possible. The transformation to $\mathcal{SNF}$ is based on the renaming technique [Fis91, Fis92].

A separated normal form $\mathcal{SNF}$ is a triple $< \mathcal{S}, \mathcal{U}, \mathcal{T} >$ where $\mathcal{S}$ and $\mathcal{U}$ are a *universal part* and an *initial part*, respectively, given by finite sets of nontemporal first-order formulas (that is, without temporal operators), and $\mathcal{T}$ is a *temporal part* given by a finite set of *temporal clauses*. All formulas are written in $\mathcal{L}$ extended by a set of predicate and propositional symbols. A temporal clause has one of the following forms:

$$P(\bar{x}) \Rightarrow \bigcirc \bigwedge_{i=1}^{n} Q_i(\bar{x}) \quad (\textit{predicate step clause}), \qquad p \Rightarrow \bigcirc \bigwedge_{j=1}^{m} q_j \quad (\textit{proposition step clause}),$$
$$P(\bar{x}) \Rightarrow \bigcirc\Diamond Q(\bar{x}) \quad (\textit{predicate eventuality clause}), \qquad p \Rightarrow \bigcirc\Diamond q \quad (\textit{ proposition eventuality clause})$$

where P, Q, Q_i are predicate symbols, p, q, q_j are propositional symbols, and $\Rightarrow$ is a substitute for the implication. Sometimes temporal clauses are called *temporal rules* to make distinctions between their left- and right-hand sides. Without loss of generality we can suppose that there are no two different temporal step (or two eventuality) rules with the same left-hand sides. Semantics of a formula in $\mathcal{SNF}$ is defined under the supposition that universal and temporal parts are closed by the outermost prefixes $\Box\forall$, the initial part is closed only by universal quantifiers, and we do not make any difference between a finite set of formulas Γ and the conjunction $\bigwedge\Gamma$ of formulas in it.

Below we present main points of a first-order temporal resolution method which gives a decision procedure for so called *ground eventuality* monodic fragment $\mathcal{L}_1^0 \subset \mathcal{L}_1$ defined as follows. A formula φ, $\varphi \in \mathcal{L}_1$,that is all predicates in predicate step clauses are *monadic*, belongs to $\mathcal{L}_1^0$ if the temporal part of its $\mathcal{SNF}$ does not contain predicate eventuality clauses.

Example Let $\varphi = \forall x((\Diamond\forall z P(x,z)) \supset (Q(x,x)\,\mathcal{U}\,\exists y R(c,y)))$, then its $\mathcal{SNF}$ is the following

$$\mathcal{S} = \{\ s1.\quad P_1(x) \vee P_2(x)\ \} \qquad \mathcal{T} = \left\{ \begin{array}{ll} t1.\quad Q_2(x) \Rightarrow \bigcirc(Q_1(x) \wedge Q_2(x)), & t3.\quad p \Rightarrow \bigcirc\Diamond q. \\ t2.\quad Q_3(x) \Rightarrow \bigcirc(P_4(x) \wedge Q_4(x)), & \end{array} \right\}$$

$$\mathcal{U} = \left\{ \begin{array}{llllll} u1. & P_1(x) \supset (Q_1(x) \wedge Q_2(x)), & u4. & P_4(x) \supset (P_3(x) \vee q), & u7. & P_3(x) \supset Q(x,x), \\ u2. & P_2(x) \supset ((Q_3(x) \wedge P_3(x)) \vee q), & u5. & Q_4(x) \supset (Q_3(x) \vee q), & u8. & q \supset \exists y R(c,y). \\ u3. & P_2(x) \supset (p \vee q), & u6. & Q_1(x) \supset \exists z \neg P(x,z), & & \end{array} \right\}$$

Here p, q are new propositional symbols, $P_i, Q_i, 1 \leq i \leq 4$, are new (monadic) predicate symbols (new names). The transformation to $\mathcal{SNF}$ is invariant with respect to satisfiability, that is $\mathcal{SNF}$ of φ is satisfiable if, and only if, φ is satisfiable.

The key role in propositional temporal resolution is played by so called *merged step clauses*. It appeared that in the case of $\mathcal{L}_1^0$ we can define an analogue of propositional merged step clauses. That gives a possibility to formulate for $\mathcal{L}_1^0$ a complete calculus which is quite analogous to the propositional temporal resolution.

