Space efficient resolution decision procedures for guarded formulae

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Abstract. In previous work [8] we have devised a hyperresolution-based decision and model-building procedure for natural subfragments of the guarded fragment. We have studied the computational properties of the procedure and the range of its applicability. The aims of our current work are, first, to construct an efficient decision procedure for subfragments of the guarded fragment which requires polynomial space and effectively corresponds to tableaux-based polynomial space algorithms for testing the concept satisfiability of description logics. Secondly, we attempt to devise a minimal model generation procedure for clausal classes induced by guarded formulae.

Our work investigates the use of hyperresolution as a decision procedure and model builder for guarded formulae and explores the boundaries of a range of applications which go beyond the guarded fragments. The initial motivation of our study has been the importance of the guarded fragment of first-order logic in relation to modal and description logics. The guarded fragment was introduced in an attempt to explain the pleasant properties of modal logics, including decidability of the satisfiability problem [1]. Various decision procedures for the fragment and its extensions have been developed. They utilise different techniques such as ordered resolution, alternating automata or embedding into second-order logic [7, 4, 9, 10]. What unites them is the range of their applicability. Any decision procedure developed for the guarded fragment can be applied to a variety of extended modal and description logics. However, the devised decision procedures also have some drawbacks. In particular, they exhibit double exponential worst-case time and space complexity [4, 9] which is in contrast to the low complexity of the satisfiability problem of modal formulae. Moreover, extensions of the guarded fragment with transitivity or number restrictions lead to undecidability, even though modal logics extended with transitivity or number restrictions are decidable. This shows the guarded fragment as a whole is too general and expressive and cannot thoroughly explain the good computational properties of modal and description logics. A natural question arises whether there are restricted subfragments of the guarded fragment which provide a more suitable framework with tractable algorithms that employ prominent techniques for modal and description logics. Identifying such fragments and developing theorem proving techniques for them was started in [13]. Continuing this line of research, we have taken a new, broader approach. Our investigations are situated in the framework of resolution [2, 14]. The resolution calculus and its refinements are well-studied and well-mechanised. The practical advantage of decision procedures based on resolution are that standard theorem provers and optimisations can be used without adaptation. The framework of resolution also allows for the investigation and comparison of the computational properties of developed decision procedures. Moreover, the development and investigation of resolution-based techniques could lead to progress not only in automated theorem proving but also in other areas, i.e. deductive databases, systems based on logic programming, deduction-based program synthesis.

We focussed on the study and implementation of the hyperresolution decision procedure described in [8]. The decision procedure p-simulates the tableaux-based algorithms for testing satisfiability for modal logics and concept satisfiability for description logics [5, 11, 12]. Tableaux-based algorithms for description logics, complete for polynomial space are presented in [6, 15]. Polynomial space complexity for such algorithms is obtained by utilising the trace technique. Sources of complexity are the increasing of the number of different completions when applications of a non-deterministic rule are possible and increasing the size of the constraint system. The sources of complexity are preserved in our decision procedure. Due to eager splitting we nondeterministically select a branch of the resolution derivation tree for expansion. Even though we use subsumption for deletion of redundant clauses, the initial clausal set expands during the derivation. Motivated by the trace technique, we are working on the development of an appropriate technique which will provide an efficient hyperresolution-based algorithm that requires less space. We are aiming at preserving the essential structure of the main inference loop as implemented in SPASS [16] and thus making the decision procedure easily implementable.

Besides, we are also working on the development of a minimal model generation procedure for clausal classes induced by guarded formulae. The procedure uses complement splitting and hyperresolution, and returns only minimal models and all minimal models for guarded formulae. It exploits the approach of [3].

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