Given the set C of constants occurring in a formula φ and the temporal part of some $\mathcal{SNF}$ of φ, (over)merged temporal clauses for temporal resolution over $\mathcal{SNF}$ are defined as follows.

Let $\{P_i(x) \Rightarrow \bigcirc R_i(x), \mid 1 \leq i \leq k\}$ and $\{p_j \Rightarrow \bigcirc r_j \mid 1 \leq j \leq l\}$ be the sets of predicate step rules and proposition step rules of $\mathcal{T}$, respectively. Let Λ be the powerset of $\{1, \dots, k\}$ and Δ be the powerset of $\{1, \dots, l\}$. For any $\lambda \in \Lambda$ let form the pair of conjunctions $A_\lambda(x) = \bigwedge_{i \in \lambda} P_i(x)$ and $B_\lambda(x) = \bigwedge_{i \in \lambda} R_i(x)$. As usual, if $\lambda = \emptyset$ then $A_\lambda(x) = B_\lambda(x) = \mathbf{true}$.

Let Λ_i be a subset of Λ, $\Lambda_i \subset \Lambda$, δ be an element of Δ, $\delta \in \Delta$, and π be a function from C to Λ_i, $\pi : C \to \Lambda_i$. Then an *overmerged rule* corresponding to (Λ_i, δ, π) is $\mathcal{L}_{(i,\delta,\pi)} \Rightarrow \bigcirc \mathcal{R}_{(i,\delta,\pi)}$ where

$$\begin{aligned} \mathcal{L}_{(i,\delta,\pi)} &= \bigwedge_{\lambda \in \Lambda_i} \exists x A_\lambda(x) \wedge \bigwedge_{j \in \delta} p_j \wedge \bigwedge_{c \in C} A_{\pi(c)}(c) \wedge \forall x \bigvee_{\lambda \in \Lambda_i} A_\lambda(x), \\ \mathcal{R}_{(i,\delta,\pi)} &= \bigwedge_{\lambda \in \Lambda_i} \exists x R_\lambda(x) \wedge \bigwedge_{j \in \delta} r_j \wedge \bigwedge_{c \in C} R_{\pi(c)}(c) \wedge \forall x \bigvee_{\lambda \in \Lambda_i} R_\lambda(x). \end{aligned}$$

In this setting the *temporal resolution rule* [DF00] gets the following form:

$$\frac{\mathcal{L}_{(i_1,\delta_1,\pi_1)} \Rightarrow \bigcirc \mathcal{R}_{(i_1,\delta_1,\pi_1)}, \dots, \mathcal{L}_{(i_n,\delta_n,\pi_n)} \Rightarrow \bigcirc \mathcal{R}_{(i_n,\delta_n,\pi_n)} \qquad p \Rightarrow \bigcirc \Diamond q}{\neg (\bigvee_{j=1}^{n} \mathcal{L}_{(i_j,\delta_j,\pi_j)}) \vee \neg p} \ (\Diamond_{res})$$

where the following *loop side conditions* have to be satisfied

$$\begin{aligned} &\mathcal{U} \cup \{\mathcal{R}_{(i_m,\delta_m,\pi_m)}\} \vdash \neg q \quad \text{ for all } 1 \leq m \leq n, \\ &\mathcal{U} \cup \{\mathcal{R}_{(i_m,\delta_m,\pi_m)}\} \vdash \bigvee_{j=1}^{n} \mathcal{L}_{(i_j,\delta_j,\pi_j)} \quad \text{ for all } 1 \leq m \leq n. \end{aligned}$$

Under the side conditions given above $\bigvee_{j=1}^{n} \mathcal{L}_{(i_j,\delta_j,\pi_j)}$ can be considered as an *invariant formula* that provides derivability of $\Box \bigcirc \neg q$ from $\mathcal{U}$. Test of whether side conditions are satisfied does not involve temporal reasoning and can be given to any first-order proof search procedure.

During carrying out the procedure the temporal and the initial parts of a given problem are left unchanged, and only the universal part is extended by derived sentences. The procedure terminates when $\mathcal{S} \cup \mathcal{U}$ becomes unsatisfiable.

References

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